MENSURATION

Concepts Covered

- **Introduction to different geometrical figures**
- **Number of edges, vertices and faces of a prism**
- **Surface area and volume of a Cube and a Cuboid**
- **Surface area and volume of a Right Circular Cylinder**
- **Surface area and volume of a Right Circular Cone**
- **Surface area and volume of Sphere, Hemisphere and Hemispherical Shell**
- **Combination and recasting of solids**
- **Cone Frustum**

Introduction

If you can draw any shapes and figures on a plane (i.e., Paper, Notebook etc.) then these shapes and figures are called as **plane figures** or **two-dimensional shapes**. We already know about two-dimensional shapes like squares, rectangles, circles, etc. We also know about the **area** and **perimeter** and how to find them. It would be interesting to see what happens if we cut out many of these plane figures of the same shape and size from cardboard sheets and stack them up in a vertical pile. Through this process, we shall obtain some solid figures such as a prism, a cuboid, a cylinder, etc.

Surface Area and Volume

Any object which occupies space is called a **solid**. The total area enclosed by all the bounding surfaces of the solid is called the **total surface area** of the solid. It is measured in square units. Also, the amount of space enclosed by the bounding surface or surfaces of a solid is called the **volume** of the solid. It is measured in cubic units.

Lateral Surface Area

The lateral surface area of an object is the area of all the surfaces of the object, excluding its base and top (if there are any).

Curved Surface Area

The total area of all the curved surfaces in an object is called its curved surface area.

Surface Area and Volume of a Cube and a Cuboid

Cuboid

Cuboid

There are 3 dimensions in all solids. The three dimensions of a cuboid are its length (l), breadth (b) and height (h).

1. The lateral surface area of a cuboid $= \mathbf{2(l + b)h}$.
2. The total surface area of a cuboid $= \mathbf{LSA + 2(Base\ area) = 2(l + b)h + 2lb = 2(lb + bh + lh)}$.
3. The volume of a cuboid $= \mathbf{(lb)h = lbh}$.
4. Diagonal of a cuboid $= \sqrt{\mathbf{l^2 + b^2 + h^2}}$.

Cube

In a cuboid, if its length, breadth and height are equal, then it is called a cube. All the edges of a cube are equal. Thus, the edge in itself is completely sufficient to determine the size of the cube.

If the edge of a cube is a unit, then

Cube

1. The lateral surface area of the cube = $\mathbf{4a^2}$.
2. The total surface area of the cube = $\mathbf{LSA + 2(Area\ of\ base) = 4a^2 + 2a^2 = 6a^2}$.
3. The volume of the cube = $\mathbf{a^3}$.
4. The diagonal of the cube = $\sqrt{\mathbf{a^2+a^2+a^2}} = \sqrt{\mathbf{3a^2}}$.

Example:

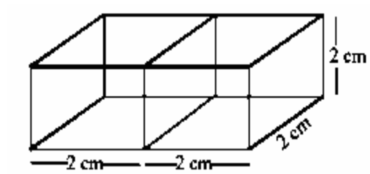
A box is in the form of a cube. Its outer edge is 5 m long. Find:
(a) The total length of the edges.
(b) Cost of painting outside of the box, on all surfaces, at the rate of Rs. 5 per m^2.

Solution: (a) Length of edges = Number of edges in base × 3 × Length of each edge = 4 × 3 × 5 = 60 m.
(b) To find the cost of painting the box, we need to find the total surface area.
TSA = $6a^2 = 6 \times 5^2 = 6 \times 25 = 150\ m^2$.
∴ Cost of painting = 150 × 5 = Rs 750.

Example:

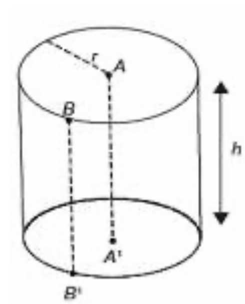
Two cubes of sides 2 cm each are joined end to end, find the volume of the cuboid so formed.

Solution: When two cubes of side 2 cm each are joined end to end then,
l = (2 + 2) = 4 cm, b = 2 cm & h = 2 cm
∴ Volume = l × b × h = 4 × 2 × 2 = 16 cm^3



Surface Area and Volume of a Right Circular Cylinder

A right circular cylinder consists of two identical parallel circular regions at the top and the bottom which are connected by a curved surface. The axis which connects the centres of both circles is perpendicular to the planes in which the circles lie. Each of the circular regions is called the base of the cylinder. A chapati roller, pipe, and electric wires are some of the cylinder-shaped objects.



Right Circular Cylinder

Here in the given figure, a right circular cylinder is shown. Let $\mathbf{A}$ be the centre of the top face and $\mathbf{A'}$ be the centre of the base. The line joining the centres (here it is $\mathbf{AA'}$) is called the axis (height) of the cylinder. The length $\mathbf{AA'}$ is called the height of the cylinder.

The radius $\mathbf{r}$ of the base of the cylinder and the height $\mathbf{h}$, completely describe the cylinder.

Lateral (curved) surface area = Perimeter of base × Height = $\mathbf{2\pi rh}$
Total surface area = $\mathbf{LSA} + \mathbf{2}$(Base area) = $\mathbf{2\pi rh + 2(\pi r^2) = 2\pi r(h + r)}$
Volume = Area of base × Height = $\mathbf{\pi r^2 h}$.

Example:

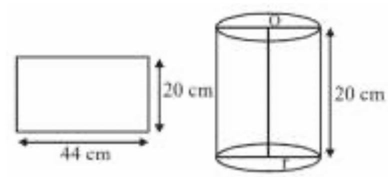
A closed cylindrical container, the radius of which is 7 cm and height 10 cm is to be made out of a metal sheet.
Find: (a) The area of the metal sheet required.
(b) The volume of the cylinder made.
(c) The cost of painting the lateral surface of the cylinder at the rate of Rs. 4 per cm^2 (Assuming that the thickness of the metal sheet is negligible).

Solution: (a) The area of the metal sheet required = The total surface area of the cylinder = 2πr(r + h)
= $2 \times \frac{22}{7} \times 7 \times (7 + 10) = 44 \times 17 = 748\ cm^2$.
(b) Volume = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 10 = 22 \times 70 = 1540\ cm^3$.
(c) To find the cost of painting the lateral surface, we need to find the curved (lateral) surface area.
∴ CSA = $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 10 = 440\ cm^2$.
Cost of painting = 440 × 4 = Rs. 1760.

Example:

A rectangular sheet of aluminium foil is 44 cm long and 20 cm wide. A cylinder is made by rolling the foil along its length. Find the volume of the cylinder so formed.

