COORDINATE GEOMETRY

Concepts Covered

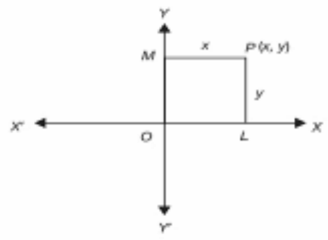
- **Introduction to 2D plane, Terminologies associated with coordinate geometry, Cartesian plane, and coordinate axes, ordered pair, Relationship between the signs of the coordinates of a given point, Quadrant in which a point lies, Plotting points in the Cartesian plane, Distance between two points, Section Formula, Area of Triangle and Quadrilateral.**

Introduction

Coordinate geometry is a branch of geometry in which the position of the points on the plane is defined with the help of an ordered pair of numbers also known as **coordinates.** It describes the links between geometry and algebra. This is done by using graphs that involve lines and curves. Thus, this gives us a geometrical understanding of algebra and enables us to solve problems related to geometry.
The use of coordinate geometry is found in engineering and physics. This, in turn, is used in aviation, aeronautical science and rocketry. The use of coordinate geometry is also seen in mobile devices, laptops and computers where it is used to find the cursor. It is also used in mapping geographical locations and distances and also used in global positioning systems or GPS.

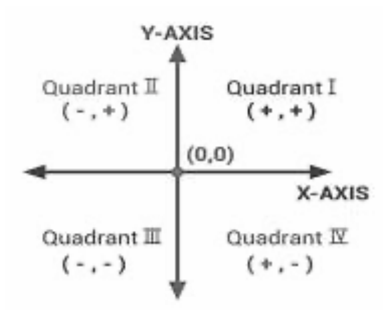
The Coordinate Plane

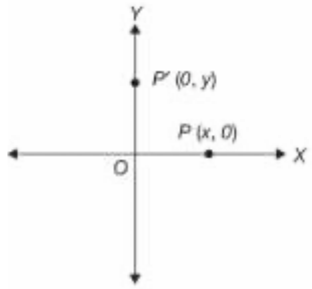
Let XOX' and YOY' be two mutually perpendicular lines intersecting at point O & dividing the plane into four quadrants. XOY, YOX', X'OY', Y'OX are respectively called the first, the second, the third and the fourth quadrants. We assume the directions of OX, OY as positive while the directions of OX', OY' as negative.
Axis of x: The line XOX' is called axis of x or x-axis.
Axis of y: The line YOY' is called axis of y or y-axis.
Co-ordinate axes: x-axis and y-axis together are called axes of co-ordinates or axes of reference.
Origin: The intersection of coordinate axes is known as the origin and the coordinates of origin is O (0,0).
Oblique axes: If both the axes are not perpendicular then they are called oblique axes.



Points to Remember

1. The system used for describing the position of a point in a plane is called the Cartesian system.
2. To locate the position of a point in a plane, we require two perpendicular lines.
3. The plane is called the Cartesian or Coordinate plane and the lines are called the coordinate axes.
4. The horizontal line is called the x-axis, and the vertical line is called the y–axis.
5. The coordinate axes divide the plane into four parts called quadrants.
6. The point of intersection of axes is called the origin.
7. The distance of a point from y–axis is called x–coordinate or abscissa, and the distance of the point from x–axis is called y–coordinate or ordinate.
8. If the abscissa of a point is x and the ordinate is y, then (x, y) are called the coordinates of the point.
9. The coordinate of a point on the x–axis is of the form (x, 0) and that of the point on the y–axis is (0, y).





Convention of Signs

Quadrant	x-coordinate	y-coordinate	Point
First quadrant	+	+	(+,+)
Second quadrant	–	+	(–,+)
Third quadrant	–	–	(–,–)
Fourth quadrant	+	–	(+,–)

Example:

In which quadrant do the following points lie?
(i) (3, 2) (ii) (–2, 1) (iii) (–1, –3) (iv) (5, –1)

Solution: (i) In the point (3, 2) abscissa and ordinate are both positive. So, it lies in the first quadrant.
(ii) In the point (–2, 1) abscissa is negative and ordinate is positive. So, it lies in the second quadrant.
(iii) In the point (–1, –3) abscissa and ordinate are both negative. So, it lies in the third quadrant.
(iv) In the point (5, –1) abscissa is positive and ordinate is negative. So, it lies in the fourth quadrant.

