

LAWS OF MOTION

1. First law of motion

According to this law, 'every body continues to remain in its state of rest or uniform in a straight line unless it is compelled by an external force to change that state'.

- (i) This law is also called the **law of inertia**. Inertia is a property by virtue of which a body opposes the change in the state of rest or motion.
- (ii) Force is such a factor, which is essential for change in translatory motion of a body.
- (iii) The first law of motion defines the force.

Examples :

- (a) The dust particles get detached from a cloth by shaking it.
- (b) Forward jerk to the passengers, sitting in a bus on applying sudden brakes.

2. Second law of motion

According to this law, 'the rate of change of momentum (mass × velocity) of a body is proportional to the Force applied and changes take place in the direction of the force'.

Mathematically $\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow \vec{F} = k \frac{d\vec{p}}{dt} = \frac{d\vec{p}}{dt}$ (Defining unit of force in such a way that $k=1$)

$$\vec{F} = \frac{d}{dt} (m \vec{v}) = m \frac{d\vec{v}}{dt} \text{ (if mass is constant)}$$

$$\boxed{\vec{F} = m \vec{a}}$$

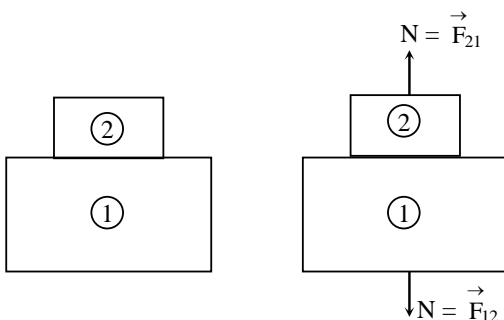
In scalar form, $F = ma$

3. Third law of motion

According to this law, "Every action has its equal and opposite reaction".

When two bodies A and B exert force on each other, the force (action) of A on B $\left(\vec{F}_{BA}\right)$, is always equal and opposite to the force of B on A $\left(\vec{F}_{AB}\right)$.

Thus, $\vec{F}_{AB} = -\vec{F}_{BA}$



(i) This law expresses the nature of force.

(ii) Action and reaction always acts on different bodies $\vec{F}_{12} = -\vec{F}_{21}$

4. Impulse

If a force \vec{F} acts on a body for a short duration Δt , then impulse is defined as product of force and its time of action.

i.e. Impulse = Force \times Duration

$$\Rightarrow \Delta \vec{p} = \vec{F} \times \Delta t$$

By Newton's second law,

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

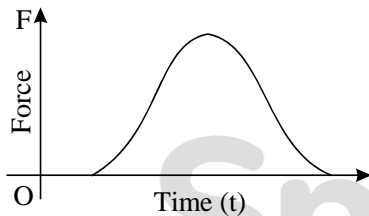
$$\vec{F} \Delta t = \Delta \vec{p} = \vec{p}_f - \vec{p}_i$$

Where \vec{p}_i and \vec{p}_f are initial and final momenta of the body respectively.

Thus impulse of force $= \vec{F} \Delta t = \Delta \vec{p} = \vec{p}_f - \vec{p}_i$

Graphically, impulse is the area under the force vs time graph, as shown in the figure.

The SI unit of impulse is Ns.



5. Problem solving strategy

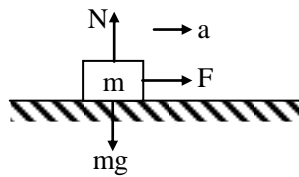
- (1) Make simple sketch of different bodies under consideration.
- (2) Identify the forces acting on the bodies. Draw arrows on your sketch to show direction of each force acting on the body, in other words, make the **free body diagram**.
- (3) Choose a coordinate system and resolve the forces into components that are parallel to the coordinate axes.
- (4) Assume the direction of acceleration of each block and relate them either logically, for simpler problems or by using Constraint Relation technique for complicated problems.
- (5) Use Newton's 2nd law and frame equations along each axis for all the bodies.
- (6) Solve the equation for the required unknowns.

Now we will deal with various simple situations and subsequently discuss the constraint relation technique in the next article.

A. Motion of a Block on a Horizontal Smooth Surface.

Case (i): When subjected to a horizontal pull

The distribution of forces on the body are shown in the image.

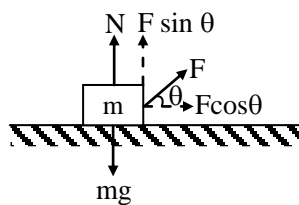


As there is no motion along vertical direction, hence, $N = mg$

For horizontal motion $F = ma$ or $a = F/m$

Case (ii): When subjected to a pull acting at an angle (θ) to the horizontal

Now 'F' has to be resolved into two components, ' $F\cos\theta$ ' along the horizontal and ' $F\sin\theta$ ' along the vertical direction.



For no motion along the vertical direction,

we have $N + F \sin \theta = mg$

or $N = mg - F \sin \theta$

Note:

Hence $N \neq mg$, $N < mg$

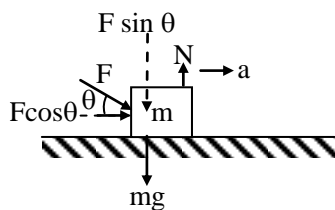
For horizontal motion

$$F \cdot \cos \theta = ma, \quad a = \frac{F \cos \theta}{m}$$

Case(iii) : When the block is subjected to a push acting at an angle θ to the horizontal: (downward)

The force equation in this case is

$$N = mg + F \sin \theta$$



Note: $N \neq mg$, $N > mg$

For horizontal motion

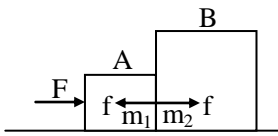
$$F \cdot \cos \theta = ma,$$

$$a = \frac{F \cos \theta}{m}$$

B. Motion of bodies in contact.

Case (i): Two body system

Let a force 'F' be applied on mass 'm₁'.



Free body diagrams:

Net horizontal force $F - f = m_1 a$	Net horizontal force $f = m_2 a$

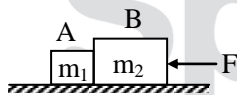
$$\Rightarrow a = \frac{F}{m_1 + m_2} \text{ and } f = \frac{m_2 F}{m_1 + m_2}$$

(i) Here 'f' is known as force of contact.

