PROBABILITY

Concepts Covered

- Introduction to Probability
- Concepts and Terms related to Probability
- Approaches of Probability
- Types of Events
- Remarks about Coins, Dice, Playing cards and Envelopes

Introduction

In our daily life we frequently use some statements like the following:

(1) I will probably get a scholarship for my performance.
(2) I doubt that I will catch the train.
(3) The chances of the prices of all commodities going up are very high.
(4) There is a fifty-fifty chance of Sania winning the match.
(5) I will most probably return from my official tour in three days.
(6) The chances of Sachin making a century in his next match are very few.

In the above statements the words probably, chances, doubt and most probably involve uncertainty. The uncertainty of 'probably' etc. can be measured numerically using **'probability'**.

Definition: Probability is a concept that numerically measures the degree of certainty of the occurrence of events.

Some important Concepts/Terms

(I) Experiment: An action or operation which can produce some well-defined result is known as an experiment.

(II) Deterministic Experiment: The experiments that when repeated under identical conditions produce the same result or outcome.
For Example: If we mark head (H) on both sides of a coin and it is tossed, then we always get the same outcome assuming that it does not stand vertically.

(III) Random or Probabilistic Experiment: If an experiment, when repeated under identical conditions, does not produce the same outcome every time. But if the outcome in a trial is one of the several possible outcomes, then it is known as a random or probabilistic experiment.
For Example: When tossing a coin one is not sure if a head (H) or tail (T) will be obtained, so it is a random experiment. Similarly, rolling an unbiased die is an example of a random experiment.

(IV) Outcomes: The possible results of a random experiment are called **outcomes.**

(V) Trial and Event

Trial: When an experiment is repeated under similar conditions and it does not give the same result each time but may result in any one of the several possible outcomes, the result is called a **trial.**

Event: The collection of all or some outcomes of a random experiment is called an **event.**

For Example:
(i) Participation of player in the game to win a game, is a trial but winning or losing is an event.
(ii) Tossing of a fair coin is a trial and turning up head or tail are events.
(iii) Throwing of a dice is a trial and the occurrence of number 1 or 2 or 3 or 4 or 5 or 6 are events.
(iv) Drawing a card from a pack of playing cards is a trial and getting an ace or a queen is an event.

(VI) Sample Space
The set of all possible outcomes of a trial is called its sample **space**. It is generally denoted by S and each outcome of the trial is said to be a point of a sample of S.
For Example:
(i) If a die is thrown once, then its sample space: S = {1, 2, 3, 4, 5, 6}
(ii) If two coins are tossed together then its sample space: S = {HT, TH, HH, TT}.

(VII) Compound event or composite event or mixed event:
An event associated with a random experiment and obtained by combining two or more simple events associated with the same random experiment is called a **compound event**.

For Example: If we throw a die, then the event E of getting an odd number is a compound event because the event E contains three elements 1, 3 and 5, which is a compound of three simple events E_1, E_2, and E_3 containing 1, 3 and 5 respectively.

(VIII) Favourable Events
Those outcomes of a trial in which a given event may happen, are called **favourable cases** for that event.

For Example:
(i) If a coin is tossed then favourable cases of getting H is 1.
(ii) If a dice is thrown then the favourable case for getting 1 or 2 or 3 or 4 or 5 or 6, is 1.
(iii) If two dice are thrown, then favourable cases of getting a sum of numbers as 9 are four i.e. (4, 5), (5, 4), (3, 6), (6, 3).

(IX) Equally likely events
The outcomes of an experiment are said to be equally likely events if the chances of their happenings are neither less nor greater than the other.

For Example: When a coin is tossed, both outcomes H and T are equally likely to appear. Thus, H and T are equally likely outcomes. Similarly, when we throw a die then the outcomes, 1, 2, 3, 4, 5 and 6 are equally likely.

Approaches of Probability

(I) Experimental or Empirical or observed frequency approach.
(II) Classical approach
(III) Axiomatic approach

(I) Experimental Probability: The experiment or empirical probability $P(E)$ of an event is defined as

$$P(E) = \frac{\text{Number of trials in which the event happened } (m)}{\text{Total number of trials } (n)}$$

i.e., $P(E) = \frac{m}{n}$

Note:
(i) These probabilities are based on the results of an actual experiment.
(ii) These probabilities are only 'estimates', i.e., we may get different probabilities for the same event in various experiments.

(II) Theoretical (Classical) Probability: The theoretical or classical probability of an event E, written as $P(E)$, is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes of the experiment}}$$

Where the outcomes of the experiment are equally likely.

An important result: The probability of an event always lies between 0 and 1. i.e., $0 \le P(E) \le 1$

Proof: Let m be the number of favourable outcomes of an event E and n be the total number of outcomes. Then, $0 \le m \le n$ [m cannot be a negative integer and m cannot be greater than n]

$$\Rightarrow 0 \le \frac{m}{n} \le \frac{n}{n} \Rightarrow 0 \le \frac{m}{n} \le 1 \Rightarrow 0 \le P(E) \le 1$$

Thus, the probability of happening an event always lies between 0 and 1. If P(A) = 1, then A is called a certain event and A is known as an impossible event, If P(A) = 0.
Further, if $\bar{A}$ denotes negative of A i.e., event that A doesn't happen, then for above cases m, n; we shall have

$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$

$\therefore P(A) + P(\bar{A}) = 1$

Notes:
(i) $0 \le P(E) \le 1$
(ii) Let $E_1, E_2, E_3, \dots, E_n$ be the n elementary events associated with a random experiment having exactly n outcomes.
Then, $P(E_1) + P(E_2) + P(E_3) + \cdots + P(E_n) = 1$

Example:

A bag contains 3 red balls, 4 white balls and 5 green balls. A ball is drawn at random. Find the probability of the ball being red, white or green.