Example:

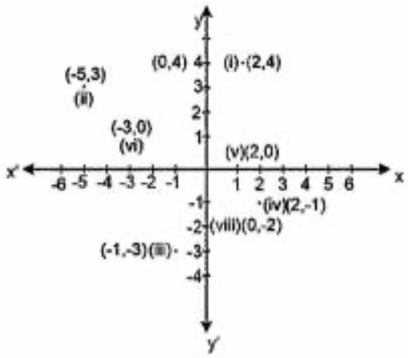
Which of the following points lie on the x–axis?
(i) (1, 1) (ii) (1, 0) (iii) (0, 1) (iv) (0, 0)
(v) (–1, 0) (vi) (0, –1) (vii) (4, 0) (viii) (0, –7).

Solution: Points of the form (a, 0), i.e., the points in which ordinate is 0, lies on the x–axis. And the points in which abscissa is 0, lies on the y–axis.
(ii) (1, 0) (iv) (0, 0), (v) (–1, 0), (vii) (4, 0). All these points have their ordinate 0, so they lie on x–axis.

Example:

Plot the following points on a graph paper.
(i) (2, 4) (ii) (–5, 3) (iii) (–1, –3) (iv) (2, –1)
(v) (2, 0) (vi) (–3, 0) (vii) (0, 4) (viii) (0, –2).

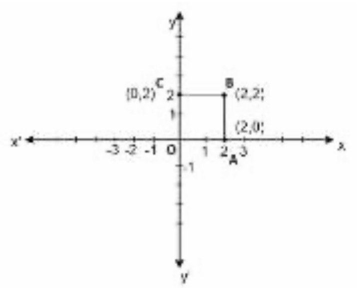
Solution:



Example:

Plot the points A (2, 0), B (2, 2), C (0, 2) and draw the line segments OA, AB, BC and CO. What figure do you obtain?

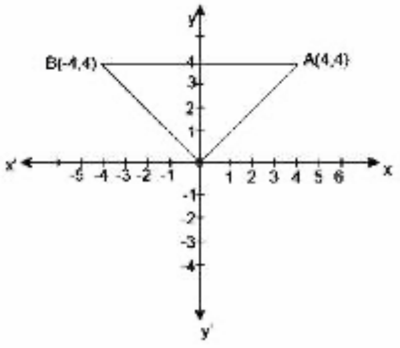
Solution:



On joining OA, AB, BC and CO, we get a square of side 2 units.

Example:

Plot the points A (4, 4) and B (–4, 4) and join the lines OA, OB and BA. Which figure do you obtain?
Solution:



Joining OA, OB and BA, we get an isosceles triangle.

Check Your Concept - 1

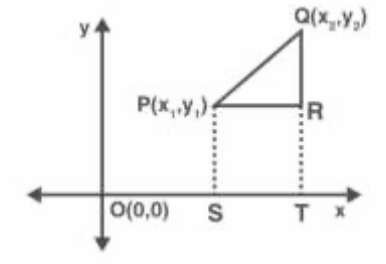
(i) Which of the following points lie on the x–axis?
(i) (1,9) (ii) (4, 0) (iii) (0, 6)
(iv) (3, 0) (v) (–7, 0) (vi) (0, –5)

(ii) Plot the points P (7, 7) and B (7, -7) and join the lines OP, OQ and PQ. Which figure do you obtain?

(iii) Plot the points P (5, 0), Q (5, 7), R (0, 7) and draw the line segments OP, PQ, QR and RO. What figure do you obtain?

Distance Formula

Let us now find the distance between any two points P (x_1, y_1) and Q (x_2, y_2). Draw PS and QT perpendicular to the x–axis. A perpendicular from the point P on QT is drawn to meet it at the point R (see Fig.).
Then OS = x_1, OT = x_2. So, ST = OT – OS = $x_2 - x_1$ = PR.
Also, QT = y_2, RT = PS = y_1. So, QR = QT – RT = $y_2 - y_1$.
Now,
applying the Pythagoras theorem in ΔPTQ, we get
$PQ^2 = PR^2 + QR^2$
$= (x_2 - x_1)^2 + (y_2 - y_1)^2$
Therefore,
$PQ = \pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Note that since distance is always non–negative, we take only the positive square root. So, the distance between the points P (x_1, y_1) and Q (x_2, y_2) is
$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
which is called the distance formula.



Example:

Find the distance between the points (8, -2) and (3, -6).
Solution: Let the points (8, -2) and (3, -6) be denoted by P and Q, respectively.
Then, by distance formula, we obtain the distance PQ as
$PQ = \sqrt{(3-8)^2 + (-6+2)^2}$
$= \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$ unit

Example:

Prove that the points $(1, -1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and $(1, 2)$ are the vertices of an isosceles triangle.

Solution: Let the point $(1, -1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and $(1, 2)$ be denoted by P, Q and R, respectively.
Now

$$PQ = \sqrt{\left(-\frac{1}{2} - 1\right)^2 + \left(\frac{1}{2} + 1\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$QR = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$

$PR = \sqrt{(1-1)^2 + (2+1)^2} = \sqrt{9} = 3$

From the above, we see that $PQ = QR$ ∴ The triangle is isosceles.