(ii) Acceleration of system can be found simply by

$$a = \frac{\text{force}}{\text{total mass}}$$

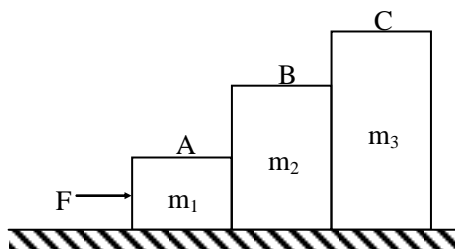
Note:



If force F be applied on m₂, the acceleration will remain the same, but force of contact will be different i.e.

$$f' = \frac{m_1 F}{m_1 + m_2}$$

Case (ii): Three body system



Free body diagrams:

For A	For B	For C
$F - f_1 = m_1 a$	$f_1 - f_2 = m_2 a$	$f_2 = m_3 a$

$$\Rightarrow a = \frac{F}{m_1 + m_2 + m_3}$$

and
$$f_1 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

$$f_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

Where, f_1 = contact force between masses m_1 and m_2 .

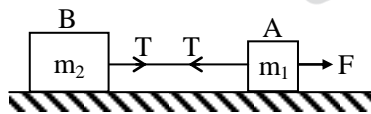
f_2 = contact force between masses m_2 and m_3 .

Remember:

Contact force is different if force 'F' will be applied on mass m_3 .

C. Motion of connected bodies

Case (i): For Two Bodies



'F' is the pull on body A of mass ' m_1 '. The pull of A on B is exercised as tension through the string connecting

A and B. The value of tension throughout the string is 'T' only. But this manifests as a pull 'T' on B and reaction pull 'T' on A.

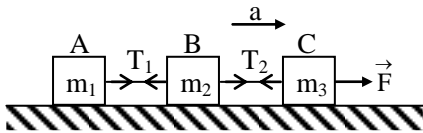
Note: The string is massless

Free body diagrams:

For body A	For body B
$N_1 = m_1 g$	$N_2 = m_2 g$
$F - T = m_1 a$	$T = m_2 a$

$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

Case (ii): For Three bodies



Free body diagrams:

For A	For B	For C
$N_1 = m_1g$	$N_2 = m_2g$	$N_3 = m_3g$
$T_1 = m_1a$	$T_2 - T_1 = m_2a$ $\Rightarrow T_2 = m_2a + T_1$ $T_2 = (m_2 + m_1)a$	$F - T_2 = m_3a$ $\Rightarrow F = m_3a + T_2$ $= m_3a + (m_1 + m_2)a$ $F = (m_1 + m_2 + m_3)a$

$$\Rightarrow a = \frac{F}{m_1 + m_2 + m_3}$$

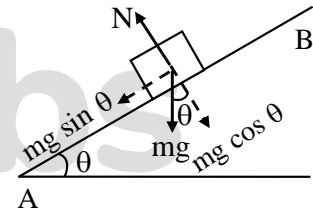
D. Motion of a body on a stationary smooth inclined plane.

A body is placed on a smooth inclined plane AB which makes an angle θ with the horizontal. The forces acting on body are:

- (i) Weight of the body ' mg ' acting vertically down wards.
- (ii) Normal reaction ' N ' acting perpendicular to the inclined plane.

The weight ' mg ' of the body is resolved into two components.

- (1) $mg \sin \theta$ (Parallel to the plane)
- (2) $mg \cos \theta$ (Perpendicular to the plane)



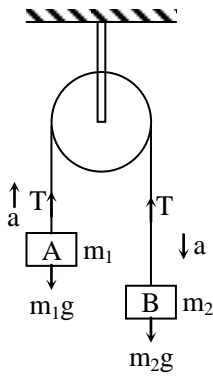
$$\text{Thus } ma = mg \sin \theta \Rightarrow a = g \sin \theta \quad \dots(i)$$

$$N = mg \cos \theta \quad \dots(ii)$$

E. Motion of two bodies connected by a string

Case (i): Motion of unequal masses suspended from a light frictionless pulley

A and B are two bodies of mass ' m_1 ' and ' m_2 ' respectively suspended by means of a light string passing over a smooth pulley P.



Let $m_2 > m_1$. If the string is light and continuous a tension 'T' exists all along the string.

The forces acting on A and B are clearly shown. Let A moves up with an acceleration 'a' and B move down with the same magnitude of acceleration 'a'.

For the motion of A,

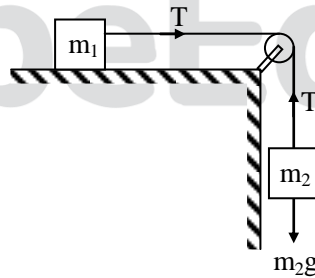
$$T - m_1g = m_1a \quad \dots(i)$$

For the motion of B

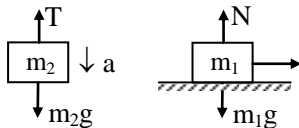
$$m_2g - T = m_2a \quad \dots(ii)$$

$$\text{Solving, } a = \frac{(m_2 - m_1)}{(m_1 + m_2)}g \text{ and } T = \frac{2m_1m_2}{m_1 + m_2}g$$

Case (ii) : Let us consider the case of a body of mass (m_1), to which a light string is attached and it rests on a smooth horizontal plane. The string passes over a frictionless pulley fixed at the end of plane. Another end of the string carries a mass (m_2) as shown in fig. Our aim is to calculate the acceleration of the system and tension in the string



Free body diagram



(i)

(ii)

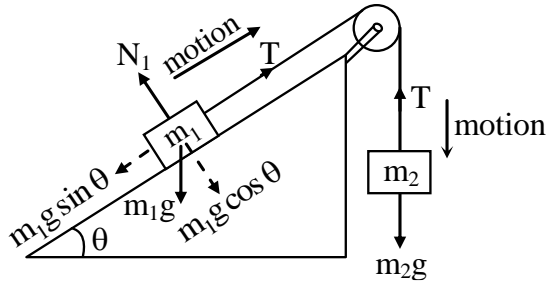
$$\text{From (i), } (m_2g - T) = m_2 a \quad \dots(1)$$

$$\text{From (ii), } T = m_1 a \quad \dots(2)$$

Solving these equations we have,

$$a = \left[\frac{m_2}{(m_1 + m_2)} \right] g \quad \text{and} \quad T = m_1 a = \left[\frac{m_1 \cdot m_2}{(m_1 + m_2)} \right] g$$

Case (iii) : Here we shall consider the above case with a difference that (m_1) placed on smooth inclined plane making an angle(θ) with horizontal as shown in figure.