Solution: Let R = the event of getting a red ball,
W = the event of getting a withe ball,
and G = the event of getting a green ball,
Here, the total number of balls (outcomes) $= 3 + 4 + 5 = 12$

Then, $P(R) = \frac{3}{12}$ [Number of favourable outcomes $= 3$]

$P(W) = \frac{4}{12}$ [Number of favourable outcomes $= 4$]

And, $P(G) = \frac{5}{12}$ [Number of favourable outcomes = 5]

Clearly, $\frac{3}{12}+\frac{4}{12}+\frac{5}{12}=\frac{12}{12}=1$
i.e., $P(R)+P(W)+P(G)=1$

Some Special Events

(1) Impossible event (or null event): An event is said to be an impossible event when none of the outcomes is favourable to the event. The probability of an impossible event $=0$.

Example:

What is the probability of getting a number 8 in a single throw of a die?
Solution: The possible outcomes are $1, 2, 3, 4, 5,$ and 6.
Let $E=$ the event of getting a number 8 in a single throw of a die.
Clearly, the number of outcomes favourable to E is 0 and the total number of possible outcomes is 6.
Therefore, $P(E)=\frac{0}{6}=0$
Here, E is an impossible event.

(2) Sure (or certain) event: An event is said to be a sure (or certain) event when all possible outcomes are favourable to the event. The probability of a sure event is 1.

Example:

What is the probability of getting a number less than 7 in a single throw of a die?
Solution: The possible outcomes are: $1, 2, 3, 4, 5, 6$.
Let $E=$ the event of getting a number less than 7 in a single throw of a die. Clearly, the number of outcomes favourable to E are $1, 2, 3, 4, 5, 6$. i.e., the number of outcomes favourable to E is 6.
Therefore, $P(E)=\frac{6}{6}=1$
Here, E is an impossible event.

(3) Complement of an event: Corresponding to every event E associated with a random experiment, there is an event 'not E', which occurs only when E does not occur.
The event $\bar{E}$, representing 'not E ', is called the complement of the event E.
E and $\bar{E}$, are also called complementary events.
In general, $P(E)+P(\bar{E})=1$ i.e., $P(\bar{E})=1-P(E)$ or $P(\text{ not } E)=1-P(E)$

Some important remarks about Coins, Dice, Playing cards and Envelopes

Coins: A coin has a head side and a tail side. If an experiment consists of more than a coin, then coins are considered to be distinct if not otherwise stated.
Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = 2^n.

Dice: A die (cubical) has six faces marked 1, 2, 3, 4, 5, 6. We may have tetrahedral (having four faces 1, 2, 3, 4) or pentagonal (having five faces 1, 2, 3, 4, 5) die. As in the case of coins, if we have more than one die, then all dice are considered to be distinct if not otherwise stated.
Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n times) = 6^n.

Playing cards: A pack of playing cards usually has 52 cards. There are 4 suits (Spade, Heart, Diamond and Club) each having 13 cards. There are two colours red (Heart and Diamond) and black (Spade and Club) each having 26 cards.
In the thirteen cards of each suit, there are 3 face cards namely king, queen and jack. So, there are in all 12 face cards (4 kings, 4 queens and 4 jacks). Also, there are 16 honour cards, 4 of each suit namely ace, king, queen and jack.

Example:

Two fair coins are tossed. What is the probability that at least one head occurs?
Solution: The sample space 'S' for this experiment is, S = {HH, HT, TH, TT}
As the coin is fair all these outcomes are equally likely. Let 'w' be the weight assigned to any sample point then w + w + w + w = 1 ⇒ w = $\frac{1}{4}$
If 'A' is the event representing the event of at least one head occurring,
Then A = {HT, TH, HH} ⇒ P(A) = $\frac{3}{4}$

Example:

From a set of 17 cards 1, 2, 3, ……16, 17, one is drawn at random. Find the probability that the number on the drawn card would be divisible by 3 or 7.
Solution: Numbers which are divisible by three are 3, 6, 9, 12 and 15. Similarly, numbers which are divisible by 7 are 7, 14.
⇒ Probability of the number written on the card being divisible by 3 or 7 is
= $\frac{7}{17}$.

Example:

A coin is tossed 500 times with the following frequencies of two outcomes:
Head: 240 times, tail: 260 times. Find the probability of occurrence of each of these events.

Solution: It is given that the coin is tossed 500 times.

∴ Total number of trials = 500

Let us denote the event of getting a head and of getting a tail by A and B respectively. Then,
Number of trials in which the event A happens = 240.
and Number of trials in which the event B happens = 260.

∴ $P(A) = \frac{\text{Number of trials in which the event A happens}}{\text{Total number of trials}}$

$= \frac{240}{500} = 0.48$

∴ $P(B) = \frac{\text{Number of trials in which the event B happens}}{\text{Total number of trials}}$

$= \frac{260}{500} = 0.52$

Note: We note that P(A) + P(B) = 0.48 + 0.52. Therefore, A and B are the only two possible outcomes of trials.