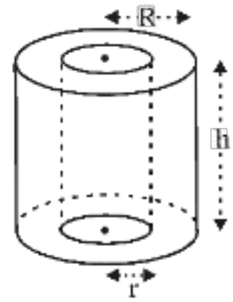
Solution: When the rectangular sheet is rolled to make a cylinder, the length of the rectangle will become the circumference of the cylinder and the breadth will become the height of the cylinder.
∴ $2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7$ cm.



Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 20 = 22 \times 140 = 3080\ cm^3$.

Example:

A hollow cylinder is 35 cm in length (height). Its internal and external diameters are 8 cm and 8.8 cm respectively. Find its:
(a) Outer curved surface area
(b) Inner curved surface area
(c) Area of cross-section
(d) Total surface area



Solution: The height of the cylinder h = 35 cm

The internal radius $r = \frac{8}{2} = 4$cm, The external radius $R = \frac{8.8}{2} = 4.4$ cm

(a) Outer curved surface area = $2\pi Rh = 2 \times \frac{22}{7} \times 4.4 \times 35 = 968\ cm^2$

(b) Inner curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 4 \times 35 = 880\ cm^2$

(c) The cross-section of a hollow cylinder is like a ring with external radius R = 4.4 cm and internal radius r = 4 cm.

∴ Area of cross-section = $\pi(R^2 - r^2) = \frac{22}{7} \times (4.4^2 - 4^2)$

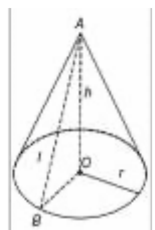
$= \frac{22}{7}(4.4 + 4)(4.4 - 4) = \frac{22}{7} \times 8.4 \times 0.4 = 10.56\ cm^2$.

(d) Total surface area of a hollow cylinder = (curved surface area of outer cylinder + curved surface area of inner cylinder + area of ring) = $2\pi Rh + 2\pi rh + \pi(R^2 - r^2) = 968 + 880 + 10.56 = 1858.56\ cm^2$.

Surface Area and Volume of a Right Circular Cone

The solid generated by the rotation of a right-angled triangle about one of the sides containing the right angle is called a **right circular cone.** Point A is the vertex of the cone. Its base is a circle with centre O and radius OB.

The length OA is the height of the cone and the length AB is called its slant height.
Clearly, ∠ AOB = 90°.
If radius of base = r units, height = h units and slant height = l units.
Then, $l^2 = (h^2 + r^2)$ and $l = \sqrt{h^2 + r^2}$.
For a right circular cone of radius r, height h and slant height l:



Right Circular Cone

(i) Curved surface area of a cone = $\pi r l$.

(ii) Total surface area of a cone = Curved surface area + Area of base

$$= \pi r l + \pi r2 = \pi r(r + l).$$

(iii) Volume of a cone = $\frac{1}{3}\pi r^2 h$.

Example:

A cone-shaped of radius 7 cm and height 24 cm. Find the area of the paper required to make the cap.

Solution: Area of the paper required = Curved surface area of the cap (or cone) = $\pi r l$.

Now, $l = \sqrt{h^2 + r^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm.

⇒ Curved surface area = $\frac{22}{7} \times 7 \times 25 = 550\ cm^2$.

∴ Area of the paper required = $550\ cm^2$.

Example:

The inner diameter of an ice cream cone is 7 cm and its height is 12 cm. Find the volume of ice cream that the cone can contain.

Solution: Volume of ice cream (cone) = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 22 \times 7 = 154\ cm^3$.

Surface Area of a Sphere and Hemisphere

Objects like football, tennis ball, marbles etc., are said to have the shape of a sphere. A sphere is a collection of points in space, equidistant from a fixed point.

The fixed point is called the **centre** and the fixed distance is called the **radius** of the sphere. A segment joining two points on the sphere and passing through its centre is called the **diameter** of the sphere.


Sphere

A sphere is a solid generated by the revolution of a semicircle about its diameter. The centre and radius of the semicircle are respectively the centre and radius of the sphere.
The surface area of a sphere = $4\pi r^2$

The volume of a sphere = $\frac{4}{3}\pi r^3$

Hemisphere

If a sphere is cut into two halves by a plane passing through the centre of the sphere, then each of the halves is called a hemisphere.

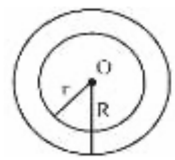
1. Curved surface area of a hemisphere = $2\pi r^2$.
2. Total surface area of a hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$.
3. Volume of a sphere = $\frac{2}{3}\pi r^3$.

Hemisphere

Spherical Shell

The region bounded by the space between two concentric solid spheres is called a spherical shell. The thickness of the shell is given by the difference in the radii of the two spheres.
Thickness = R - r, where R = outer radius and r = inner radius.

1. Outer surface area = $4\pi R^2$.
2. Inner surface area = $4\pi r^2$.
3. Volume of material = $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(R^3 - r^3)$.

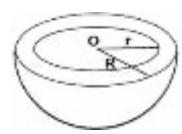


Spherical Shell

Hemispherical Shell

A hemispherical shell is referred to as a 3D figure made by cutting a spherical shell into two halves.
Thickness = R - r, where R = outer radius and r = inner radius.

1. Outer curved surface area = $2\pi R^2$ **sq. units.**
2. Inner curved surface area = $2\pi r^2$ **sq. units.**
3. Area of Ring = $\pi(R^2 - r^2)$.
4. Volume of material = $\frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(R^3 - r^3)$ **cubic units.**
5. Total surface area = (curved surface area of outer hemisphere + curved surface area of inner hemisphere + area of ring) = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) = 3\pi R^2 + \pi r^2$.



Hemispherical Shell

Example:

The radius of a spherical balloon increases from 10 cm to 15 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in both cases.

Solution: Required ratio $= \frac{\text{Surface area of the balloon in } 1^{st} \text{ case}}{\text{Surface area of the balloon in } 2^{nd} \text{ case}} = \frac{4\pi r^2 \text{ in 1st case}}{4\pi r^2 \text{ in 2nd case}} = \frac{4\times\pi\times10\times10}{4\times\pi\times15\times15} = \frac{4}{9}$ = 4 : 9.

Example:

A hollow hemispherical bowl, of thickness 1 cm has an inner radius of 6 cm. Find the volume of metal required to make the bowl.

Solution: Inner radius, r = 6 cm; thickness, t = 1 cm.

∴ Outer radius, R = r + t = 6 + 1 = 7 cm.

∴ Volume of steel required $= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3}\pi(7^3 - 6^3) = \frac{2}{3} \times \frac{22}{7} \times 127 = \frac{5588}{21}$ cm³.