In this case,

$$T - m_1 g \sin \theta = m_1 a$$

$$\text{and} \quad (m_2 g - T) = m_2 a$$

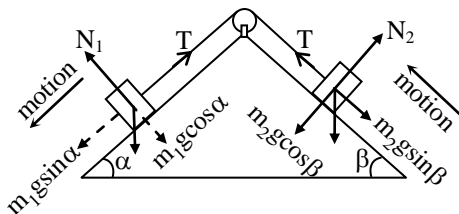
$$\text{solving we get, } a = \frac{(m_2 - m_1 \sin \theta)g}{(m_1 + m_2)}$$

$$\text{and} \quad T = m_2 g - m_2 a$$

$$\text{or} \quad T = m_2 g \left[1 - \frac{(m_2 - m_1 \sin \theta)}{(m_1 + m_2)} \right]$$

$$= \frac{m_1 m_2 g}{m_1 + m_2} (1 + \sin \theta)$$

Case (iv) : Let us consider the case when masses (m_1) and (m_2) are on inclined plane making angles (α) and (β) with horizontal respectively as shown in figure



$$\text{We have, } m_1 g \sin \alpha - T = m_1 a$$

$$\text{and} \quad T - m_2 g \sin \beta = m_2 a$$

solving these equations we get,

$$a = \frac{g(m_1 \sin \alpha - m_2 \sin \beta)}{(m_1 + m_2)}$$

$$\text{and } T = m_2 \frac{g(m_1 \sin \alpha - m_2 \sin \beta)}{(m_1 + m_2)} + m_2 g \sin \beta$$

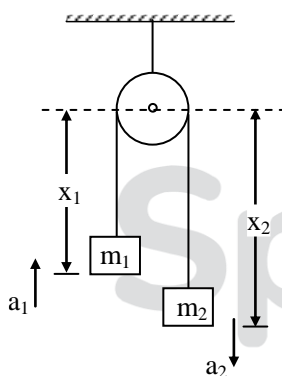
$$\text{or } T = \frac{m_1 m_2}{m_1 + m_2} (\sin \alpha + \sin \beta) g$$

6. Constraint Relation

- It is a relation (usually between displacement, velocity, or the acceleration of different masses connected in some way) that one must think of whenever one finds the number of equations less than the number of unknowns. In all the previous problems we have learned till now, we have obtained the constraint relation by experience and judgment. Now we will explain the mathematical procedure for this.

How to Determine Constraint Relation:

- Assume the direction of acceleration of each block e.g. a_1 (upward) and a_2 (downward), [$m_1 < m_2$]
 - Locate the position of each block from a fixed point [depending on your convenience], e.g. centre of pulley in this case.
 - Identify the constraint and write down the equations of constraint in terms of distance.
- In the chosen problem, the length of the string remains constant is the constraint or the restriction.



$$\therefore x_1 + x_2 = \text{constant}$$

differentiating both sides w.r.t. time, we get

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0 \quad \dots\dots(1)$$

$$\Rightarrow v_1 + v_2 = 0$$

v_1 and v_2 are velocities of the block 1 and 2.

Again differentiating equation (1) w.r.t. time we get

$$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

Since block m_1 is assumed to be moving upward ($\because x_1$ decreasing with time)

$$\therefore \frac{d^2x_1}{dt^2} = -a_1,$$

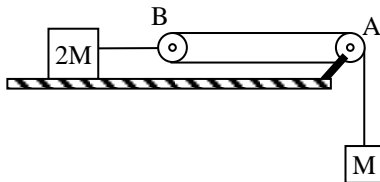
and block m_2 is assumed moving downward

$$\therefore \frac{d^2 x_1}{dt^2} = + a_2 \text{ (} x_2 \text{ increasing with time)}$$

Thus, $-a_1 + a_2 = 0 \Rightarrow a_1 = a_2 = a$

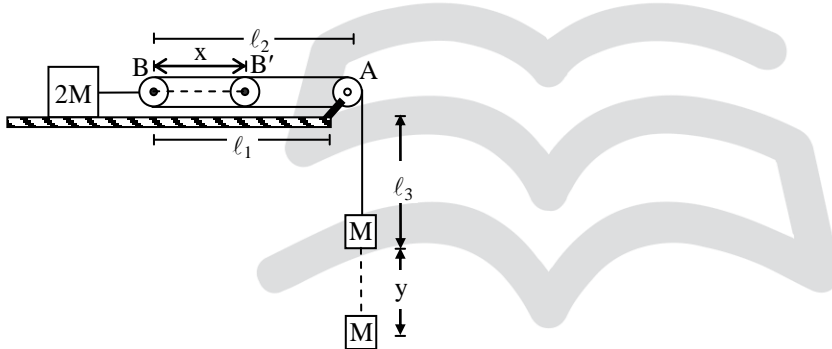
Note: We have used this relation from our judgment in the pulley mass system in the earlier problems.

3. Consider the figure and assume both the pulleys and the strings are light and all the surfaces are frictionless.



Let the initial total length of the string (as shown) = $l_1 + l_2 + l_3$

When the system is released, let the pulley B comes to B' (by a distance x) and mass 'M' moves down by distance 'y'.



when B moves to B then, length of string is-

$$l_1 - x + l_2 - x + l_3 + y$$

\therefore Total length of string is constant

$$\therefore l_1 + l_2 + l_3 = l_1 - x + l_2 - x + l_3 + y \Rightarrow y = 2x$$

differentiating w.r.t. to time $\frac{dy}{dt} = 2 \frac{dx}{dt}$

differentiating again w.r.t. to time again,

$$\frac{d^2 y}{dt^2} = 2 \frac{d^2 x}{dt^2}$$

$$\Rightarrow a_M = 2a_{2M} = 2a \quad \dots(i)$$

[a = acceleration of mass 2M]

Note: The above relation becomes the constraint relation.

7. Reference frames: Inertial and Non-inertial

A frame which is either at rest or moving with constant velocity for a stationary observer is known as

Inertial Reference frame. Newton's first law is valid in inertial reference frame.

An accelerated frame of reference is known as **Non - Inertial frame.** Objects in non - inertial frame do not obey Newton's 1st law.

8. Pseudo force

- (i) It is an imaginary force which acts on a body when body is observed from a non-inertial frame of reference.
- (ii) It is not experienced when observer in inertial frame.
- (iii) The direction of pseudo force on a body is opposite to the direction of acceleration of the non-inertial reference frame from which the body is being observed.

$$\vec{F}_p = -m \vec{a}$$

m = mass of body

\vec{a} = acceleration of frame of reference

(-)ve sign indicates pseudo force acts in the opposite direction of the acceleration of the frame of reference.

- (iv) Magnitude of the of the pseudo force is

$$|F_p| = ma$$

- (v) Its action does not have the reaction required by the third law.