Example:

Three unbiased coins are tossed together. Find the probability of getting
(i) One head (ii) Two heads (iii) All heads (iv) At least two heads

Solution: If three unbiased coins are tossed together, then all possible outcomes are:

HHH, HHT, THH, HTT, THT, TTH, TTT

Total number of possible outcomes = 8

(i) Let A_1 = the event of getting one head.
Then, favourable outcomes are HTT, THT, TTH.
Number of favourable outcomes = 3.
Hence, required probability = P (getting one head) $= P(A_1) = \frac{3}{8}$

(ii) Let A_2 = the event of getting two heads.
Then, favourable outcomes are HHT, HTH, THH.
Number of favourable outcomes = 3.
Hence, required probability = P (getting two heads) $= P(A_2) = \frac{3}{8}$

(iii) Let A_3 = event of getting all heads.
Then, favourable outcomes are HHH
Number of favourable outcomes = 1
Hence, required probability = P (getting all heads) $= P(A_3) = \frac{1}{8}$

(iv) Let A_4 = event of getting at least two heads.
Then, favourable outcomes are HHT, HTH, THH, HHH
Number of favourable outcomes = 4
Hence, required probability = P(getting at least two heads) $= P(A_4) = \frac{4}{8} = \frac{1}{2}$

Example:

A die is thrown once. Find the probability of getting
(i) A prime number (ii) A number lying between 2 and 6 (iii) An odd number.

Solution: If a die is thrown, then all possible outcomes are 1, 2, 3, 4, 5 and 6.

Total number of possible outcomes = 6

(i) Let A_1 = event of getting a prime number.
Then, the favourable outcomes are 2, 3, 5.
Number of favourable outcomes = 3
Hence, required probability = P (getting a prime number) $= P(A_1) = \frac{3}{6} = \frac{1}{2}$

(ii) Let A_2 = event of getting a number lying between 2 and 6.
Then, the favourable outcomes are 3, 4, 5.
Number of favourable outcomes = 3
Hence, required probability = P (getting a number lying between 2 and 6) $= P(A_2) = \frac{3}{6} = \frac{1}{2}$

(iii) Let A_3 = event of getting an odd number.
Then, the favourable outcomes are 1, 3, 5.
Number of favourable outcomes = 3
Hence, required probability = P (getting an odd number) $= P(A_3) = \frac{3}{6} = \frac{1}{2}$

Example:

A die is thrown twice. What is the probability that
(i) 5 will not come up either time? (ii) 5 will come up at least once?

Solution: If a die is thrown twice, then all the possible outcomes are:
$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6).$

(i) Let A_1 = event of getting 5 neither time.
Then, the favourable outcomes are:
$(1,1),(1,2),(1,3),(1,4),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,6),$
$(6,1),(6,2),(6,3),(6,4),(6,6).$
Number of favourable outcomes $= 25$

Hence, required probability $=$ P(5 will not come up either time) $= P(A_1) = \frac{25}{36}$

(ii) Let A_2 = event of getting 5 at least once
Then, the favourable outcomes are:
$(1,5),(2,5),(3,5),(4,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,5).$
Number of favourable outcomes $= 11$

Hence, required probability $=$ P(5 will not come up at least once) $= P(A_2) = \frac{11}{36}$

Example:

Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (NCERT)
(i) 8 (ii) 13 (iii) Less than or equal to 12?

Solution: If two dice, one blue and one grey, are thrown at the same time, then all possible outcomes are:
$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6).$
Total number of possible outcomes $= 36$
(i) Let A_1 = event of getting a sum of two numbers appearing on the top of the dice is 8 .
Then, the favourable outcomes are $(2,6),(3,5),(4,4),(5,3),(6,2).$
Number of favourable outcomes $= 5.$

Hence, required probability $= P(A_1) = \frac{5}{36}$

(ii) Let A_2 = event of getting a sum of two numbers appearing on the top of dice is 13.
Then, the favourable outcomes $= 0.$

Hence, required probability $= P(A_2) = \frac{0}{36} = 0$

(iii) Let A_3 = event of getting a sum of two numbers appearing on the top of the dice is less than or equal to 12.
Then, the favourable outcomes = all the possible outcomes $= 36.$

Hence, required probability $= P(A_3) = \frac{36}{36} = 1.$

Example:

A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card drawn is
(i) A red card (ii) A non-ace (iii) A king or a jack (iv) Neither a king nor a queen.

Solution: If a card is drawn at random from a well-shuffled deck of 52 cards, then the total number of possible outcomes
$= 52$
(i) Let A_1 = event of getting a red card.
Then, the favourable outcomes $= 26.$

Hence, required probability $=$ P (getting a red card) $= P(A_1) = \frac{26}{52} = \frac{1}{2}$

(ii) Let A_2 = event of getting a non-ace
Then, the favourable outcomes $= 48.$ [there are 4 aces in a pack of playing cards]

Hence, required probability $=$ P (getting a non-ace) $= P(A_2) = \frac{48}{52} = \frac{12}{13}$

(iii) Let A_3 = event of getting a king or a jack.
There are 4 king cards and 4 jack cards.
Hence, required probability $= P(A_3) =$ P (getting a king or a jack) $=$ P (getting a king) $+$P (getting a jack)

$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

(iv) Let A_4 = event of getting neither a king nor a queen.
There are 4 king cards and 4 queen cards.
Hence, required probability $= P(A_4) = P$ (getting neither a king nor a queen)
$= 1 - P$ (getting a king or a queen)
$= 1 - [P$ (getting a king) $+P$ (getting a queen)]

$= 1 - \left(\frac{4}{52} + \frac{4}{52}\right) = 1 - \frac{8}{52} = \frac{44}{52} = \frac{11}{13}$

Alternative: Let A_4 = event of getting neither king nor queen.
∴ no. of favourable outcomes
i.e., neither king nor queen cards $= 52 - 8 = 44$

$\frac{44}{52} = \frac{11}{13}$

Example:

All three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting
(i) A black face card
(ii) A queen
(iii) A black card

Solution: If all three face cards of spades are removed from a well-shuffled pack of 52 cards, then there are 49 cards left in the pack.