Check Your Concept - 1

(i) Find the lateral surface area and the length of the diagonal of a cube of side 8 cm.

(ii) If the volume of a solid hemisphere and its total surface area are numerically equal, then find its radius.

(iii) The heights of two cylinders are equal and their radii are in the ratio of 3 : 2. The ratio of their curved surface areas is?

(iv) If the radius of a sphere is increased by 25%, then the percentage increase in its volume is (approximately)?

Surface Area of a Combination of Solids

Combination of Solids

A combination of solid is a figure which is formed by combining two or more different solids. Two cubes may combine to form a cuboid, while a cone over a cylinder might fuse to form a tent. In our daily lives, we see many shapes that are a blend of different shapes.

A Circus Tent or a Hut

A circus tent is a combination of a cylinder and a cone. Some circus tents also constitute a cuboid and a cone. A hut is a kutcha house and has a tent-like structure.

An Ice Cream Cone

An ice cream cone is a combination of a cone and a hemisphere.

Ice-cream Cone

A Dome on any Solid Shape

A dome is generally built on buildings or tents. A dome is the upper half of the hemisphere.
Now if a building has a dome structure, then we combine the solid shape of that building with the dome shape.

When working with calculations of the surface area of a solid combination, the initial step is to figure out what shapes have combined to form the structure. You must combine the surface areas of the constituting structures to determine the surface area of a solid structure created by combining two or more solids.

For example: To calculate the surface area of a circus tent, combine the cone and cylinder surface areas. In a tent made up of a cone and a cylinder, we first compute the surface area of each cone and cylinder separately, then combine them.

Example:

Consider three cubes each of $5\ \mathrm{cm}$ edges are joined end to end. Find the surface area of the resulting cuboid.

Solution: If three cubes are joined end to end, we get a cuboid such that,

Length of the resulting cuboid, $\mathbf{l} = \mathbf{5\ cm} + \mathbf{5\ cm} + \mathbf{5\ cm} = \mathbf{15\ cm}$.

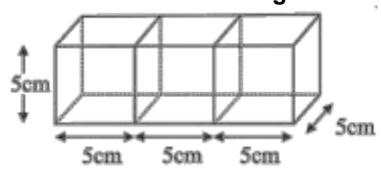
Breadth of the resulting cuboid, $\mathbf{b} = \mathbf{5\ cm}$.

Height of the resulting cuboid, $\mathbf{h} = \mathbf{5\ cm}$.

The surface area of the cuboid $= \mathbf{2(lb + bh + lh)}$

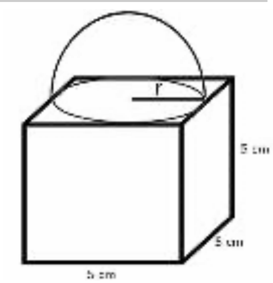
$= \mathbf{2(15 \times 5 + 5 \times 5 + 5 \times 15)}$

$= \mathbf{2(75 + 25 + 75) = 2(175) = 350\ cm^2}$.



Example:

A block is made up of two solids such as cube and hemisphere as shown in the figure. The base of the block is a cube with an edge 5 cm and the hemisphere is fixed on the top such that it touches the midpoints of the sides of the top face of the cube. Find the total surface area of a block (Take π = 22/7).



Solution: Edge of the cube, a = 5 cm. The total surface area of the cube = $6a^2$ square units.

TSA of cube = $6(5)^2 = 6(25) = 150\ \mathrm{cm}^2$.

From the given figure, it is observed that a hemisphere is attached to the face of the cube.

Hence, that part of the face of the cube is not included in the surface area.

Therefore, the formula to calculate the surface area of the block is given as:

Surface Area of the block = TSA of cube – Base area of Hemisphere + CSA of hemisphere.

Surface area of the block = $150 - \pi r^2 + 2\pi r^2 = 150 + \pi r^2$

= 150 + [(22/7)(2.5)(2.5)] [Since, the diameter of the hemisphere is 5 cm]

= $150 + 19.64 = 169.64\ \mathrm{cm}^2$

Therefore, the surface area of the block is $169.64\ \mathrm{cm}^2$.

Example:

A capsule is formed like a cylinder with two hemispheres attached to each end. What is the surface area of the capsule if the length of the capsule is 10 mm and the width is 6 mm?

Solution: Given, the diameter of hemisphere $= \mathbf{6\ mm}$, the radius of hemisphere $= \mathbf{\frac{6}{2} = 3\ mm}$.

Height of the entire capsule $= \mathbf{10\ mm}$,

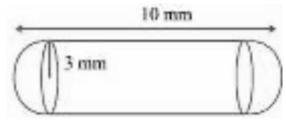
Height of the cylinder $= \mathbf{10 - 6 = 4\ mm}$.

The surface area of the capsule $= \mathbf{2} \times$ Surface area of hemisphere + Surface area of the cylinder

$= \mathbf{2 \times 2\pi r^2 + 2\pi r(r + h)}$

$= \mathbf{4 \times \frac{22}{7} \times 3^2 + 2 \times \frac{22}{7} \times 3(3 + 4)}$

$= \mathbf{113.14 + 132 = 245.14\ mm^2}$.



Volume of a Combination of Solids

Volume is a physical quantity that measures the space occupied by a solid. It is measured in **cubic units**. The volume of a combination of solids is given by adding up the volumes of each individual solid in the combination of solids. Thus, **Volume of the new solid = Sum of the Volume of the individual solids.**

For example: When a solid wooden toy is formed by mounting a right circular cone on a hemisphere, then the volume of the wooden toy is given by,

Volume of wooden toy = Volume of the conical part + Volume of the hemisphere

= Volume of cone + Volume of hemisphere

Example:

A cube is of side 2 cm. Eight of them are joined together to form a bigger cube. Calculate the volume of the bigger cube.

Solution: The volume of a cube can be determined by the following formula: $\mathbf{s^3 = 8\ cm^3}$

The volume of a combination of solids is the sum of the volumes of each individual solid used in making it.

The large cube is a combination of 8 smaller cubes. So, the volume of the large cube can be calculated as:

Volume of the small cube $\times \mathbf{8 = 8 \times 8 = 64\ cm^3}$

Therefore, the volume of the large cube is $\mathbf{64\ cm^3}$.

Example:

An ice-cream cone is of radius 7 cm. It has a total length of 21 cm. Find the volume of the ice-cream held in this cone.