SOLVED EXAMPLES

Ex. 1 A boy standing on a weighing machine observes his weight as 200 N. When he suddenly jumps upwards, his friend notices that the reading increased to 400 N. The acceleration by which the boy jumped will be-

Sol. Force causing the acceleration = $400 - 200 = 200\text{N}$

$$\text{Mass of the boy} = \frac{200}{9.8}$$

$$\text{Hence acceleration} = \frac{F}{m} = \frac{200}{200} \times 9.8 = 9.8 \text{ m/s}^2$$

Ex. 2 A force of $(6\hat{i} + 8\hat{j})$ N acted on a body of mass 10kg. The displacement after 10sec, if it starts from rest, will be -

Sol. Acceleration = $\frac{\vec{F}}{m} = \frac{6\hat{i} + 8\hat{j}}{10}$ in the direction of force and displacement

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$= 0 + \frac{1}{2}\left(\frac{6\hat{i} + 8\hat{j}}{10}\right)100 = 30\hat{i} + 40\hat{j}$$

So, the displacement is 50m along $\tan^{-1} \frac{4}{3}$ with x-axis.

Ex. 3 A fireman has to carry an injured person of mass 40kg from the top of a building with the help of the rope which can withstand a load of 100kg. The acceleration of the fireman if his mass is 80 kg, will be-

Sol. Total mass = $80 + 40 = 120 \text{ kg}$

The rope cannot withstand this load so the fire man should slide down the rope with some acceleration

\therefore The maximum tension = $100 \times 9.8 \text{ N}$

$m(g - a) = \text{tension}$,

$$120(9.8 - a) = 100 \times 9.8 \Rightarrow a = 1.63 \text{ m/s}^2$$

Ex. 4 A body of 0.02 kg falls from a height of 5 m into a pile of sand. The body penetrates the sand a distance of 5 cm before stopping. What force has the sand exerted on the body?

Sol. Suppose the velocity of the body at the instant when it reaches the pile of sand be 'v'. Then

$$v^2 = 0 + 2(9.8) \times (5 \text{ metre}) = 98 \quad (\because v^2 = u^2 + 2as)$$

$$a = -\frac{98}{2 \times (0.05)} = -980 \text{ m/sec}^2$$

Now, retarding force

$$F = \text{mass} \times \text{acceleration} = 0.02 \text{ kg} \times (-980 \text{ m/sec}^2) = -19.6 \text{ N}$$

Ex. 5 The total mass of an elevator with a 80 kg man in it is 1000 kg. This elevator moving upward with a speed of 8 m/sec, is brought to rest over a distance of 16 m. The tension 'T' in the cables supporting the elevator and the force exerted on the man by the elevator floor will respectively be-

Sol. (a) The elevator having an initial upward speed of 8 m/sec is brought to rest within a distance of 16m.

Hence,

$$0 = (8)^2 + 2a(16) \quad (\because v^2 = u^2 + 2as),$$

$$a = -\frac{8 \times 8}{2 \times 16} = -2 \text{ m/sec}^2.$$

Resultant upward force on elevator = $T - mg$.

$$T - mg = ma$$

$$\text{or } T = mg + ma = m(g + a)$$

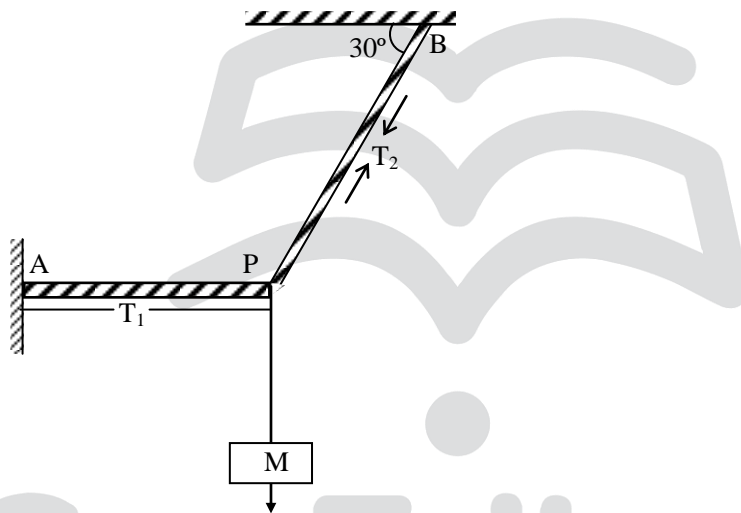
$$= 1000(9.8 - 2) = 7800\text{N}.$$

(b) Let 'P' be the upward force exerted on the man by the elevator floor, If m' be the mass of the man, then, weight of the man acting downward = $m'g$

Upward force on the man = $P - m'g$

According to Newton's 2nd law. $P - m'g = m'a$ or $P = m'(a + g) = (-2 + 9.8) = 624\text{N}$.

Ex. 6 A mass 'M' is hung with a light inextensible string as shown in figure. Find the tension horizontal strings will be –



Sol. As there is a load at P so tension in AP and PB will be different. Let these be T_1 and T_2 respectively. For vertical equilibrium of P

$$T_2 \cos 60^\circ = Mg \Rightarrow T_2 = 2Mg \quad \dots\dots (1)$$

and for horizontal equilibrium of P

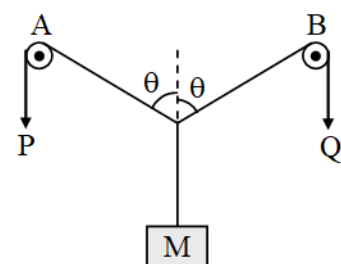
$$T_1 = T_2 \sin 60^\circ = T_2 (\sqrt{3}/2) \quad \dots\dots (2)$$

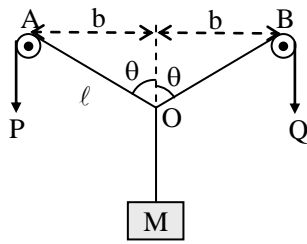
Substituting the value of T_2 from

$$\text{equation (1) in (2) } T_1 = (2Mg) \times \frac{\sqrt{3}}{2} = \sqrt{3} Mg$$

Ex. 7 In the arrangement shown in figure, the ends P and Q of an unstretchable string move downwards with uniform speed 'U'. Pulleys A and B are fixed. Then, find the speed by which mass 'M' moves upwards.

Sol. As P and Q move down, the length 'l' decreases at the rate of Um/s .