(i) Let A_1 = event of getting a black face card.
There are 3 black face cards left (face cards of club).
Hence, required probability $= P(A_1) = P$ (getting a black face card) $= \frac{3}{49}$

(ii) Let A_2 = event of getting a queen.
There are three queens left.
Hence, required probability $= P(A_2) = P$ (getting a queen) $= \frac{3}{49}$

(iii) Let A_3 = event of getting a black card.
There are 23 black cards left.
Hence, required probability $= P(A_3) = P$(getting a black card) $= \frac{23}{49}$

Example:

Five cards, the ten, jack, queen, king and ace of diamonds, are well-shuffled with their faces downwards. One card is then picked up at random.
(i) What is the probability that the card drawn is the queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is
(a) An ace **(b) A queen**

Solution: There are five cards- the ten, jack, queen, king and ace of diamond.

(i) Let A = event of getting a queen
There is only one queen out of the five cards.
Hence, required probability $= P(A) = P$ (getting a queen) $= \frac{1}{5}$

(ii) When a queen is drawn and put aside, four cards- the ten, jack, king and ace are left. Therefore.
(a) Required probability $= P$ (getting an ace) $= \frac{1}{4}$
(b) Required probability $= P$ (getting a queen) $= \frac{0}{4} = 0$.

Example:

A box contains 5 red, 4 green and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is
(i) White **(ii) Neither red nor white**

Solution: Total number of balls in the box $= 5 + 4 + 7 = 16$.
Let A_1 = event of getting a red ball
A_2 = event of getting a white ball.
(i) There are 7 white balls in the box.
Hence, required probability $= P(A_2) = P$ (getting a white ball) $= \frac{7}{16}$

(ii) There are 7 white and 5 red balls in the box.
Hence, required probability $= P$ (getting neither a red nor a white ball)
$= 1 - P$ (getting either red or white ball)
$= 1 - [P$ (getting a red) $+P$ (getting a white ball)]

$= 1 - \left(\frac{5}{16} + \frac{7}{16}\right) = 1 - \frac{12}{16} = \frac{4}{16} = \frac{1}{4}$

Example:

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of the red ball, determine the number of blue balls in the bag.

Solution: There are 5 red balls in a bag.

Let the number of blue balls be x.

Let A_1 = event of getting a red ball and A_2 = event of getting a blue ball.

$P(A_1) = P(\text{getting a red ball}) = \frac{5}{x+5}$

$P(A_2) = P\text{ (getting a blue ball)} = \frac{x}{x+5}$

$2P(A_1) = P(A_2) \Rightarrow \frac{2 \times 5}{x+5} = \frac{x}{x+5} \Rightarrow 10 = x \Rightarrow x = 10$

Hence, the required number of blue balls = 10.

Example:

A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be

(i) Red

(ii) White

(iii) Not green?

Solution: Total number of marbles in the box $= 5 + 8 + 4 = 17$.

Let A_1 = event of getting a red marble and A_2 = event of getting a white marble

(i) There are 5 red marbles in the box.

Hence, required probability $= P(A_1) = P\text{ (getting a red marble)} = \frac{5}{17}$

(ii) There are 8 white marbles in the box.

Hence, required probability $= P(A_2) = P\text{ (getting a white marble)} = \frac{8}{17}$

(iii) There are 4 green marbles in the box.

$\therefore P(A_3) = P\text{ (getting a green marble)} = \frac{4}{17}$

Hence, required probability $= P\text{ (not getting a green marble)}$

$= 1 - P\text{ (getting a green marble)} = 1 - P(A_3) = 1 - \frac{4}{17} = \frac{13}{17}$

Example:

A box contains 19 balls bearing numbers $1, 2, 3, \dots\dots, 19$ respectively. A ball is drawn at random from the box. Find the probability that the number on the ball is

(i) A prime number **(ii) Even number**

(iii) Divisible by 3 or 5 **(iv) Neither divisible by 5 nor by 10.**

Solution: Total number of balls in the box $= 19$

$\therefore$ Number of all possible outcomes $= 19$

(i) Let A_1 = event of getting a prime number.

Then, the favourable outcomes are $2, 3, 5, 7, 11, 13, 17, 19$.

Number of favourable outcomes $= 8$.

Hence, required probability $= P\text{ (getting a prime number)} = P(A_1) = \frac{8}{19}$.

(ii) Let A_2 = event of getting an even number

Then, the favourable outcomes are 2, 4, 6, 8, 10, 12, 14, 16, 18.

Number of favourable outcomes $= 9$.

Hence, required probability $= P\text{ (getting an even number)} = P(A_2) = \frac{9}{19}$.

(iii) Let A_3 = event of getting a number divisible by 3 or 5.

Then, the favourable outcomes are $3, 5, 6, 9, 10, 12, 15, 18$.

Number of favourable outcomes $= 8$.

Hence, required probability $= P\text{ (getting a number divisible by 3 or 5)} = P(A_3) = \frac{8}{19}$.

(iv) Let A_4 = event of getting a number divisible by 5 or 10.