Example:

Using distance formula, show that the points $(-3,2), (1,-2)$ and $(9,-10)$ are collinear.

Solution: Let the given points $(-3,2), (1,-2)$ and $(9,-10)$ be denoted by A, B and C, respectively.

Points A, B and C will be collinear, if the sum of the lengths of two line-segments is equal to the third.

Now, $AB = \sqrt{(1+3)^2 + (-2-2)^2} = \sqrt{16+16} = 4\sqrt{2}$

$BC = \sqrt{(9-1)^2 + (-10+2)^2} = \sqrt{64+64} = 8\sqrt{2}$

$AC = \sqrt{(9+3)^2 + (-10-2)^2} = \sqrt{144+144} = 12\sqrt{2}$

Since, $AB + BC = 4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} = AC$,

the, points A, B and C are collinear.

Example:

Find a point on the X-axis which is equidistant from the points $(5,4)$ and $(-2,3)$.

Solution: Since the required point (say P) is on the X-axis, its ordinate will be zero.

Let the abscissa of the point be x.

Therefore, coordinates of the point P are $(x, 0)$.

Let A and B denote the points $(5,4)$ and $(-2,3)$, respectively.

Since we are given that $AP = BP$, we have

$AP^2 = BP^2$

i.e. $(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$

or $x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$

or $-14x = -28$

or $x = 2$

Thus, the required point is $(2,0)$.

Example:

The vertices of a triangle are $(-2,0), (2,3)$ and $(1,-3)$. Is the triangle equilateral, isosceles or scalene?

Solution: Let the points $(-2,0), (2,3)$ and $(1,-3)$ be denoted by A, B and C respectively.

Then,

$AB = \sqrt{(2+2)^2 + (3-0)^2} = 5$

$BC = \sqrt{(1-2)^2 + (-3-3)^2} = \sqrt{37}$

and $AC = \sqrt{(1+2)^2 + (-0-0)^2} = 3\sqrt{2}$

Clearly, $AB \neq BC \neq AC$

Therefore, ABC is a scalene triangle.

Example:

The distance between the points $(0,5)$ and $(-5,0)$ is

(A) 5 (B) $5\sqrt{2}$

(C) $2\sqrt{5}$ (D) 10

Solution: (B)

Example:

If the distance between the points $(4,p)$ and $(1,0)$ is 5, then the value of p is

(A) 4 only (B) ± 4

(C) -4 only (D) 0

Solution: (B)

Example:

If the point (x,y) be equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, then

(A) $ax + by = 0$ (B) $ax - by = 0$

(C) $bx + ay = 0$ (D) $bx - ay = 0$

Solution: (D) Let points $P(x,y), A(a+b, b-a), B(a-b, a+b)$

According to Question, $PA = PB$, i.e., $PA^2 = PB^2$

$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow (a+b)^2 + x^2 - 2x(a+b) + (b-a)^2 + y^2 - 2y(b-a)$

$= (a-b)^2 + x^2 - 2x(a-b) + (a+b)^2 + y^2 - 2y(a+b)$

$\Rightarrow 2x(a-b-a-b) = 2y(b-a-a-b)$

$\Rightarrow -4bx = -4ay \Rightarrow bx - ay = 0$

Check Your Concept - 2

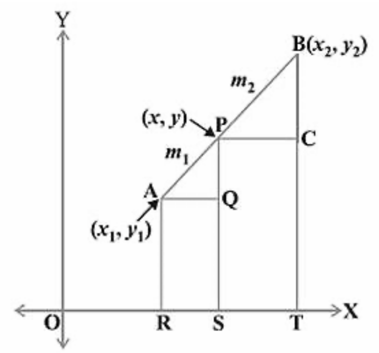
(i) Find the distance between the points (7, -3) and (5, -7).

(ii) If the distances of P (x, y) from the points A (3,6) and B (-3,4) are equal prove that 3x + y = 5.