From figure, $l^2 = b^2 + y^2$

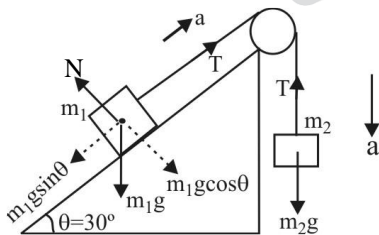
Differentiating with respect to time

$$2l \frac{dl}{dt} = 2y \frac{dy}{dt} \quad (\because b \text{ is constant})$$

$$\therefore \frac{dy}{dt} = \frac{l}{y} \cdot \frac{dl}{dt} = \frac{1}{\cos \theta} \cdot \frac{dl}{dt} = \frac{U}{\cos \theta}$$

Ex.8 A block of mass $m_1 = 4\text{kg}$ on a smooth inclined plane of 30° is connected by a cord over a small, frictionless pulley to a second block of mass $m_2 = 5\text{kg}$ hanging vertically. The acceleration with which the block moves and the tension in the cord will respectively be- (Take $g = 10\text{m/sec}^2$)

Sol. The different forces acting on the masses are shown in figure.



$$\text{We have } T - m_1 g \sin \theta = m_1 a \quad \dots(1)$$

$$\text{and } m_2 g - T = m_2 a \quad \dots(2)$$

Solving equations (1) and (2),

$$\text{we get } a = \frac{(m_2 - m_1 \sin \theta) g}{(m_1 + m_2)} \quad \dots(3)$$

$$\text{and } T = m_2 g \left[1 - \frac{(m_2 - m_1 \sin \theta)}{(m_1 + m_2)} \right]$$

$$\text{or } T = \frac{m_1 m_2 (1 + \sin \theta) g}{(m_1 + m_2)} \quad \dots(4)$$

Here $m_1 = 4\text{kg}$, $m_2 = 5\text{kg}$, $\theta = 30^\circ$ and $g = 10\text{m/s}^2$

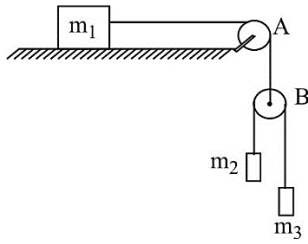
Substituting these values in equation (3), we get

$$a = \frac{[5 - (4 \times \frac{1}{2})] \times 10}{(4 + 5)} = \frac{30}{9} = 3.3\text{m/sec}^2$$

Further substituting the values in eq. (4), we get

$$T = \frac{4 \times 5 (1 + \frac{1}{2}) \times 10}{(4 + 5)} = \frac{300}{9} = 33.33\text{N.}$$

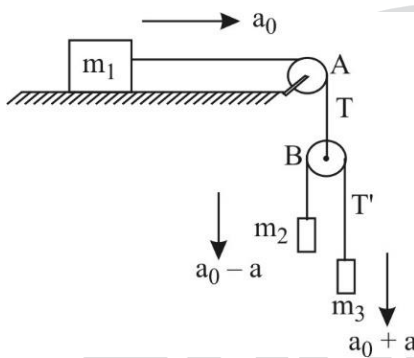
Ex. 9 Three blocks of mass m_1 , m_2 and m_3 are connected as shown in the figure. All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of m_1 .



Sol. Suppose the acceleration of m_1 is ' a_0 ' towards right. That will also be the downward acceleration of the pulley B because the string connecting m_1 and B is constant in length. Also the string connecting m_2 and m_3 has a constant length. This implies that the decrease in the separation between m_2 and B equals the increase in the separation between m_2 and B. So, the upward acceleration of m_2 with respect to B equals the downward acceleration of m_3 with respect to B. Let this acceleration be ' a '.

The acceleration of m_2 with respect to the ground = $a_0 - a$ (downward) and the acceleration of m_3 with respect to the ground = $a_0 + a$ (downward).

These accelerations will be used in Newton's laws. Let the tension be T in the upper string and T' in the lower string. Consider the motion of the pulley B.



The forces on this light pulley are

- (a) T upwards by the upper string, and
- (b) $2T'$ downward by the lower strings.

As the mass of the pulley is negligible,

$$2T' - T = 0$$

$$\text{giving } T' = T/2 \quad \dots\text{(i)}$$

Motion of m_1 :

The acceleration is a_0 in the horizontal direction. The forces on m_1 are-

- (a) T by the string (horizontal)
- (b) m_1g by the earth (vertically downwards) and
- (c) N by the table (vertically upwards)

In the horizontal direction, the equation is

$$T = m_1a_0 \quad \dots\text{(ii)}$$

Motion of m_2 : acceleration is $a_0 - a$ in the downward direction. The forces on m_2 are-

(a) m_2g downward by the earth and

(b) $T' = T/2$ upward by the string

Thus, $m_2g - T/2 = m_2(a_0 - a)$ (iii)

Motion of m_3 : The acceleration is $(a_0 + a)$ downward. The forces on m_3 are

(a) m_3g downward by the earth and

(b) $T' = T/2$ upward by the string. Thus

$$m_3g - T/2 = m_3(a_0 + a) \quad \dots\text{(iv)}$$

We want to calculate a_0 , so we shall eliminate T and a from (ii), (iii) and (iv)

Putting T from (ii) in (iii) and (iv)

$$a_0 - a = \frac{m_2g - m_1a_0/2}{m_2} = g - \frac{m_1a_0}{2m_2} \quad \text{and}$$

$$a_0 + a = \frac{m_3g - m_1a_0/2}{m_3} = g - \frac{m_1a_0}{2m_3}$$

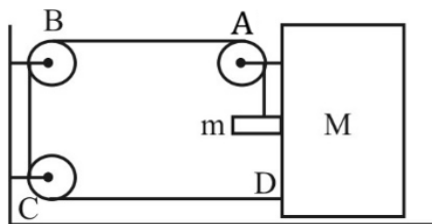
$$\text{Adding } 2a_0 = 2g - \frac{m_1a_0}{2} \left(\frac{1}{m_2} + \frac{1}{m_3} \right)$$

$$\text{or } a_0 = g - \frac{m_1a_0}{4} \left(\frac{1}{m_2} + \frac{1}{m_3} \right)$$

$$\text{or } a_0 \left[1 + \frac{m_1}{4} \left(\frac{1}{m_2} + \frac{1}{m_3} \right) \right] = g$$

$$\text{or } a_0 = \frac{g}{1 + \frac{m_1}{4} \left(\frac{1}{m_2} + \frac{1}{m_3} \right)}$$

Ex. 10 In the given figure find the acceleration of m assuming that there is no friction between m and M , and all other surfaces are smooth and pulleys are light.