Then, the favourable outcomes are $5, 10, 15$.

Number of favourable outcomes $= 3$.

$\therefore P(\text{getting a number divisible by 5 or 10}) = P(A_4) = \frac{3}{19}$

Hence, required probability $= P\text{ (getting a number neither divisible by 5 nor by 10)}$

$= 1 - P(\text{getting a number divisible by 5 and 10}) = 1 - \frac{3}{19} = \frac{16}{19}$

Example:

Seventeen cards numbered $1, 2, 3, 4, \ldots\ldots, 16, 17$ are put in a box and mixed thoroughly. One person has drawn a card from the box. Find the probability that the number on the card is
(i) Odd
(ii) A prime number
(iii) Divisible by 3
(iv) Divisible by 2 and 3 both

Solution: There are seventeen cards in the box.
∴ Number of all possible outcomes $= 17$.
(i) Let A_1 = event of getting an odd number.
Then, the favourable outcomes are $1, 3, 5, 7, 9, 11, 13, 15, 17$.
Number of favourable outcomes $= 9$.
Hence, required probability $= P$ (getting an odd number) $= P(A_1) = \frac{9}{17}$.

(ii) Let A_2 = event of getting a prime number.
Then, the favourable outcomes are $2, 3, 5, 7, 11, 13, 17$.
Number of favourable outcomes $= 7$.
Hence, required probability $= P$ (getting a prime number) $= P(A_2) = \frac{7}{17}$.

(iii) Let A_3 = event of getting a number divisible by 3.
Then, the favourable outcomes are $3, 6, 9, 12, 15$.
Number of favourable outcomes $= 5$.
Hence, required probability $= P(A_3) = \frac{5}{17}$.

(iv) Let A_4 = event of getting a number divisible by 2 and 3 both.
Then, the favourable outcomes are 6,12.
Number of favourable outcomes $= 2$.
Hence, required probability $= P(A_4) = \frac{2}{17}$.

Example:

A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears
(i) A two-digit number
(ii) A perfect square number
(iii) A number divisible by 5.

Solution: The total number of discs $= 90$. Therefore, the number of possible outcomes $= 90$.
(i) Let A_1 = event of getting a two-digit number.
There are 9 single-digit numbers and 81 two-digit numbers.
Then, the number of favourable outcomes $= 81$.
Hence, required probability $= P(A_1) = P$ (getting a two-digit number) $= \frac{81}{90} = \frac{9}{10}$.

(ii) Let A_2 = event of getting a perfect square number.
Then, the number of favourable outcomes are $1, 4, 9, 16, 25, 36, 49, 64$, and 81.
Number of favourable outcomes $= 9$.
Hence, required probability $= P(A_2) = P$ (getting a perfect square number) $= \frac{9}{90} = \frac{1}{10}$.

(iii) Let A_3 = event of getting a number divisible by 5.
Then, the number of favourable outcomes are
$5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90$.
Number of favourable outcomes $= 18$.
Hence, required probability $= P(A_3) = P$ (getting a number divisible by 5) $= \frac{18}{90} = \frac{1}{5}$.

Check Your Concept - 1

(i) Three dice are thrown once. The probability that all the dice show different faces is

(A) $\frac{5}{18}$ (B) $\frac{2}{9}$

(C) $\frac{8}{15}$ (D) $\frac{5}{9}$

(ii) A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card is $\frac{3}{13}$.

(A) True (B) False

(C) Can't say (D) Partially true/false

(iii) A card is selected from a deck of 52 cards, then the probability of its being a red face card is

(A) $\frac{3}{26}$ (B) $\frac{3}{13}$

(C) $\frac{2}{13}$ (D) $\frac{1}{2}$

Solved Examples

(1) The percentage of marks obtained by a student in the monthly unit tests are given below:

Unit test	I	II	III	IV	V
Percentage of marks obtained	58	64	76	62	85

Find the probability that the student gets:
(i) a first-class i.e., at least 60% marks.
(ii) marks between 70% and 80%.
(iii) a distinction i.e., 75% or above.
(iv) less than 65% marks.

Solution: Total number of unit tests held = 5

(i) Number of unit tests in which the student gets the first class i.e., at least 60% marks = 4.

∴ Probability that the student gets the first class

$= \frac{4}{5}$ = **0.8**

(ii) Number of tests in which the student gets between 70% and 80% = 1.

∴ Probability that the student gets marks between 70% and 80% = $\frac{1}{5}$ = **0.2.**

(iii) Number of tests in which the student gets distinction = $\frac{2}{5}$ = **0.4**

(iv) Number of tests in which the student gets less than 65% marks = 3

∴ Probability that a student gets less than 65% marks = $\frac{3}{5}$ = **0.6.**

(2) On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in the table below:

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

A number is chosen at random, find the probability that the digit at its unit's place is:
(i) 6
(ii) A non-zero multiple of 3
(iii) A non-zero even number
(iv) An odd number.

Solution: We have,

Total number of selected telephone numbers = 200

(i) It is given that the digit 6 occurs 14 times at the unit's place.

∴ Probability that the digit at the unit's place is 6 = $\frac{14}{200}$ **= 0.07**

(ii) A non-zero multiple of 3 means 3, 6 and 9.

Number of telephone numbers in which unit's digit is either 3 or 6 or 9 = 22 + 14 + 20 = 56.