(iii) The coordinate of the vertices of a rectangle whose length and breadth are 6 and 4 units, respectively. It's one vertex is at the origin. The longer side is on the X-axis and one of the vertices lies in second quadrant is
(A) (0,0), (6,4), (6,0), (0,4)
(B) (0,0), (0,4), (6,0), (6,4)
(C) (0,0), (6,4), (-6,0), (6,4)
(D) (0,0), (0,4), (-6,4), (-6,0)

Section Formula

Consider any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ and assume that $P(x, y)$ divides AB internally in the ratio $m_1 : m_2$, i.e., $\frac{PA}{PB} = \frac{m_1}{m_2}\cdot$ (see fig.)
Draw AR, PS and BT perpendicular to the x-axis. Draw AQ and PC parallel to the x-axis. Then, by the AA similarity criterion,
$\triangle PAQ \sim \triangle BPC$
Therefore, $\frac{PA}{BP} = \frac{AQ}{PC} = \frac{PQ}{BC}$......(i)
Now,
$AQ = RS = OS - OR = x - x_1$
$PC = ST = OT - OS = x_2 - x$
$PQ = PS - QS = PS - AR = y - y_1$
$BC = BT - CT = BT - PS = y_2 - y$
Substituting these values in (i), we get
$\frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$
Taking
$\frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x}$, we get $x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$
Similarly, taking
$\frac{m_1}{m_2} = \frac{y - y_1}{y_2 - y}$, we get $y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$
So, the coordinates of the point P (x, y) which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2), internally, in the ratio $m_1 : m_2$ are
$\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$ .... (ii)



This is known as the section formula.
This can also be derived by drawing perpendiculars from A, P and B on the y–axis and proceeding as above.

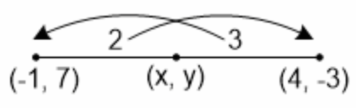
If the ratio in which P divides AB is k: 1, then the coordinates of the point p will be
$\mathbf{\frac{kx_2 + x_1}{k + 1}, \frac{kx_2 + y_1}{k + 1}}$

Co–ordinates of the Mid-point
If P be the mid–point of AB, then it will divide AB in the ratio of 1: 1, then the co–ordinates of P are
$\frac{1 \times x_1 + 1 \times x_2}{1 + 1}, \frac{1 \times y_1 + 1 \times y_2}{1 + 1} = \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$

Example:

Find the coordinates of the point which divides the line joining points (–1, 7) and (4, –3) in the ratio 2: 3.
Solution: The co–ordinates of the point which divides the line joining points (-1, 7) and (4, -3) internally in the ratio 2: 3 are (x, y) such that

$$x = \frac{2 \cdot 4 + 3(-1)}{2+3}$$
$$= \frac{8-3}{5} = \frac{5}{5} = 1$$



And $y = \frac{2(-3)+3(7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$

Hence, the coordinates of the point of division are $(1, 3)$

Example:

Find the middle point of the line joining $(-3, -6)$ and $(1, -2)$.
Solution: The co-ordinates of the mid-point of the join of $(-3, -6)$ and $(1, -2)$ are

$$x = \frac{x_1 + x_2}{2} = \frac{(-3)+1}{2} = \frac{-2}{2} = -1$$
$$y = \frac{y_1 + y_2}{2} = \frac{(-6)+(-2)}{2} = \frac{-8}{2} = -4$$

Hence, the mid-point is $(-1, -4)$.

Example:

Find the ratio in which the y–axis divides the line segment joining the points (5, –6) and (–1, –4). Also find the point of intersection.
Solution: Let the ratio be k: 1. Then by the section formula, the coordinates of the point which divides AB in the ratio k: 1 are $\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}$

This point lies on the y–axis, and we know that on the y–axis the abscissa is 0.
Therefore,

$$\frac{-k+5}{k+1} = 0$$

So, k = 5That is, the ratio 5: 1. Putting the value of k = 5, we get the point of intersection as $(0, \frac{-13}{3})$.

Check Your Concept - 3

(i) Find the coordinates of the point which divides the line joining points (7, –1) and (3, 4) in the ratio 2: 3.

(ii) The coordinates of the point which divides the line segment joining the points (4, -3) and (9, 7) internally in the ratio 3: 2 is

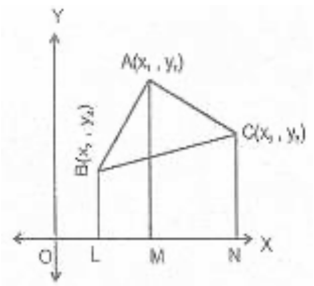
(A) (7, 3) (B) (3, 7)
(B) (35, 15) (D) (27, 21)

(iii) The point which divides the line segment joining the points (7, -6) and (3, 4) in ratio 1: 2 internally lies in the

(A) I quadrant (B) II quadrant
(C) III quadrant (D) IV quadrant

Area of a Triangle

Let ABC be any triangle whose vertices are A (x_1, y_1) B (x_2, y_3) and C (x_3, y_3).
Draw BL, AM and CN perpendicular from B, A and C respectively, to the X-axis. ABLM, AMNC and BLNC are all trapeziums.
Area of $\triangle$ ABC = Area of trapezium ABLM + Area of trapezium AMNC - Area of trapezium BLNC We know that, Area of trapezium $= \frac{1}{2}$ (Sum of parallel sides) (distance b/w them)
Therefore