Sol. Let X be the leftward displacement of A and ' x ' and ' y ' be the leftward and downward displacements of ' m '. Then by using constraint application, we get,

$$x = X \Rightarrow x = X \Rightarrow a_x = A_x$$

$$\text{and } l_1 - x + l_2 + l_3 - x + l_4 + y$$

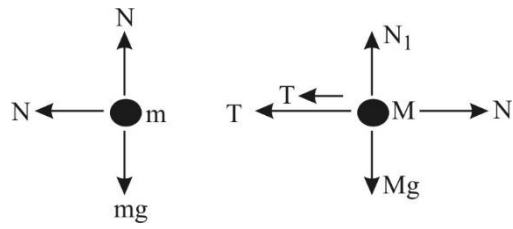
$$= l_1 + l_2 + l_3 + l_4$$

where l_1, l_2, l_3, l_4 are the instantaneous lengths of the segments of the string.

$$\Rightarrow 2x = y \Rightarrow 2x = y \Rightarrow 2a_x = a_y$$

$$N = ma_x \text{ and } mg - T = ma_y$$

$$\text{and } 2T - N = MA_x = Ma_x$$



Eliminating T , A_x and N in the above equation, we get

$$a_x = \frac{2mg}{M + 5m} \text{ and } a_y = \frac{4mg}{M + 5m}$$

$$\therefore a = \sqrt{a_x^2 + a_y^2} = \frac{2\sqrt{5}mg}{M + 5m}$$



SpeEdLabs

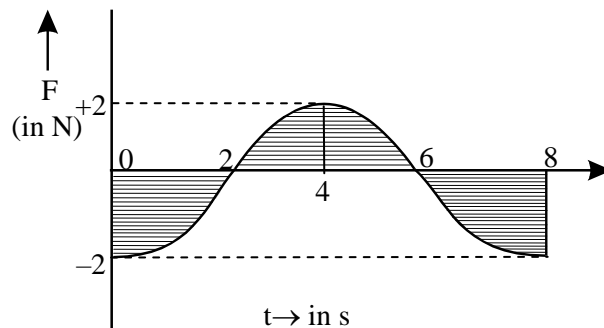
EXERCISE – 1

Laws of motion: Theoretical Description

- Q.1** In a cricket match the fielder draws his hands backward after receiving the ball in order to take a catch because -
- (A) his hands will be saved from getting hurt.
 - (B) he deceives the player.
 - (C) it is a fashion.
 - (D) he catches the ball firmly.
- Q.2** A boy sitting on the top most berth in the compartment of a train which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother situated vertically below his hands at a distance of about 2m. The apple will fall -
- (A) in the hand of his brother.
 - (B) slightly away from the hands of his brother in the direction of motion of the train.
 - (C) slightly away from the hands of his brother in the direction opposite to the direction of motion of the train.
 - (D) none of the above.

Application of $\vec{F} = \frac{d\vec{p}}{dt}$, Impulse

- Q.3** In a legend the hero kicked a body pig so that he is projected with a speed greater than that of his cry. If the weight of the body pig is assumed to be 5kg and the time of contact 0.01 sec., the force with which the hero kicked him was -
(Speed of cry = 330 m/s)
- (A) 5×10^{-2} N
 - (B) 2×10^5 N
 - (C) 1.65×10^5 N
 - (D) 1.65×10^3 N
- Q.4** A force - time graph for the motion of a body is shown in figure. Change in linear momentum between 0 and 8s is -



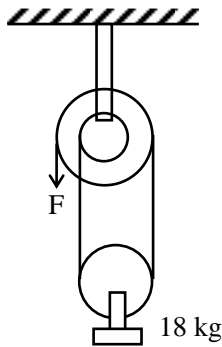
- (A) zero
- (B) 4 N-s
- (C) 8 N-s
- (D) None

Statics and Dynamics involving single system

Q.5 A particle is acted upon by two mutually perpendicular forces of 3N and 4N. In order that the particle remains stationary, the magnitude of the third force that should be applied is -

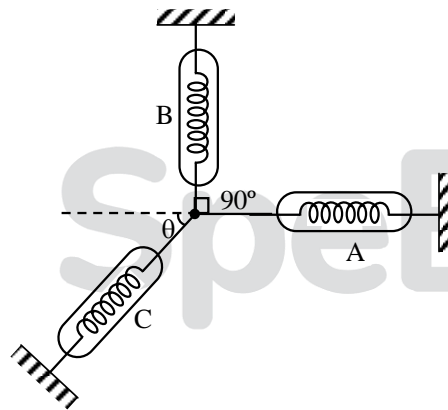
- (A) 12 N (B) 5 N
(C) 8 N (D) 7 N

Q.6 In the figure at the free end a force F is applied to keep the suspended mass of 18 kg at rest. The value of F is-



- (A) 180 N (B) 90 N
(C) 60 N (D) 30 N

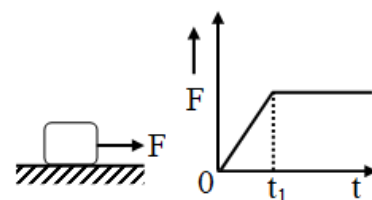
Q.7 Three spring balances are attached to the ring as shown in the figure. There is an angle of 90° between the balance A and balance B. There is a reading of 5 N on balance A and 12 N on the balance B. Then of the following options are correct.



- (A) Reading in the balance C is 13 N and angle θ is 67.4°
(B) Reading in the balance C is 13 N and angle θ is 22.6°
(C) Reading in the balance C is 5 N and angle θ is 67.4°
(D) Reading in the balance C is 5 N and angle θ is 22.6°

Q.8 A particle is on a smooth horizontal plane. A force F is applied whose F - t graph is given. Then-

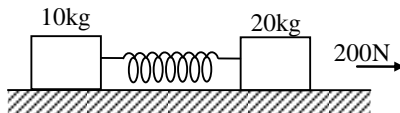
- (A) between 0 & t_1 , acceleration is constant.
(B) initially body must be in rest.
(C) after t_1 acceleration is constant.
(D) finally acceleration is zero.



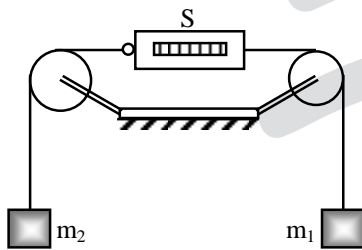
- Q.9** A stretching force of 1000 Newton is applied at one end of a spring balance and an equal stretching force is applied at the other end at the same time. The reading of the balance will be -
- (A) 2000 N (B) 0 N
(C) 1000 N (D) 500 N

Dynamics of Multi system

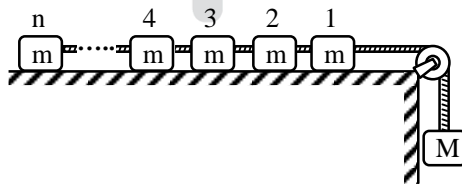
- Q.10** The masses of 10 kg and 20 kg respectively are connected by massless spring as shown in the figure. A force of 200 N acts on the 20 kg mass. At the instant shown, the 10 kg mass has acceleration of 12 m/s^2 . What is the acceleration of 20 kg mass ?