Solution: The volume of a hemisphere is half the volume of a sphere i.e. volume of a hemisphere can be calculated by using the following formula:

$$\left(\frac{2}{3}\right)\pi r^3 = \left(\frac{2}{3}\right) \times \left(\frac{22}{7}\right) \times (7)^3 = \left(\frac{2}{3}\right) \times (22) \times (7)^2 = \left(\frac{2156}{3}\right) = 718.67\ cm^3.$$

We know that the total length of the solid is $\mathbf{21\ cm}$. The radius is $\mathbf{7\ cm}$. The radius is the same even if we take it from top to bottom. So, the height of the cone will be calculated as:

Height of Cone = Total Height - Radius of Hemisphere = $\mathbf{21 - 7 = 14\ cm}$.

The height of the cone is $\mathbf{14\ cm}$. Now the volume of the cone can be calculated using the following formula:

$$\left(\frac{1}{3}\right)\pi r^2 h = \left(\frac{1}{3}\right)\left(\frac{22}{7}\right) \times (7)^2 \times 14 = \left(\frac{1}{3}\right) \times (22) \times 7 \times 14 = \left(\frac{2156}{3}\right) = 718.67\ cm^3.$$

The volume of a combination of solids is the sum of the volumes of each individual solid used in making it.

Volume of Ice-cream cone = Volume of Hemisphere + Volume of Cone.

$$= 718.67\ cm^3 + 718.67\ cm^3 = 1437.34\ cm^3.$$

Therefore, the volume of the ice-cream cone is $\mathbf{1437.34\ cm^3}$.

This is the maximum amount of ice-cream it can hold at any point of time.

Conversion of Solid from One Shape to Another

Each and every solid that exists occupies some volume. When you convert one solid shape into another, its volume remains the same, no matter how different the new shape is i.e., the volume of the original, as well as the new solid, remains the same. If you melt one large cylindrical candle into three small cylindrical candles, the sum of the smaller candles' volumes equals the larger candle's volume, assuming there is no wastage while converting.

Also, Number of new solids obtained by recasting $= \dfrac{\text{Volume of the solid that is melted}}{\text{Volume of the new solid made}}$

Example:

Three solid spheres of radii 1 cm, 6 cm and 8 cm are melted and recast into a single sphere. Find the radius of the larger sphere obtained.

Solution: Volume of larger sphere = sum of the volumes of all the smaller spheres

Let radius of the larger sphere obtained = R cm.

$$\text{So, } \frac{4}{3} \times \pi R^3 = \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3$$

$R^3 = 1 + 216 + 512$

$R = (729)^{1/3}$

R = 9 cm.

Example:

Two solid metallic cuboids with dimensions $15cm \times 8\ cm \times 5\ cm$ and $20\ cm \times 5\ cm \times 4\ cm$, are melted together and re-casted into solid cubes each of side 5 cm. Find the number of solid cubes so formed.

Solution: Dimensions of the first cuboid = $\mathbf{15\ cm \times 8\ cm \times 5\ cm}$

Dimensions of second cuboid = $\mathbf{20\ cm \times 5\ cm \times 4\ cm}$

Side of a cube = 5 cm

Number of solid cubes $= \dfrac{\text{the volume of a first cuboid} + \text{the volume of a second cuboid}}{\text{Volume of the cube}}$

$$= \frac{(15 \times 8 \times 5) + (20 \times 5 \times 4)}{(5 \times 5 \times 5)} = \frac{600 + 400}{125} = \frac{1000}{125} = 8$$

Hence, 8 cubes are formed.

Example:

64 solid iron spheres, each of radius r and surface area S, are melted to form a sphere with surface area S'. Find the:

(i) Radius r' of the new sphere,

(ii) Ratio of S and S'.

Solution: (i) Volume of the bigger solid sphere formed $= 64 \times$ vol. of each solid sphere melted.

$\Rightarrow \frac{4}{3}\pi(r')^3 = 64 \times \frac{4}{3}\pi r^3$.

$\Rightarrow (r')^3 = 64r^3 = (4r)^3 \Rightarrow r' = 4r$.

(ii) $S =$ surface area of each sphere melted $= 4\pi r^2$.
and, $S' =$ Surface area of the sphere formed $= 4\pi(r')^2 = 4\pi(4r)^2 = 64\pi r^2$.

$\therefore$ The ratio of S and $S' = \frac{S}{S'} = \frac{4\pi r^2}{64\pi r^2} = \frac{1}{16} = 1:16$.

Check Your Concept – 2

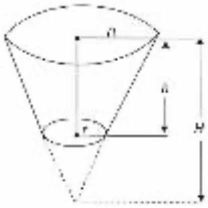
(i) Three cubes of sides 3 cm, 4 cm and 5 cm respectively, are melted and formed into a larger cube. What is the side of the cube formed?

(ii) Fifteen identical spheres are made by melting a solid cylinder of radius 10 cm and height 5.4 cm. Find the diameter of each sphere.

(iii) Find the number of soaps of size 2.1 cm × 3.7 cm × 2.5 cm that can be put in a cuboidal box of size 6.3 cm × 7.4 cm × 5 cm.

(iv) A cylinder-shaped tank is surmounted by a cone of equal radius. The height of the cone is 6 m and the total height of the tank is 18 m. Find the volume of the tank if the base radius of the cylinder is 5 m.

Cone Frustum (or a Conical Bucket)



If a right circular cone is cut by a plane perpendicular to its axis (i.e., a plane parallel to the base), then the solid portion containing the base of the cone is called the **frustum** of the cone. From the figure, we observe that a frustum is in the shape of a bucket. Let,
Radius of upper base be R, Radius of the lower base $= r$.
Height of frustum $= h$, Slant height of frustum $= l$.

1. Curved surface area of a frustum $= \pi l(R + r)$.
2. Total surface area of a frustum = Curved surface area + Area of upper base + Area of lower base $= \pi l(R + r) + \pi r^2 + \pi R^2$.
3. Volume of a frustum $= \frac{1}{3}\pi h(R^2 + Rr + r^2)$.
4. Slant height (l) of a frustum $= \sqrt{(R - r)^2 + h^2}$.

Example:

A joker's cap is in the form of a cone of radius 7 cm and height 24 cm. Find the area of the cardboard required to make the cap.

Solution: Area of the cardboard required = Curved surface area of the cap (or cone) $= \pi rl$.

Now, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm.

$\Rightarrow$ Curved surface area $= \frac{22}{7} \times 7 \times 25 = 550\ cm^2$

$\therefore$ Area of the cardboard required $= 550\ cm^2$.

Example:

The diameter of an ice-cream cone is 7 cm and its height is 12 cm. Find the volume of ice-cream that the cone can contain.

Solution: Volume of ice-cream $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 12 = 22 \times 7 = 154\ cm^3$.