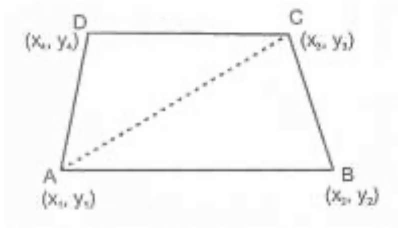
Area of $\Delta ABC = \frac{1}{2}(BL + AM)(LM) + \frac{1}{2}(AM + CN)MN - \frac{1}{2}(BL + CN)(LN)$

Area of $\triangle ABC = \frac{1}{2}(y_2 + y_1)x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$

Area of $\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y) + x_3(y_1 - y_2)]$

Condition for collinearity
Three points $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if Area of $\triangle ABC = 0$.

Area of Quadrilateral



Let the vertices of Quadrilateral $ABCD$ are $A(x_1, y_1), B(x_2, y_2, C(x_3, y_3)$ and $D(x_4, y_4)$
So, Area of quadrilateral $ABCD$ = Area of $\triangle ABC$ + Area of $\triangle ACD$

Example:

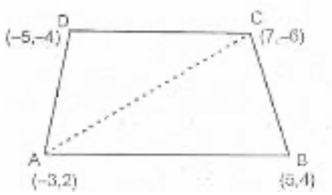
The vertices of $\triangle ABC$ are $(-2, 1), (5, 4)$ and $(2, -3)$ respectively. Find the area of triangle.
Solution: $A(-2, 1), B(-2, 1)$ and $C(2, -3)$ be the vertices of triangle.
So, $x_1 = -2, y_1 = 1; x_2 = 5, y_2 = 4; x_3 = 2y_3 = -3$
Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
$= \frac{1}{2}|[(-2)(4 + 3) + (5)(-3 - 1) + 2(1 - 4)]|$
$= \frac{1}{2}|[-14 + (-20) + (-6)]|$
$= \frac{1}{2}| - 40|$
$= 20$ Sq. unit.

Example:

The area of a triangle is 5. Two of its vertex's areas $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. Find the third vertex.
Solution: Let the third vertex be (x_3, y_3) area of triangle
$= \frac{1}{2}[[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
As $x_1 = 2y_1 = 1; x_2 = 3, y_2 = -2;$ Area of $\Delta = 5$ sq. unit
$\Rightarrow 5 = \frac{1}{2}|2(-2 - y_3) + 3(y_3 - 1) + x_3(1 + 2)|$
$\Rightarrow 10 = |3x_3 + y_3 - 7|$
$\Rightarrow 3x_3 + y_3 - 7 = \pm 10$
Taking positive sign
$3x_3 + y_3 - 7 = 10 \Rightarrow 3x \therefore + y_3 = 17$ ...(i)
Taking negative sing
$\Rightarrow 3x_3 + y_3 - 7 = -10$
$\Rightarrow 3x_{\therefore} + y_3 = -3$ ...(ii)
Given that (x_3, y_3) lies on $y = x + 3$
So, $-x \therefore + y_3 = 3$ ...(iii)
Solving eq. (i) & (iii)
$x_3 = \frac{7}{2}, y_3 = \frac{13}{2}$
Solving eq. (ii) & (iii)
$x_3 = \frac{-3}{2}, y_3 = \frac{3}{2}$
So, the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

Example:

Find the area of quadrilateral whose vertices, taken in order, are A (−3, 2), B (5, 4), C (7, −6) and D (−5, −4).
Solution: Area of quadrilateral = Area of $\triangle ABC$ + Area of $\triangle ACD$



So, Area of $\triangle ABC = \frac{1}{2}|(-3)(4 + 6) + 5(-6 - 2) + 7(2 - 4)|$
$= \frac{1}{2}| - 30 - 40 - 14|$
$= \frac{1}{2}| - 84| = 42$ Sq. units
Area of $\Delta ACD = \frac{1}{2}| - 3(-6 + 4) + 7(-4 - 2) + (-5)(2 + 6)|$
$= \frac{1}{2}| + 6 - 42 - 40| = \frac{1}{2}| - 76| = 385$ q. units
So, Area of quadrilateral $ABCD = 42 + 38 = 80$ Sq. units.