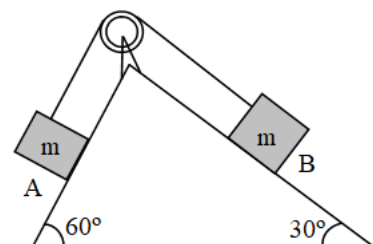
- (A) 12 m/s^2 (B) 4 m/s^2
(C) 10 m/s^2 (D) zero
- Q.11** In the arrangement shown, the pulleys are fixed and ideal, the strings are light, $m_1 > m_2$ and S is a spring balance which is itself massless. The reading of S (in units of mass) is-



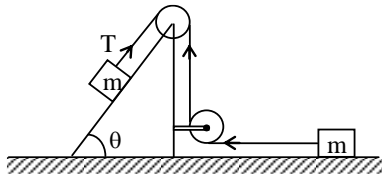
- (A) $m_1 - m_2$ (B) $\frac{1}{2} (m_1 + m_2)$ (C) $\frac{m_1 m_2}{m_1 + m_2}$ (D) $\frac{2m_1 m_2}{m_1 + m_2}$
- Q.12** In the given arrangement, n number of equal masses are connected by strings of negligible masses. The tension in the string connected to n^{th} mass is-



- (A) $\frac{mMg}{nm + M}$ (B) $\frac{mMg}{nM + m}$ (C) mg (D) mng
- Q.13** Two blocks each of mass m are resting on a frictionless inclined plane as shown in figure. Then -
- (A) Block A moves down the plane
(B) Block B moves down the plane
(C) Both the blocks remains at rest
(D) Both the blocks moves down the plane

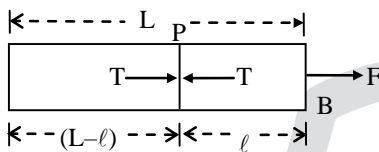


Q.14 For the system shown in the figure, the pulleys are light and frictionless. The tension in the string will be-



- (A) $\frac{2}{3} mg \sin \theta$ (B) $\frac{3}{2} mg \sin \theta$
(C) $\frac{1}{2} mg \sin \theta$ (D) $2mg \sin \theta$

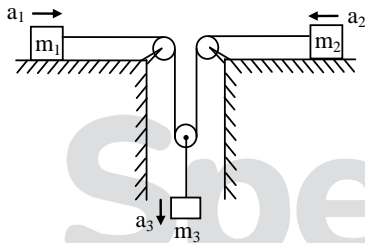
Q.15 A uniform rope of length L , resting on a frictionless horizontal surface is pulled at one end by a force F . What is the tension of the rope at a distance l from the end where the force is applied?



- (A) $F(1 - l/L)$ (B) $F(1 + l/L)$
(C) $F/(1 - l/L)$ (D) $F/(1 + l/L)$

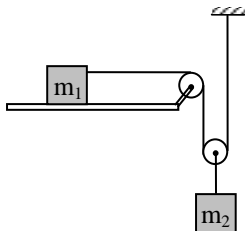
Constraint Relation

Q.16 In the figure shown the relation between acceleration is –



- (A) $a_1 + a_2 + 2a_3 = 0$ (B) $a_1 + a_2 = 2a_3$
(C) $a_1 + a_2 = a_3$ (D) $a_1 + a_2 + a_3 = 0$

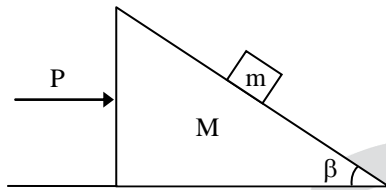
Q.17 If the surface is smooth, the acceleration of the block m_2 will be –



- (A) $\frac{m_2 g}{4m_1 + m_2}$ (B) $\frac{2m_2 g}{4m_1 + m_2}$
(C) $\frac{2m_2 g}{m_1 + 4m_2}$ (D) $\frac{2m_1 g}{m_1 + m_2}$

Non-Inertial Reference frame and Pseudo force

- Q.18** A man goes up in a uniformly accelerating lift. He returns downward with the lift accelerating at the same rate. The ratio of apparent weights in the two cases is 2 : 1. The acceleration of the lift is -
 (A) $g/3$ (B) $g/4$ (C) $g/5$ (D) $g/6$
- Q.19** A block can slide on a smooth inclined plane of inclination θ kept on the floor of a lift. When the lift is descending with a retardation a , the acceleration of the block relative to incline is -
 (A) $(g + a) \sin \theta$ (B) $(g - a)$
 (C) $g \sin \theta$ (D) $(g - a) \sin \theta$
- Q.20** Two wooden blocks are moving on a smooth horizontal surface such that the mass m remains stationary with respect to block of mass M as shown in figure. The magnitude of force P is -

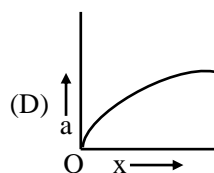
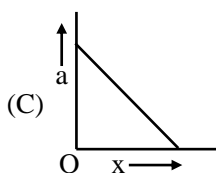
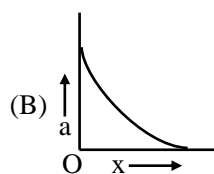
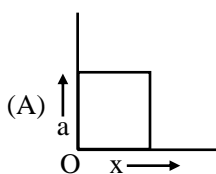
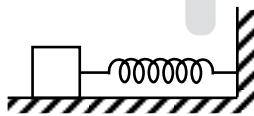


- (A) $(M + m) g \tan \beta$ (B) $g \tan \beta$ (C) $mg \cos \beta$ (D) $(M + m) \operatorname{cosec} \beta$

EXERCISE - 2

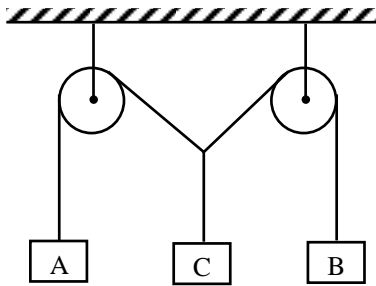
Part – A: Only single correct answer type questions

- Q.1** A light spring is compressed and placed horizontally between a vertical fixed wall and a block, free to slide over a smooth horizontal table top as shown in figure. If the system is released from rest, which of the graphs below represents the relation between the acceleration 'a' of the block and the distance 'x' travelled by it ?