Example:

The diameters of the top and bottom portions of a milk can are 56 cm and 14 cm respectively. The height of the can is 72 cm. Find the:
(a) Area of metal sheet required to make the can (without lid).
(b) Volume of milk which the container can hold.

Solution: The milk can is in the shape of a frustum with $R = 28$ cm, $r = 7$ cm and $h = 72$ cm.

(a) Area of metal sheet required = Curved surface area + Area of bottom base

$= \pi l(R + r) + \pi r^2$

$= 3\sqrt{49 + 576} = 3 \times \sqrt{625} = 3 \times 25 = 75$ cm.

$\therefore$ Area of metal sheet $= \frac{22}{7} \times 75(28 + 7) + \frac{22}{7} \times 7^2$

$= 22 \times 75 \times 5 + 22 \times 7$

$= 22(375 + 7)$

$= 22(382) = 8404\ cm^2$.

(b) Amount of milk which the container can hold $= \frac{1}{3}\pi h(R^2 + Rr + r^2)$

$= \frac{1}{3} \times \frac{22}{7} \times 72(28^2 + 7 \times 28 + 7^2)$

$= \frac{22}{7} \times 24(7 \times 4 \times 28 + 7 \times 28 + 7 \times 7)$

$= \frac{22}{7} \times 24 \times 7(112 + 28 + 7)$

$= 22 \times 24 \times (147) = 77616\ cm^3$.

Solved Examples

(1) A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely? [CBSE Delhi 2014]

Solution: Given, Diameter of $tank = 10\ m$

Depth of $tank(H) = 2\ m$

Internal diameter of pipe $= 20\ cm = \frac{2}{10}\ m$

Rate of flow of water, $v = 4\ km/h = 4,000\ m/h$

Internal radius of the pipe, $r = \frac{1}{10}\ m$

Let ' t ' be the time taken to fill the tank.

$\therefore$ Water flowing through a pipe in t hours = Volume of tank

$\pi r^2 \times v \times t = \pi R^2 H$

$\frac{1}{10} \times \frac{1}{10} \times 4,000 \times t = 5 \times 5 \times 2$

$t = \frac{5 \times 5 \times 2}{40} = \frac{5}{4} = 1\frac{1}{4}$ hours.

(2) A hemispherical bowl of internal diameter $36\ cm$ contains liquid. This liquid is filled into 72 cylindrical bottles of diameter $6\ cm$. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

Solution: Radius of hemispherical bowl, $R = \frac{36}{2} = 18\ cm$ *[CBSE (AI) 2015]*

Radius of the cylindrical bottle, $r = 3\ cm$

Let the height of the cylindrical bottle $= h$

Since 10% of liquid is wasted, therefore only 90% liquid is filled into 72 cylindrical bottles.

$\therefore$ Volume of 72 cylindrical bottles $= 90\%$ of the volume in the bowl

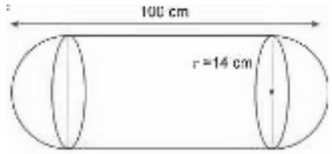
$\Rightarrow 72 \times \pi r^2 h = 90\%$ of $\frac{2}{3}\pi R^3$

$72 \times \pi \times 3 \times 3 \times h = \frac{90}{100} \times \frac{2}{3} \times \pi \times 18 \times 18 \times 18$

$h = \frac{90 \times 2 \times \pi \times\times 18 \times 18 \times 18}{100 \times 3 \times \pi \times 72 \times 3 \times 3}$

$h = \frac{27}{5} = 5.4\ cm$.

(3) A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is $100\ cm$ and the diameter of the hemispherical ends is $28\ cm$. Find the cost of polishing the surface of the solid at the rate of 5 paise per sq. cm.



Solution: We have,

$r =$ radius of cylinder $=$ radius of hemispherical ends $= \frac{28}{2}\ cm$

$h =$ height of the cylinder $= 100 - 2 \times 14 = 100 - 28 = 72\ cm$.

Total surface area

$=$ curved surface area of cylinder $+ 2 \times$ surface area of hemispherical ends

$= 2\pi rh + 2 \times (2\pi r^2)$

$= 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 14(72 + 28)$

$= 2 \times \frac{22}{7} \times 14 \times 100 = 8800\ cm^2$

Rate of polishing $= 5$ paise per sq. cm, Cost of polishing the surface $= \frac{8800 \times 5}{100} = ₹440$.

(4) A hemispherical tank, of diameter $3\ \text{m}$, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half-empty? (Use $\pi = \frac{22}{7}$) [CBSE (F) 2016]

Solution: Radius of hemispherical tank $= \frac{3}{2}\ \text{m} = 150\ \text{cm}$

Volume of water in the hemispherical $\textbf{tank} = \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 150 \times 150 \times 150\ \text{cm}^3$

Volume of water to be emptied $= \frac{1}{2} \times \frac{2}{3} \times \frac{22}{7} \times \frac{150\times150\times150}{1000}$ L

Time taken to empty the tank$= \frac{22}{7} \times \frac{5\times15\times15\times7}{60\times25}$ min

$= \frac{33}{2}\text{minutes} = 16\frac{1}{2}\text{minutes}.$

(5) The radius and height of a solid right circular cone are in the ratio of $5:12$. If its volume is $314\ \text{cm}^3$, find its total surface area. [Take $\pi = 3.14$] [CBSE (F) 2017]

Solution: Given $r : h = 5:12$

Let $r = 5x$ and $h = 12x$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$314 = \frac{1}{3} \times 3.14(5x)^2 \times 12x$

$\Rightarrow x^3 = \frac{314 \times 3}{3.14 \times 25 \times 12} \Rightarrow x^3 = 1$

$\Rightarrow x = 1$

Total surface area of a right circular cone, $A = \pi r\,(r + \sqrt{h^2 + r^2})$

$= 3.14 \times 90 = 282.6\ \text{cm}^2.$

(6) A circus tent is in the shape of a cylinder, up to a height of $8\ \text{m}$, surmounted by a cone of the same radius $28\ \text{m}$. If the total height of the tent is $13\ \text{m}$, find:

(i) Total inner curved surface area of the tent.

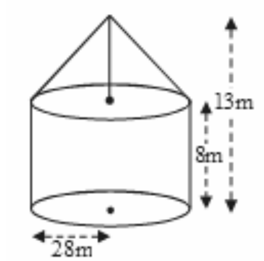
(ii) Cost of painting its inner surface at the rate of Rs. 3.50 per m^2.