Check Your Concept - 4

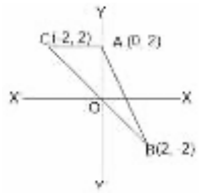
(i) Find the area of the triangle whose vertices are A (1, 2), B (–2, 3) and C (–3, –4)
(A) 11sq. units (B) 22 sq. units
(B) 7 sq. units (D) 6.5 sq. units

(ii) The area of triangle formed by the points (p, 2 - 2p), (l - p, 2p) and (-4 - p, 6 - 2p) is 70 sq. units. How many integral values of p are possible?
(A) 2 (B) 3
(C) 4 (D) none of these

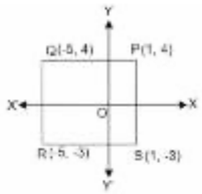
(iii) Find the area of quadrilateral, the coordinates of whose angular points are (1, 1), (3, 4), (5, −2) and (4, −7).
(A) 20.5 sq.units (B) 41 sq.units
(C) 82 sq.units (D) 61.5 sq.units

Solved Example

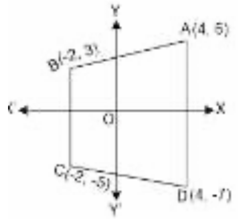
(1) Draw a triangle ABC where vertices A, B and C are (0, 2), (2, –2) and (–2, 2) respectively.
Solution: Plot the point A by taking its abscissa = 0 and ordinate = 2.
Similarly, plot points B and C taking abscissa 2 and – 2 and ordinates – 2 and 2 respectively. Join A, B and C. This is the required triangle.



(2) Draw a rectangle PQRS in which vertices P, Q, R and S are (1, 4), (–5, 4), (–5, –3) and (1, –3) respectively.
Solution: Plot the point P by taking its abscissa 1 and ordinate 4.
Similarly, plot the points Q, R and S taking abscissa as –5, –5 and 1 and ordinates as 4, – 3 and –3 respectively.
Join the points P, Q, R and S. PQRS is the required rectangle.

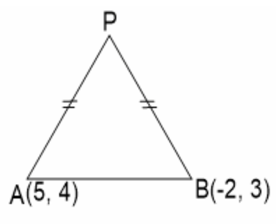


(3) Draw a trapezium ABCD in which vertices A, B, C and D are (4, 6), (–2, 3), (–2, –5) and (4, –7) respectively.
Solution: Plot the point A taking its abscissa as 4 and ordinate as 6.
Similarly, plot the points B, C and D taking abscissa as –2, –2 and 4 and ordinates as 3, –5, and –7 respectively. Join A, B, C and D. ABCD is the required trapezium.

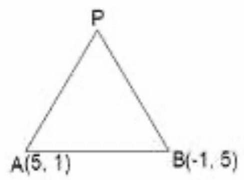


(4) Find the length of the line AB formed by two points $A(4,10)$ and $B(7,-6)$.
Solution: We know that distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
∴ Length of line AB = Distance between the points A and B
$= \sqrt{(7-4)^2 + (-6-10)^2}$
$= \sqrt{(3)^2 + (-16)^2} = \sqrt{9 + 256}$
$= \sqrt{265}$

(5) Find a point on the x-axis which is equidistant from the points $(5, 4)$ and $(-2, 3)$.
Solution: Since, the required point (say P) is on the x-axis, its ordinate will be zero. Let the abscissa of the point be x.
Therefore, the coordinates of the point P are $(x, 0)$.
Let A and B denote the points $(5, 4)$ and $(-2, 3)$, respectively
Since, given that $AP = BP$, we have
$AP^2 = BP^2$
i.e., $(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$
$\Rightarrow x^2 + 25 - 10x + 16 = x^2 + 4x + 4 + 9$
$\Rightarrow -14x = -28$
$\Rightarrow x = 2$
Thus, the required point is $(2, 0)$.



(6) If the distance of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, prove that $3x = 2y$.
Solution: $P(x, y), A(5, 1)$ and $B(-1, 5)$ are the given points.
$\therefore AP^2 = BP^2$
$\Rightarrow AP^2 - BP^2 = 0$
$\Rightarrow \{(x-5)^2 + (y-1)\}^2 - \{(x+1)^2 + (y-5)^2\} = 0$
$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y - x^2 - 1 - 2x - y^2 - 25 + 10y = 0$
$\Rightarrow -12x + 8y = 0$
$\Rightarrow 3x = 2y$



(7) Check whether $(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.
Solution: Let the points $(5, -2), (6, 4)$ and $(7, -2)$ be denoted by A, Q and B respectively.

Now,

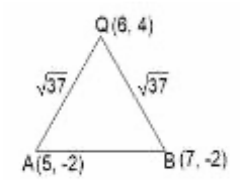
$AQ = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$

$QB = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$

$AB = \sqrt{(5-7)^2 + (-2+2)^2} = \sqrt{4} = 2$

From the above, we see that $AQ = QB$

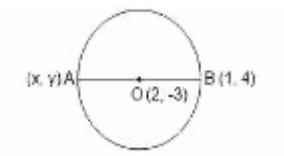
Hence, the triangle is isosceles.