∴ Probability of getting a telephone number having a multiple of 3 at unit's place = $\frac{56}{200}$ **= 0.28**

(iii) Number of telephone numbers having an even number (0 or 2 or 4 or 6 or 8) at unit's place = 22 + 22 + 20 + 14 + 16 = 94

∴ Probability of getting a telephone number having an even number at unit's place = $\frac{94}{200}$ = **0.47**

(iv) Number of telephone numbers having an odd digit (1 or 3 or 5 or 7 or 9) at unit's place
= 26 + 22 + 10 + 28 + 20 = 106

∴ Probability of getting a telephone number having an odd number at unit's place = $\frac{106}{200}$ = **0.53**

(3) Find the probability that a number selected at random from the numbers 1 to 25 is not a prime number when each of the given numbers is equally likely to be selected.

Solution: Here S = {1, 2, 3, 4, ..., 25}

Let E = event of getting a prime number = {2, 3, 5, 7, 11, 13, 17, 19, 23}.

Then, n (E) = 9

∴ P(E) = $\frac{n(E)}{n(S)} = \frac{9}{25}$.

Required probability = 1 – P(E) = $\left(1 - \frac{9}{25}\right) = \mathbf{\frac{16}{25}}$.

(4) Eleven bags of wheat flour, each marked 5 kg actually, contained the following weights of flour (in kg): 4.97, 5.05, 5.08, 5.03, 5.00, 5.06, 5.08, 4.98, 5.04, 5.07, 5.00. Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Solution: Total number of bags = 11

Number of bags containing more than 5 kg of flour = 7

Therefore, the probability of bags containing more than 5 kg of flour

= $\frac{\text{Number of bags containing more than 5kg flour}}{\text{Total number of bags}} = \mathbf{\frac{7}{11}}$.

(5) The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct?

(ii) What is the probability that it was not correct on a given day?

Solution: The total number of days for which the record is available = 250

(i) P (correct forecast) = $\frac{\text{Number of days when the forecast was correct}}{\text{Toal number of days for which the record is available}} = \frac{175}{250}$ = **0.7.**

(ii) The number of days when the forecast was not correct = 250 – 175 = 75.

P (not correct forecast) = $\frac{75}{250}$ = **0.3**

(6) If the probability of winning a game is 0.3, what is the probability of losing it?

Solution: Probability of winning a game = 0.3.

Probability of losing it = q (say).

⇒ 0.3 + q = 1 ⇒ q = 1 – 0.3

⇒ q = **0.7**

(7) Two coins are tossed simultaneously. Find the probability of getting-

(i) Two heads **(ii) At least one head** **(iii) No head**

Solution: Let H denote the head and T denote tail.

∴ On tossing two coins simultaneously, all the possible outcomes are (HH, HT, TH, TT)

(i) The probability of getting two heads = P(HH)

= $\frac{\text{Event of occurrence of two heads}}{\text{Total number of possible outcomes}} = \mathbf{\frac{1}{4}}$.

(ii) The probability of getting at least one head = P (HT or TH or HH)

= $\frac{\text{Event of occurrence at least one head}}{\text{Total number of possible outcomes}} = \mathbf{\frac{3}{4}}$.

(iii) The probability of getting no head = P(TT)

= $\frac{\text{Event of occurrence of no head}}{\text{Total number of possible outcomes}} = \mathbf{\frac{1}{4}}$.

(8) On tossing three coins at a time, find -

(i) All possible outcomes.

(ii) Events of occurrence of 3 heads, 2 heads, 1 head and 0 head.

(iii) Probability of getting 3 heads, 2 heads, 1 head and no head.

Solution: Let H denotes head and T denote tail. On tossing three coins at a time,

(i) All possible outcomes = **{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}**.

(ii) An event of occurrence of 3 heads = (HHH) = **1.**

An event of occurrence of 2 heads = {HHT, HTH, THH} = **3.**

An event of occurrence of 1 head = {HTT, THT, TTH} = **3.**

An event of occurrence of 0 head = {TTT} = **1.**

(iii) Now, probability of getting 3 heads = P (HHH) = $\frac{\text{Event of occurrence of 3 heads}}{\text{Total number of possible outcomes}} = \mathbf{\frac{1}{8}}$.

Probability of getting 2 heads = P (HHT or THH or HTH) = $\frac{\text{Event of occurrence of 2 heads}}{\text{Total number of possible outcomes}} = \mathbf{\frac{3}{8}}$.

Probability of getting one head = P (HTT or THT or TTH) = $\frac{\text{Event of occurrence of 1 head}}{\text{Total number of possible outcomes}} = \mathbf{\frac{3}{8}}$.

Probability of getting no head = P (TTT)= $\frac{\text{Event of occurrence of no heads}}{\text{Total number of possible outcomes}} = \frac{1}{8}$.

(9) A bag contains 12 balls out of which x are white,
(i) If one ball is drawn at random, what is the probability that it will be a white ball?
(ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will double of what it was before. Find x.

Solution: Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

∴ Total number of possible outcomes = 12

Number of white balls = x

(i) Out of total 12 outcomes, favourable outcomes = x

P (White ball) = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x}{12}$

(ii) If 6 more white balls are put in the bag, then

Total number of white balls = x + 6

Total number of balls in the bag

= 12 + 6 = 18

P (White ball) = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x+6}{12+6}$

According to the question,

Probability of drawing a white ball in the second case

= 2 × probability drawing of white ball in the first case

$\Rightarrow \frac{x+6}{18} = 2\left(\frac{x}{12}\right)$

$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$

⇒ 6x + 36 = 18x

⇒ 12x = 36

⇒ x = 3

Hence, the number of white balls = **3.**

(10) What is the probability that a leap year, selected at random will contain 53 Sundays?