Solution: According to the given statement, the rough sketch of the circus tent will be as shown:

(i) For the cylindrical portion: $r = 28\ \text{m}$ and $h = 8\ \text{m}$

$\therefore$ Curved surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 28 \times 8\ \text{m}^2 = 1408\ \text{m}^2$



For conical portion: $r = 28\ \text{m}$ and $h = 13\ \text{m} - 8\ \text{m} = 5\ \text{m}$

$\therefore l^2 = h^2 + r^2 \Rightarrow l^2 = 5^2 + 28^2 = 809$

$\Rightarrow l = \sqrt{809}\ \text{m} = 28.4\ \text{m}$

$\therefore$ Curved surface area $= \pi r l$

$= \frac{22}{7} \times 28 \times 28.4\ \text{m}^2 = 2499.2\ \text{m}^2$

$\therefore$ Total inner curved surface area of the tent.

= C.S.A. of the cylindrical portion + C.S.A. of the conical portion

$= 1408\ \text{m}^2 + 2499.2\ \text{m}^2 = 3907.2\ \text{m}^2$

(ii) Cost of painting the inner surface $= 3907.2 \times$ Rs. $3.50 =$ Rs. 13675.20

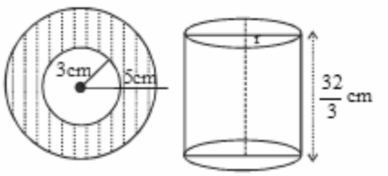
(7) The radius of the internal and external surface of a metallic spherical shell are $3\ \text{cm}$ and $5\ \text{cm}$ respectively. It is melted and re-casted into a solid right circular cylinder of height $10\frac{2}{3}$ cm. Find the diameter of the base of the cylinder.

Solution: Here, radius of the internal and external surfaces of a metallic spherical shell are $3\ \text{cm}$ and $5\ \text{m}$ respectively.

So, its volume $= \left[\frac{4}{3}\pi(5^3 - 3^3)\right]\text{cm}^3 = \left[\frac{4}{3}\pi \times (125 - 27)\right]\text{cm}^3 = \left(\frac{4}{3}\pi \times 98\right)\text{cm}^3.$

Let r be the radius of the right circular cylinder of height $\frac{32}{3}$ cm.

Its volume $= \pi r^2 h = \left(\pi \times r^2 \times \frac{32}{3}\right)\text{cm}^3.$



We have, Volume of the spherical shell = volume of the right circular cylinder

$\Rightarrow \frac{4}{3}\pi \times 98 = \pi \times r^2 \times \frac{32}{3}$

$\Rightarrow 392 = 32r^2 \Rightarrow r^2 = \frac{392}{32} = \frac{49}{4} \Rightarrow r = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5\ \text{cm}.$

Hence, the diameter of the right circular cylinder $= 2r = 2 \times 3.5\ \text{cm} = 7\ \text{cm}.$

(8) A hemispherical bowl of internal radius 15 cm is full of a liquid. The liquid is to be filled into some bottles of cylindrical shape whose diameters and heights are 5 cm and 6 cm respectively. Find the number of bottles necessary to empty the bowl.

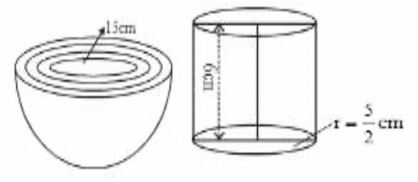
Solution: We have the internal radius of the hemispherical bowl = R = 15 cm.

So, its volume $= \frac{2}{3}\pi \times R^3 = \left[\frac{2}{3} \times \pi \times (15)^3\right] cm^3 = \left(\frac{2}{3} \times \pi \times 15 \times 15 \times 15\right) cm^3$

$= 10 \times 15 \times 15 \times \pi\ cm^3 = 2250\pi\ cm^3$

So, volume of the entire liquid $= 2250\pi\ cm^3$

The liquid is to be filled into some bottles of cylindrical shape whose diameters and heights are 5 cm and 6 cm respectively.



So, radius of the cylindrical bottle = $\frac{5}{2}$ cm and height of it $= 6$ cm

So, volume of one cylindrical bottle $= \pi r^2 h$

$= \left(\pi \times \frac{5}{2} \times \frac{5}{2} \times 6\right) cm^3 = \left(\frac{75\pi}{2}\right) cm^3$

So, the number of bottles necessary to empty the hemispherical bowl

$= \frac{\text{Volume of the entire liquid in the bowl}}{\text{Volume of one cylindrical bottle}} = \frac{2250\ cm^3}{\frac{75\pi}{2}\ cm^3} = \frac{2250 \times 2}{75\pi} = 60.$

(9) A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Solution: Volume of water that flows through a river, canal or pipe, etc., in unit time

= Area of cross-section × Speed of water through it.

Since the area of cross-section of the river = depth × width

$= 3\ m \times 40\ m = 120\ m^2$

And, speed of flow of water through the river

$= 2\ km/hr = 2 \times \frac{5}{18}\ m/s = \frac{5}{9}\ m/s$

∴ Volume of water that flows through it in 1 second.

= Area of cross-section × speed of water through it $= 120 \times \frac{5}{9} = \frac{200}{3}\ m^3/sec$

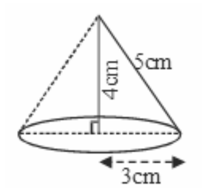
⇒ Volume of water that flows through it in 1 min. (60 sec.) $= \frac{200}{3} \times 60 = 4000\ m^3$

∴ Volume of water that will fall into the sea in a minute. $= 4000\ m^3$.

(10) A right triangle ABC has sides 3 cm, 4 cm and 5 cm. Find the:

(i) Volume of the solid obtained by revolving △ ABC about the side 4 cm.

(ii) Volume of the solid obtained by revolving △ ABC about the side 3 cm.



Solution: Since, $3^2 + 4^2 = 5^2$ ⇒ Angle opp. to 5 cm is a right angle

(i) When the Δ is revolved about the side of 4 cm, a cone is formed.

For the cone formed: $h = 4$ cm and $r = 3$ cm

∴ Volume of solid obtained $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 4 = 37.71\ cm^3$

(ii) When the Δ is revolved about the side of 3 cm, for the cone formed: $h = 3$ cm and $r = 4$ cm.

∴ Volume of solid obtained $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3\ cm^3 = 50.28\ cm^3$.

(11) Water flows out of a circular pipe, whose internal diameter is 2 cm, at the rate of 0.7 m/sec into a cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in half an hour?