(8) Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Solution: Let the coordinates of point A be (x, y). As AB is diameter and O is centre of circle. Then O will be mid-point of AB.

∴ Coordinates of O = Coordinates of mid-point of AB



$\Rightarrow (2,-3) = \frac{x+1}{2}, \frac{y+4}{2}$

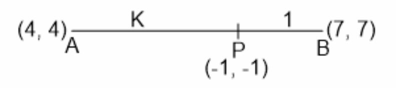
$\Rightarrow 2 = \frac{x+1}{2}$

$\Rightarrow x + 1 = 4 \Rightarrow x = 3$

and $-3 = \frac{y+4}{2} \Rightarrow y + 4 = -6 \Rightarrow y = -10$

Hence, coordinates of point A are (3, – 10).

(9) In what ratio is the line joining the points A (4, 4) and B (7, 7) is divided by P (–1, –1)?



Solution: Let P divides AB in the ratio of k: 1.

Then, co–ordinates of P are

$\frac{7k+4}{k+1}, \frac{7k+4}{k+1}$

But the coordinate of P is given as (–1, –1),

$\therefore \frac{7k+4}{k+1} = -1 \Rightarrow 7k + 4 = -k - 1 \Rightarrow 8k = -5 \Rightarrow k = \frac{-5}{8}$

Verification of y coordinate = $\frac{7\times\frac{-5}{8}+4\times 1}{\frac{-5}{8}+1} = \frac{\frac{-3}{8}}{\frac{3}{8}}$ = -1

Negative sign shows that AB is divided externally.

Hence, P divides AB externally in the ratio 5: 8

(10) The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a+1)$. Find the values of a and b.

Solution: As $(1, 2a+1)$ is the mid point of AB.

$\therefore 1 = \frac{2a-2}{2}$ and $2a + 1 = \frac{4+3b}{2}$

Now, $1 = \frac{2a-2}{2} \Rightarrow 2 = 2a - 2 \Rightarrow 2a = 4 \Rightarrow a = 2$

and $2a + 1 = \frac{4+3b}{2} \Rightarrow 2 \times 2 + 1 = \frac{4+3b}{2}$

$\Rightarrow 10 = 4 + 3b \Rightarrow 3b = 10 - 4 = 6 \Rightarrow b = 2$

$\therefore a = 2, b = 2$

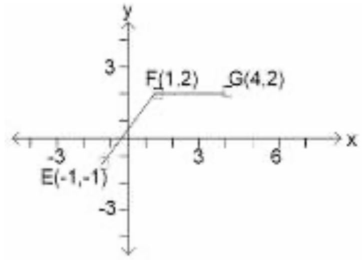
Exercise

OBJECTIVE TYPE QUESTIONS

(1) Quadrilateral formed by $A(-2,2), B(8,2), C(4,-4)$ and $D(-6,-4)$ is
(A) Rectangle
(B) Square
(C) Rhombus
(D) Parallelogram

(2) The axis on which the point $(0,-4)$ lie, is
(A) Positive x-axis
(B) Negative x-axis
(C) Positive y-axis
(D) Negative y-axis

(3) What would be the coordinates of point H for points E, F, G and H to form a parallelogram?



(A) $(2,-1)$
(B) $(3,-1)$
(C) $(6,-1)$
(D) $(-1,2)$

(4) The distance between $(5,4)$ and $(5,1)$ is
(A) 2
(B) 3
(C) 4
(D) 5

(5) If the distance between the points $(2,-2)$ and $(-1,x)$ is 5, one of the values of x is
(A) -2
(B) 2
(C) -1
(D) 1

(6) The distance of the point $P(2,3)$ from the x-axis is
(A) 2
(B) 3
(C) 1
(D) 5

(7) The distance between the points $A(0,6)$ and $B(0,-2)$ is
(A) 6
(B) 8
(C) 4
(D) 2

(8) The distance of the point $P(-6,8)$ from the origin is
(A) 8
(B) $2\sqrt{7}$
(C) 10
(D) 6

(9) The perimeter of a triangle with vertices $(0,4), (0,0)$ and $(3,0)$ is
(A) 5
(B) 12
(C) 11
(D) $7+\sqrt{5}$

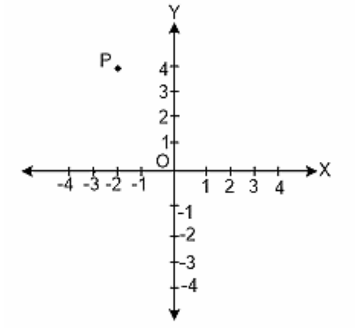
(10) On plotting the points $O(0,0), A(3,0), B(3,4), C(0,4)$ and joining OA, AB, BC and CO which of the following figure is obtained?
(A) Square
(B) Rectangle
(C) Trapezium
(D) Rhombus