Solution: Number of days in a leap year = 366 days

Now, 366 days = 52 weeks and 2 days

The remaining two days can be

(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Friday and Saturday
(vii) Saturday and Sunday

For the leap year to contain 53 Sundays, the last two days are either Sunday and Monday or Saturday and Sunday.

∴ Number of such favourable outcomes = 2

Total number of possible outcomes = 7

∴ P (a leap year contains 53 Sundays) = $\frac{2}{7}$.

(11) Three unbiased coins are tossed together. Find the probability of getting:
(i) All heads, (ii) Two heads (iii) One head (iv) At least two heads.

Solution: Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

∴ Total number of elementary events = 8.

(i) The event "Getting all heads" is said to occur, if the elementary event HHH occurs i.e., HHH is an outcome. Therefore, ∴ Favourable number of elementary events = 1

Hence, the required probability = $\frac{1}{8}$.

(ii) The event "Getting two heads" will occur, if one of the elementary events HHT, THH, HTH occurs.

∴ Favourable number of elementary events = 3

Hence, the required probability = $\frac{3}{8}$.

(iii) The events of getting one head, when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH happens.

∴ Favourable number of elementary events = 3

Hence, the required probability = $\frac{3}{8}$.

(iv) If any of the elementary events HHH, HHT, HTH and THH is an outcome, then we say that the event "Getting at least two heads" occurs.

∴ Favourable number of elementary events = 4

Hence, the required probability = $\frac{4}{8} = \frac{\mathbf{1}}{\mathbf{2}}$.

(12) A piggy bank contains hundred 50 p coins, fifty Rs. 1 coin, twenty Rs. 2 coins and ten Rs. 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin
(i) Will be a 50-p coin?
(ii) Will not be a 5 coin?

Solution: Number of 50 paise coins = 100
Number of rupee 1 coin = 50
Number of Rs. 2 coins = 20
Number of Rs. 5 coins = 10
(i) The number of favourable outcomes of 50 p coin to fall = 100
Total number of coins = 100 + 50 + 20 + 10 = 180
Total number of possible outcomes = 180

P = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

P (50 p) = $\frac{100}{180} = \frac{\mathbf{5}}{\mathbf{9}}$.

(ii) Number of favourable outcomes of 5 Rs coin to not fall = 180 – 10 = 170

P = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

P (not Rs. 5) = $\frac{170}{180} = \frac{\mathbf{17}}{\mathbf{18}}$.

(13) A box contains 20 balls bearing numbers, 1, 2, 3, 4, ... 20. A ball is drawn at random from the box. What is the probability that the number on the balls is
(i) An odd number (ii) Divisible by 2 or 3 (iii) Prime number (iv) Not divisible by 10

Solution: Total number of possible outcomes = 20

Probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

(i) Number of odds out of the first 20 numbers = 10
Favourable outcomes by odd = 10

P(odds) = $\frac{\text{Favourable outcomes of odd}}{\text{Total number of possible outcomes}} = \frac{10}{20} = \frac{\mathbf{1}}{\mathbf{2}}$.

(ii) The numbers divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.
Favourable outcomes of numbers divisible by 2 or 3 = 13
P (numbers divisible by 2 or 3)

= $\frac{\text{Favourable outcomes of divisible by 2 or 3}}{\text{Total number of possible outcomes}} = \frac{\mathbf{13}}{\mathbf{20}}$.

(iii) Prime numbers out of the first 20 numbers are 2, 3, 5, 7, 11, 13, 17, 19
Favourable outcomes of primes = 8

P (primes) = $\frac{\text{Favourable outcomes of primes}}{\text{Total number of possible outcomes}} = \frac{8}{20} = \frac{\mathbf{2}}{\mathbf{5}}$.

(iv) Numbers not divisible by 10 are 1, 2, ... 9, 11, ...19
Favourable outcomes of not divisible by 10 = 18

P (not divisible by 10) = $\frac{\text{Favourable outcomes of not divisible by 10}}{\text{Total number of possible outcomes}} = \frac{18}{20} = \frac{\mathbf{9}}{\mathbf{10}}$.

(14) A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that:
(i) She will buy it (ii) She will not buy it

Solution: There are 144 ball pens.
∴ The total number of possible outcomes = 144.
(i) Let A_1 = event of buying a pen.
There are 20 ball pens which are defective out of 144 ball pens.
∴ number of good ball pens = 144 – 20 = 124

Hence, required probability = $P(A_1)$ = P (buying a good pen) = $\frac{124}{144} = \frac{\mathbf{31}}{\mathbf{36}}$.

(ii) Let A_2 = event of not buying a pen
Then, the number of favourable outcomes = 20.

Hence, required probability = $P(A_2)$ = P (not buying a good pen) = $\frac{20}{144} = \frac{\mathbf{5}}{\mathbf{36}}$.