Solution: We have volume of water that flows out through a circular pipe in 1 second = volume of a cylinder of the base of radius 1 cm $\left(r = \frac{2}{2} = 1\ cm\right)$ and height 70 cm ($h = 0.7\ m = 70\ cm$)

$= \pi r^2 h = \left(\frac{22}{7} \times 1^2 \times 70\right) cm^3 = 220\ cm^3$

So, volume of water passing through the pipe into the cylindrical tank in 1800 seconds

$= (220 \times 1800) cm^3 = 396000\ cm^3$ $\left(\frac{1}{2}\text{ hour } = \frac{3600}{2} = 1800 sec\right)$

Thus, a rise in the level of water in 1800sec or half an hour

$= \frac{\text{Total volume of water poured int o the cylindrical tank}}{\text{Area of the base of the cylindrical tank}} = \frac{396000\ cm^3}{\pi \times 40^2\ cm^2}$

$= \left(\frac{396000\ cm^3}{\frac{22 \times 1600}{7}\ cm^2}\right) = \left(\frac{396000 \times 7}{22 \times 1600}\right) cm$

$= 78.75\ cm \cong 79\ cm$

Hence, the water rises up to 79 cm in half an hour.

(12) Find the volume of a solid cube of side 12 cm. If this cube is cut into 8 identical cubes, find :
(i) Volume of each small cube.
(ii) Side of each small cube.
(iii) Surface area of each small cube.

Solution: Since, the side (edge) of the given solid cube $= \mathbf{12\ cm}$.

∴ Volume of given solid cube $= (\text{ edge })^3 = (\mathbf{12\ cm})^3 = \mathbf{1728\ cm^3}$.

(i) As the given cube is cut into 8 identical cubes.

⇒ Vol. of 8 small cubes obtained = Vol. of given cube $= \mathbf{1728\ cm^3}$.

⇒ Volume of each small cube $= \frac{1728}{8} = \mathbf{216\ cm^3}$

(ii) If edge (side) of each small cube $= \mathbf{x\ cm}$

∴ Volume = $(\text{edge})^3$

$\Rightarrow \mathbf{x^3 = 216 = 6 \times 6 \times 6 = 6^3 \Rightarrow x = 6\ cm}$

∴ Side of each small cube $= \mathbf{6\ cm}$.

(iii) Surface area of each small cube $= \mathbf{6} \times (\text{edge})^2$

$= \mathbf{6 \times (6\ cm)^2 = 216\ cm^2}$.

(13) A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by a canvas to protect it from rain. Find the area of the canvas required.

Solution: For the conical heap :

Radius $(\mathbf{r}) = \frac{10.5}{2}\ \mathbf{m} = \mathbf{5.25\ m}$ and height $(\mathbf{h}) = \mathbf{3\ m}$.

∴ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3\ m^3 = 86.625\ m^3$

Now, $l^2 = h^2 + r^2$

$\Rightarrow l^2 = (3)^2 + (5.25)^2$

$= 9 + 27.5625 = 36.5625$

$\Rightarrow l = \sqrt{36.5625}\ m = 6.047\ m$

∴ Area of canvas required = curved surface area of the conical heap

$= \pi r l = \frac{22}{7} \times 5.25 \times 6.047\ m^2 = 99.7755\ m^2$

(14) Two spheres of same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere. [CBSE (F) 2015]

Solution: Volume of smaller sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

Volume of smaller sphere × density = mass

∴ 36π (density of metal) $= 1$

Density of metal $= \frac{1}{36\pi}$

∴ Volume of bigger sphere × density = mass

$\frac{4}{3}\pi(R)^3 \times \frac{1}{36\pi} = 7$

$R^3 = \frac{7 \times 36 \times 3}{4} = 7 \times 9 \times 3$

Volume of new sphere = volume of smaller sphere + volume of bigger sphere

$\frac{4}{3}\pi(R')^3 = \frac{4}{3}\pi r^3 + \frac{4}{3}\pi R^3$ (where $\mathbf{R'}$ is the radius of new sphere)

$\frac{4}{3}\pi(R')^3 = \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi(7 \times 9 \times 3)$ [using $(\mathbf{i})$]

$\frac{4}{3}\pi(R')^3 = \frac{4}{3}\pi[3^3 + 7 \times 9 \times 3]$

$(R')^3 = [3^3 + 7 \times 3^3]$

$(R')^3 = 3^3(1 + 7)$

$(R')^3 = 3^3 \times 8$

$(R')^3 = 3^3 \times 2^3$

$R' = 3 \times 2$

$R' = 6\ cm$

∴ Diameter of new sphere $= \mathbf{12\ cm.}$

(15) The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Solution: The capacity of the cuboidal tank is 50000 litres of water.

$\Rightarrow$ Volume of the tank $= 50000$ litres

$= \frac{50000}{1000}$ m^3 [$\because 1\ m^3 = 1000$ litres]

$= 50\ m^3$

$\Rightarrow$ Length of the tank $\times$ its breadth $\times$ its height $= 50\ m^3$

$\Rightarrow 2.5\ m \times$ breadth $\times 10\ m = 50\ m^3$

$\Rightarrow$ Breadth $= \frac{50}{2.5 \times 10}$ $m = 2\ m$.

Exercise

OBJECTIVE TYPE QUESTIONS

(1) A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find the total surface area. (use $\pi = 3\frac{1}{7}$)

(A) $241.5\ cm^2$ (B) $214.5\ cm^2$
(C) $412.5\ cm^2$ (D) $124.5\ cm^2$

(2) A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of the axis length 9 cm. Determine the capacity of the tank.

(A) $8136\ cm^3$ (B) $8163\ cm^3$
(C) $8316\ cm^3$ (D) $8631\ cm^3$

(3) If a solid sphere of radius 6 cm is melted and drawn into a wire of radius 0.2 cm, then the length of the wire is.

(A) 72 m (B) 75 m
(C) 72 cm (D) 75 cm

(4) A cone is 8.4 cm. high and the radius of its base is 2.1 cm. It is melted and re-cast into a sphere. Find the radius of the sphere.

(A) $r = 2.1$ cm (B) $r = 2.4$ cm
(C) 1.9 cm (D) 2.5 cm

(5) A boiler in the form of a cylinder 2 m long with hemispherical ends each of 2 m diameter, find the volume of the boiler.

(A) $10\frac{10}{21}\ m^3$ (B) $10\frac{20}{21}\ m^3$
(C) $9\frac{11}{21}\ m^3$ (D) $11\frac{19}{21}\ m^3$

(6) How many lead balls, each of radius 1 cm can be made from a sphere whose radius is 8 cm?

(A) 510 (B) 512
(C) 480 (D) 250

(7) The circular ends of a bucket are of radii 35 cm and 14 cm and the height of the bucket is 40 cm. Find the volume of the bucket.