(11) If $P(-1,1), Q(3,-4), R(1,-1), S(-2,-3)$ and $T(-4,4)$ are plotted on the graph paper, then the point(s) in the fourth quadrant are
(A) P and T
(B) Q and R
(C) Only S
(D) P and R

(12) If the coordinates of the two points are $P(-2,3)$ and $Q(-3,5)$, then (abscissa of P) $-$ (abscissa of Q) is
(A) -5
(B) 1
(C) -1
(D) -2

(13) If $P(5,1), Q(8,0), R(0,4), S(0,5)$ and $O(0,0)$ are plotted on the graph paper, then the point(s) on the x-axis are
(A) P and R
(B) R and S
(C) Only Q
(D) Q and O

(14) In Fig. coordinates of P are

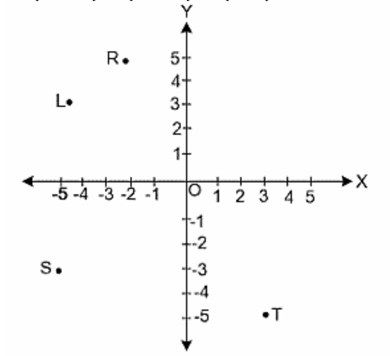


(A) (– 4, 2)
(B) (–2, 4)
(C) (4, – 2)
(D) (2, – 4)

(15) In Fig. the point identified by the coordinates (–5, 3) is
(A) T
(B) R
(C) L
(D) S

(16) The point whose ordinate is 4 and which lies on y-axis is
(A) $(4, 0)$
(B)$(0,4)$
(C) $(1,4)$
(D) $(4,2)$

(17) Which of the points $P(0, 3), Q(1, 0), R(0, -1), S(-5, 0), T(1, 2)$ do not lie on the x axis?



(A) P and R only
(B) Q and S
(C) P, R and T
(D) Q, S and T

(18) Points $(1, -1), (2, -2), (4, -5), (-3, -4)$
(A) Lie in II quadrant
(B) Lie in III quadrant
(C) Lie in IV quadrant
(D) Do not lie in the same quadrant

(19) The missing member 'x' in the ordered pair $(x, -8)$ if the ordinate is 4 more than the abscissa
(A) -4
(B) -8
(C) -12
(D) 4

(20) The mid-point of the line segment joining the points $A(-2, 8)$ and $B(-6, -4)$ is
(A) $(-4, -6)$
(B) $(2, 6)$
(C) $(-4, 2)$
(D) $(4, 2)$

(21) The points which trisect the line segment joining the points $(0,0)$ and $(9,12)$ are
(A) $(3, 4), (6, 8)$
(B) $(4, 3), (6, 8)$
(C) $(4, 3), (8, 6)$
(D) $(3, 4), (8, 6)$

(22) The points $(-5, 2)$ and $(2, -5)$ lie in the
(A) Same quadrant.
(B) II and III quadrants, respectively.
(C) II and IV quadrants, respectively.
(D) IV and II quadrants, respectively.

(23) If (3, -4) and (-6, 5) are the extremities of the diagonal of a parallelogram and (-2, 1) is third vertex, then its fourth vertex is -
(A) (-1, 0)
(B) (0, -1)
(C) (-1, 1)
(D) None of these

(24) The area of a triangle whose vertices are (a, c + a), (a, c) and (-a, c - a) are

(A) a^2 (B) b^2

(C) c^2 (D) $a^2 + c^2$

(25) The area of the quadrilateral's the coordinates of whose verticals are (1, -2,) (6, 2), (5, 3) and (3, 4) are

(A) $\frac{9}{2}$ (B) 5

(C) $\frac{11}{2}$ (D) 11

Answer Key

CHECK YOUR CONCEPT

(1)	**(i)** (4, 0), (3, 0), (-7, 0)		
(2)	**(iii)** (D)		
(3)	**(i)** (-3, 1)	**(ii)** (A)	**(iii)** (D)
(4)	**(i)** (A)	**(ii)** (D)	**(iii)** (A)

OBJECTIVE TYPE QUESTIONS

(1)	(D)	**(6)**	(B)	**(11)**	(B)	**(16)**	(B)	**(21)**	(A)
(2)	(D)	**(7)**	(B)	**(12)**	(B)	**(17)**	(C)	**(22)**	(C)
(3)	(A)	**(8)**	(C)	**(13)**	(D)	**(18)**	(D)	**(23)**	(A)
(4)	(B)	**(9)**	(B)	**(14)**	(B)	**(19)**	(C)	**(24)**	(A)
(5)	(B)	**(10)**	(B)	**(15)**	(C)	**(20)**	(C)	**(25)**	(C)