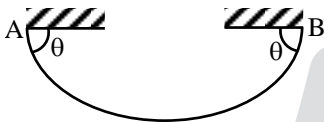


- Q.2** A particle of small mass m is joined to a very heavy body by a light string passing over a light pulley. Both bodies are free to move. The total downward force on the pulley is -
 (A) mg (B) $2 mg$ (C) $4 mg$ (D) $\gg mg$

- Q.3** Three blocks A, B and C are suspended as shown in figure. Mass of each of blocks A and B is m . If system is in equilibrium, and mass of C is M then –



- (A) $M = 2m$ (B) $M < 2m$
(C) $M > 2m$ (D) $M \geq 2m$
- Q.4** A flexible chain of weight w hangs between two fixed points A and B as shown in fig. at the same level. Then the vector force exerted by the chain on each end point, and the tension in the chain at the lowest point will be:

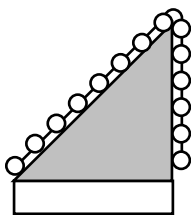


- (A) $2w \sin \theta, 2w \cot \theta$ (B) $2w \cot \theta, 2w \sin \theta$
(C) $\frac{W}{2} \sin \theta, \frac{W}{2} \cot \theta$ (D) $(w/2) \cos \theta, (w/2) \tan \theta$
- Q.5** Two objects A and B are thrown upward simultaneously with the same speed. The mass of A is greater than the mass of B. Suppose the air exerts a constant and equal force of resistance on the two bodies-
- (A) the two bodies will reach the same height
(B) A will go higher than B
(C) B will go higher than A
(D) any of the above three may happen depending on the speed with which the objects are thrown

- Q.6** A chain of mass M and length L held vertical by fixing its upper end to a rigid support. The tension in the chain at a distance y from the rigid support is –

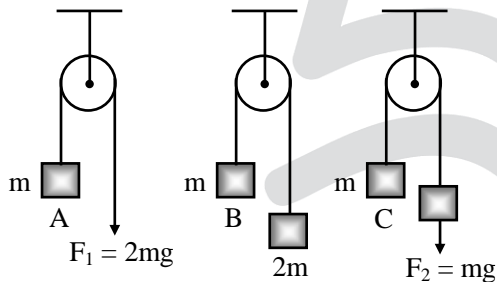
- (A) Mg (B) $Mg(L - y)/L$
(C) $MgL / (L - y)$ (D) Mgy / L

- Q.7** Sixteen beads in a string are placed on a smooth inclined plane of inclination $\sin^{-1} (1/3)$ such that some of them lie along the incline whereas the rest hang over the top of the plane. If acceleration at first bead is $g/2$, the arrangement of beads is that–

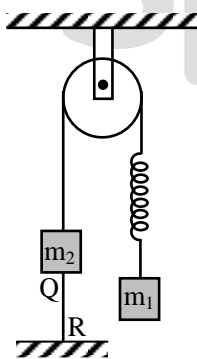


- (A) 12 hang vertically. (B) 10 lie along inclined plane.
(C) 8 lie along inclined plane. (D) 10 hang vertically.

- Q.8** A man slides down a light rope whose breaking strength is η times his weight extra space ($\eta < 1$). What should be his maximum acceleration (downward) so that the rope just breaks?
- (A) ηg (B) $g(1 - \eta)$
 (C) $\frac{g}{1 + \eta}$ (D) $\frac{g}{2 - \eta}$
- Q.9** An empty plastic box of mass m is found to accelerate up at the rate of $g/6$ when placed deep inside water. How much sand should be put inside the box so that it may accelerate down at the rate of $g/6$?
- (A) $2 m/3$ (B) $2 m/5$
 (C) $m/5$ (D) $6 m/7$
- Q.10** A spring toy of weight 1 kg rests on a weighing machine. The toy suddenly jumps and the balance reads 11 N. The acceleration of the toy just on jumping up is ($g = 10 \text{ m/s}^2$) -
- (A) 0.5 m/s^2 (B) 1 m/s^2
 (C) 1.5 m/s^2 (D) 2 m/s^2
- Q.11** In the figure, the blocks A, B and C of mass m each have accelerations a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitude $2mg$ and mg respectively.

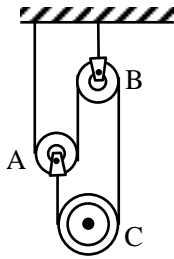


- (A) $a_1 = a_2 = a_3$ (B) $a_1 > a_3 > a_2$
 (C) $a_1 = a_2, a_2 > a_3$ (D) $a_1 > a_2, a_2 = a_3$
- Q.12** In the shown system $m_1 > m_2$. Thread QR is holding the system. If this thread is cut, then just after cutting-



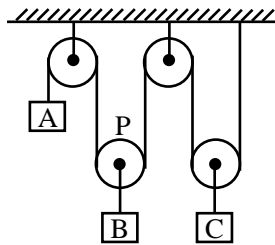
- (A) acceleration of mass m_1 is zero and that of m_2 is directed upward.
 (B) acceleration of mass m_1 is zero and that of m_2 is directed downward.
 (C) acceleration of both the blocks will be same.
 (D) acceleration of system is given by $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)kg$, where k is a spring factor.

- Q.13** In the arrangement shown in figure, pulleys A and B are massless and the thread is inextensible. Mass of pulley C is equal to m . If friction in all the pulleys is negligible, then



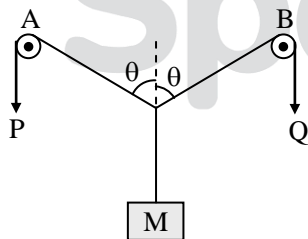
- (A) tension in thread is equal to $1/2 mg$.
 (B) acceleration of pulley C is equal to $g/2$ (downward).
 (C) acceleration of pulley A is equal to g (upward).
 (D) acceleration of pulley A is equal to $2g$ (upward).

- Q.14** In the ideal case,



- (A) magnitude of acceleration of A is sum of magnitude of acceleration of B and C.
 (B) magnitude of acceleration of A is arithmetic mean of magnitude of acceleration of B and C.
 (C) acceleration of pulley P is same as that of mass B.
 (D) if P is massless, net force on pulley is non-zero.

- Q.15** In the arrangement shown in the figure, the ends P and Q of an unstretchable string move downwards with uniform speed u , pulley A and B are fixed. Mass M move up with speed-



- (A) $2u \cos \theta$ (B) $u/\cos \theta$
 (C) $2u/\cos \theta$ (D) $u \cos \theta$

ANSWER KEY

EXERCISE - 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	A	B	C	A	B	B	A	C	C	B	D	A	A	C	A	B
Q.No.	17	18	19	20												
Ans.	A	A	A	A												

22. True

23. False

24. 100 N, 0

25. ma

EXERCISE - 2

PART-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	B	C	B	B	D	B	B	B	B	A	D	C	B



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