(A) $80010\ cm^3$ (B) $80000\ cm^3$
(C) $80080\ cm^3$ (D) $79000\ cm^3$

(8) If the radii of the circular ends of a conical bucket, which is 45 cm high are 28 cm and 7 cm. Find the capacity of the bucket.

(A) $48500\ cm^3$ (B) $48510\ cm^3$.
(C) $48310\ cm^3$ (D) $48200\ cm^3$

(9) The number of solid spheres, each of diameter 6 cm that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm is:

(A) 3 (B) 4
(C) 5 (D) 6

(10) A solid sphere of radius r is melted and cast into the shape of a solid cone of height r, then the radius of the base of the cone is:

(A) 2r (B) 3r
(C) r (D) 4r

(11) The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is:

(A) 12 (B) 18
(C) 36 (D) 40

(12) A cylinder-shaped tank is surmounted by a cone of equal radius. The height of the cone is 6 m and the total height of the tank is 18 m. Find the volume of the tank if the base radius of the cylinder is 5 m.

(A) 1650 m^3 (B) 1244 m^3
(C) 1100 m^3 (D) 2200 m^3

(13) A right circular conical tent is such that the angle at its vertex is 60° and its base radius is 14 m. Find the cost of the canvas required to make the tent at the rate of 25 per m^2.

(A) Rs. 15,400 (B) Rs. 30,800
(C) Rs. 16,400 (D) Rs. 32,800

(14) A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone is $\mathbf{6\ cm}$ and its height is $\mathbf{4\ cm}$. Calculate the surface area of the toy.

(A) $\mathbf{102.58\ cm^2}$ (B) $\mathbf{103.62\ cm^2}$
(C) $\mathbf{105.28\ cm^2}$ (D) $\mathbf{101.25\ cm^2}$

(15) 50 circular plates, each of radius $\mathbf{7\ cm}$ and thickness $\mathbf{1/2\ cm}$ are placed on top of each other to form a solid right circular cylinder. Find the total surface area and the volume of the cylinder so formed.

(A) $\mathbf{1400\ cm^2, 3850\ cm^3}$ (B) $\mathbf{1408\ cm^2, 3850\ cm^3}$
(C) $\mathbf{1432\ cm^2, 3800\ cm^3}$ (D) $\mathbf{1408\ cm^2, 3800\ cm^3}$

(16) A solid sphere of radius $\mathbf{x\ cm}$ is melted and cast into the shape of a solid cone of height $\mathbf{x\ cm}$, the radius of the base of the cone is: **[NTSE-2012]**

(A) $\mathbf{x}$ cm (B) $\mathbf{4x\ cm}$
(C) $\mathbf{3x\ cm}$ (D) $\mathbf{2x\ cm}$

(17) A copper rod of diameter $\mathbf{1\ cm}$ and length $\mathbf{8\ cm}$ is drawn into a wire of length $\mathbf{18\ m}$ of uniform thickness. The thickness of the wire is **[NTSE-2014]**

(A) $\mathbf{0.67\ mm}$ (B) $\mathbf{1/30\ cm}$
(C) $\mathbf{0.5\ mm}$ (D) $\mathbf{0.7\ cm}$

(18) A solid metallic block of volume one cubic metre is melted and re-casted into the form of a rectangular bar of length 9 metres having a square base. If the weight of the block is $\mathbf{90\ kg}$ and the biggest cube is cut off from the bar, then the weight of the cube is:

(A) $\mathbf{6\frac{1}{3}\ kg}$ (B) $\mathbf{5\frac{2}{3}\ kg}$
(C) $\mathbf{4\frac{2}{3}\ kg}$ (D) $\mathbf{3\frac{1}{3}\ kg}$

(19) A cylindrical pencil sharpened at one edge is a combination of **[NCERT Exemplar]**

(A) A cone and a cylinder (B) Frustum of a cone and cylinder
(C) A hemisphere and cylinder (D) Two cylinders

(20) A Surahi is the combination of **[NCERT Exemplar]**

(A) A sphere and cylinder (B) Frustum and a cylinder
(C) Two hemisphere (D) A cylinder and a cone

(21) A spherical ball of lead, $\mathbf{3\ cm}$ in diameter is melted and re-casted into three spherical balls. The diameter of two of these balls is $\mathbf{1.5\ cm}$ and $\mathbf{2\ cm}$ respectively. The diameter of the third ball is: **[IMO 2021-22]**

(A) $\mathbf{2.66\ cm}$ (B) $\mathbf{2.5\ cm}$
(C) $\mathbf{3\ cm}$ (D) $\mathbf{3.5\ cm}$

(22) A metallic sheet of rectangular shape have dimensions $\mathbf{48\ cm} \times \mathbf{36\ cm}$. From each one of its corners, a square of $\mathbf{8\ cm}$ is cut off. An open box is made up from the remaining sheet. Find the volume of the box. **[IMO 2020-21]**

(A) $\mathbf{4280\ cm^3}$ (B) 2050 $\mathbf{cm^3}$
(C) $\mathbf{5120\ cm^3}$ (D) $\mathbf{4690\ cm^3}$

(23) A spherical ball of radius 3 cm is melted and re-casted into three spherical balls. The radii of two of these balls are 1.5 cm and 2 cm. The radius of the third ball is

(A) 1.5 cm (B) 2 cm
(C) 3 cm (D) 2.5 cm

(24) A circus tent is cylindrical up to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, the total area of canvas required to build the tent is

(A) 7920 m^2 (B) 7820 m^2
(C) 9720 m^2 (D) 2645 m^2

(25) The material of a cone is converted into the shape of a cylinder of equal radius. If the height of the cylinder is $\mathbf{5\ cm}$, then the height of the cone is

(A) 10 cm (B) 15 cm
(C) 18 cm (D) 24 cm

Answer Key

CHECK YOUR CONCEPT

(1)	(i) $\mathbf{256\ cm^2, 8\sqrt{3}cm}$	(ii) $\mathbf{4.5\ units}$	(iii) 3 : 2	(iv) 95%
(2)	(i) 6 cm	(ii) 6 cm	(iii) 12	(iv) 1100 m^3

OBJECTIVE TYPE QUESTIONS

(1)	(B)	**(6)**	(B)	**(11)**	(C)	**(16)**	(D)	**(21)**	(B)
(2)	(C)	**(7)**	(C)	**(12)**	(C)	**(17)**	(B)	**(22)**	(C)
(3)	(A)	**(8)**	(B)	**(13)**	(B)	**(18)**	(D)	**(23)**	(D)
(4)	(A)	**(9)**	(C)	**(14)**	(B)	**(19)**	(A)	**(24)**	(A)
(5)	(A)	**(10)**	(A)	**(15)**	(B)	**(20)**	(A)	**(25)**	(B)