OBJECTIVE TYPE QUESTIONS

(1) A number is randomly chosen from the set {1, 2, 3, …. 10}. The probability that the number will be even, is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{1}{4}$

(2) In a single throw of a die, the probability of getting a multiple of 2 is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

(3) Two dice are thrown simultaneously. The probability of getting a multiple of 2 on one die and a multiple of 3 on the other is

(A) $\frac{5}{36}$ (B) $\frac{5}{12}$

(C) $\frac{11}{36}$ (D) $\frac{1}{12}$

(4) The king, queen and jack of clubs are removed from a deck of 52 cards and then will–shuffled. One card is selected from the remaining cards. The probability of getting a club is

(A) $\frac{13}{49}$ (B) $\frac{10}{49}$

(C) $\frac{3}{49}$ (D) $\frac{1}{49}$

(5) In an experiment, a coin is tossed 500 times. If the head turns up 280 times, then the experimental probability of getting (i) a head (ii) a tail is

(A) $\frac{14}{25}, \frac{11}{25}$ (B) $\frac{11}{20}, \frac{12}{20}$

(C) $\frac{12}{25}, \frac{10}{25}$ (D) $\frac{9}{25}, \frac{11}{25}$

(6) A bag contains 6 blue and 4 green marbles. If a marble is drawn at random from the bag, the probability that the marble drawn will be green is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{4}{5}$ (D) $\frac{1}{10}$

(7) A card is drawn at random from a well shuffled pack of 52 cards. The probability that the cards drawn is neither a red card nor a queen is

(A) $\frac{6}{13}$ (B) $\frac{5}{13}$

(C) $\frac{4}{13}$ (D) $\frac{2}{13}$

(8) One card is drawn from a well–shuffled deck of 52 cards. The probability of drawing an ace is

(A) $\frac{1}{12}$ (B) $\frac{1}{13}$

(C) $\frac{1}{50}$ (D) $\frac{3}{10}$

(9) A die is thrown once. The probability of getting a number greater than 3 is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) 0

(10) In a cricket match a batswoman hits a boundary 6 times out of 30 balls she plays. The probability that she did not hit a boundary is

(A) $\frac{3}{5}$ (B) $\frac{4}{5}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

(11) Two coins are tossed simultaneously. The probability of getting at least one head is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) 0

(12) Cards marked with the numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this bag. The probability that the number on the card is a prime less than 20 is

(A) $\frac{2}{25}$ (B) $\frac{3}{25}$
(C) $\frac{4}{25}$ (D) $\frac{1}{5}$

(13) A number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. The probability that $|x| < 2$ is

(A) $\frac{5}{7}$ (B) $\frac{2}{7}$
(C) $\frac{3}{7}$ (D) $\frac{1}{7}$

(14) The probability of guessing the correct answer to a certain test question is $\frac{x}{2}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then x =

(A) 2 (B) 3
(C) $\frac{2}{3}$ (D) $\frac{1}{3}$

(15) Two numbers are chosen from 1 to 5. The probability for the two numbers to be consecutive is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$
(C) $\frac{1}{10}$ (D) $\frac{2}{10}$

(16) Two dice are thrown at a time. The probability that the difference of the numbers shown on the dice will be 1 is

(A) $\frac{5}{18}$ (B) $\frac{1}{36}$
(C) $\frac{1}{6}$ (D) $\frac{2}{5}$

(17) A bag contains 3 white and 5 red balls. If a ball is drawn at random, the probability that the drawn ball to be red is

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$
(C) $\frac{3}{15}$ (D) $\frac{5}{15}$

(18) When a die is rolled, the probability of getting an even number is

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$
(C) $\frac{1}{2}$ (D) $\frac{2}{5}$

(19) A card is drawn from a packet of 100 cards numbered 1 to 100. The probability of drawing a number which is a square is

(A) $\frac{1}{10}$ (B) $\frac{9}{100}$
(C) $\frac{1}{100}$ (D) $\frac{2}{100}$

(20) The probability for a randomly selected number out of 1, 2, 3, 4, …., 25 to be a prime number is

(A) $\frac{2}{25}$ (B) $\frac{23}{25}$
(C) $\frac{10}{25}$ (D) $\frac{9}{25}$

(21) A letter is chosen at random from the letters of the word 'ASSASSINATION', then the probability that the letter chosen is a vowel and is in the form of $\frac{6}{2x+1}$, then x is equal to

(A) 5 (B) 6
(C) 7 (C) 8

(22) Two unbiased coins are tossed simultaneously then the probability of getting no head is $\frac{A}{B}$, then $(A+B)^2$ is equal to

(A) 1 (B) 4
(C) 5 (D) 25

(23) A man is known to speak the truth 3 out of 4 times. He throws a die and a number other than six comes up. Find the probability that he reports it is a six

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$
(C) $\frac{1}{2}$ (D) 1

(24) There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹50 each and 200 of them contain a cash prize of ₹10 each and the rest do not contain any cash prize. If they are well-shuffled and an envelope is picked up out, then the probability that it contains no cash prize is

(A) 0.65 (B) 0.69

(C) 0.54 (D) 0.57

(25) Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random, then the probability that the ticket has a number which is a multiple of 3 or 7 is

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$

(C) $\frac{4}{5}$ (D) $\frac{1}{5}$

Answer Key

CHECK YOUR CONCEPT

(1) **(i)** (D) **(ii)** (A) **(iii)** (A)

OBJECTIVE TYPE QUESTIONS

(1)	(B)	**(6)**	(A)	**(11)**	(C)	**(16)**	(A)	**(21)**	(B)
(2)	(A)	**(7)**	(A)	**(12)**	(A)	**(17)**	(B)	**(22)**	(D)
(3)	(C)	**(8)**	(B)	**(13)**	(C)	**(18)**	(C)	**(23)**	(B)
(4)	(B)	**(9)**	(A)	**(14)**	(C)	**(19)**	(A)	**(24)**	(B)
(5)	(A)	**(10)**	(B)	**(15)**	(B)	**(20)**	(D)	**(25)**	(A)