SEQUENCE AND SERIES

1. Definition

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a progression.

Progression can be classified into 5 parts as-

(i) Arithmetic Progression (A.P.)

(ii) Geometric Progression (G.P.)

(iii) Arithmetic Geometric Progression (A.G.P.)

(iv) Harmonic Progression (H.P.)

(v) Miscellaneous Progressions

2. Arithmetic Progression (A.P.)

If a is the first term and d is the common difference then A.P. can be written as

a + (a + d) + (a + 2d) + (a + 3d) + ...

2.1 General Term of an A.P.

General term (n^{th} term) of an AP is given by

$T_n = a + (n - 1) d$

Note :

(i) General term is also denoted by ℓ (last term).

(ii) n (no. of terms) always belongs to set of natural numbers.

(iii) Common difference can be zero, + ve or – ve.

(iv) n^{th} term from end is given by $= T_m - (n - 1) d$ or $(m - n + 1)^{th}$ term from beginning (where m is total no. of terms).

2.2 Sum of 'n' terms of an A.P.

$S_n =$ [2a + (n –1) d] or $S_n =$ [a +T_n] or $S_n =$ (a+l)

Note :

(i) General term $T_n = S_n - S_{n-1}$ where S_{n-1} is sum of (n –1) terms of A.P. and S_n is sum of n terms.

(ii) Common difference of A.P. is given by $d = S_2 - 2S_1$ where S_2 is sum of first two terms and S_1 is sum of first term or first term.

(iii) The sum of infinite terms of an A.P. is ∞ if d > 0 and $-\infty$ if d < 0.

2.3 Arithmetic Mean (A.M.)

If A is the A.M. between two given numbers a and b, then A =

$\Rightarrow 2A = a + b$

2.4 'n' AM's between two given numbers a and b

d = n ,A_1= a + d, A_2 = a + 2d,.... A_n

$= a + nd$ or $A_n = b - d$ and

$A_r = a + r\left(\frac{b-a}{n+1}\right)$ where A_r is r^{th} A.M. between a & b.

(i) Sum of n AM's inserted between a and b is equal to $\frac{n}{2}(a + b)$.

(ii) Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$a_n = 1/2\,(a_{n-k} + a_{n+k}),\ k < n$.

Where k is distance of term.

2.5 Supposition of terms in A.P.

(i) Three terms as: $a - d, a, a + d$

(ii) Five terms are: $a - 2d, a - d, a, a + d, a + 2d$

(iii) Four terms as : $a - 3d, a - d, a + d, a + 3d$

2.6 Some properties of an A.P.

(i) If each term of a given A.P. be increased, decreased, multiplied or divided by some non-zero constant number then resulting series thus obtained will also be in A.P.

(ii) In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.

(iii) Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$a_n = \frac{1}{2}(a_{n-k} + a_{n+k}),\ k < n$

(iv) If in a finite A.P. the number of terms be odd, then its middle term is the A.M. between the first and last term and its sum is equal to the product of middle term and no. of terms.

3. General term of a G.P.

General term (n^{th} term) of a G.P.

$a + ar + ar^2 + ...$ is given by $T_n = ar^{n-1}$

3.1 Sum of 'n' term of a G.P.

The sum of first n terms of a G.P. is given by

$S_n = \frac{a(1-r^n)}{1-r} = \frac{a - rT_n}{1-r}$ when $r < 1$ or

$S_n = \frac{a(r^n-1)}{r-1} = \frac{rT_n - a}{r-1}$ when $r > 1$ and

$S_n = na$ when $r = 1$

3.2 Sum of an infinite G.P.

The sum of an infinite G.P. with first term a and common ratio r ($-1 < r < 1$ i.e. $|r| < 1$) is

$S_\infty = \frac{a}{1-r}$

3.3 Geometrical Mean (G.M.)

If G is the G.M. between two numbers a and b then $G^2 = ab \Rightarrow G = \sqrt{ab}$

3.4 'n' GM's between two given numbers a & b

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ then $G_1 = ar$, $G_2 = ar^2$, $G_3 = ar^3$ ...., $G_n = ar^n$ or $G_n = \frac{b}{r}$ and $G_k = a(b/a)^{k/n+1}$. where G_k is k^{th} G.M. between a & b. Product of n GM's inserted between a and b is equal to $(ab)^{n/2}$.

3.5 Supposition of terms in G.P.

(i) Three terms as : a/r, a, ar

(ii) Five terms as : $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

(iii) Four terms as : $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3

3.6 Some properties of a G.P.

(i) If each term of a G.P. be multiplied or divided by the same non-zero quantity, then resulting series is also a G.P.

(ii) In a G.P. the product of two terms which are at equidistant from the first and the last term, is constant and is equal to product of first and last term.

(iii) If each term of a G.P. be raised to the same power, then resulting series is also a G.P.

(iv) In a G.P. every term (except first) is G.M. of its two terms which are at equidistant from it. i.e. $T_r = \sqrt{T_{r-k}T_{r+k}}$ $k < r$

(v) In a finite G.P., the number of terms be odd then its middle term is the G.P. of the first and last term.

(vi) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(vii) If $a_1, a_2, a_3, \ldots, a_n$ is a G.P. of non-zero, non negative terms, then $\log a_1, \log a_2, \ldots, \log a_n$ is an A.P. and vice-versa.

(viii) If $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3 \ldots$ are two G.P.'s then $a_1b_1, a_2b_2, a_3b_3 \ldots$ is also in G.P.

7. Some special series

(i) $1 + 2 + 3 \ldots\ldots + n = \sum_{k=1}^{n}(k) = \frac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 \Rightarrow \sum_{k=1}^{n}(k)^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 \Rightarrow \sum_{k=1}^{n}(k)^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n}k\right)^2$

(iv) Sum of first n odd natural numbers $\Rightarrow \sum_{r=1}^{n}(2r-1) = n^2$

(v) Sum of first n even natural numbers $\Rightarrow \sum_{r=1}^{n}2r = n(n+1)$

8. Properties of A.P.

(i) If for an A.P. p^{th} term is q, q^{th} term is p then m^{th} term is = p + q – m.

(ii) If for an AP sum of p terms is q, sum of q term is p, then sum of (p + q) term is – (p + q).

(iii) If for an A.P. sum of p terms is equal to sum of q terms then sum of (p + q) terms is zero.

9. Mean of 'n' numbers

(i) Single AM (A) of n positive numbers $A = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$

(ii) Single GM (G) of n positive numbers $G = (a_1 a_2 a_3 \ldots a_n)^{1/n}$

SOLVED EXAMPLES

Ex.1 The first, second and middle terms of an AP are a, b, c respectively. Their sum is–

(A) $\frac{2(c-a)}{b-a}$ (B) $\frac{2c(c-a)}{b-a}+c$

(C) $\frac{2c(b-a)}{c-a}$ (D) $\frac{2b(c-a)}{b-a}$

Sol. We have first term = a, second term = b

d = common difference = b – a

It is given that the middle term is c. This means that there are an odd number of terms in the AP. Let there be (2n+1) terms in the AP. Then $(n+1)^{th}$ term is the middle term.

$\therefore$ middle term = c $\Rightarrow$ a + nd = c

$\Rightarrow a + n(b-a) = c \Rightarrow n = \frac{c-a}{b-a}$

$$\text{Sum} = \frac{2n+1}{2}[2a + (2n+1-1)d]$$

$$= \frac{1}{2}\left\{2\left(\frac{c-a}{b-a}\right)+1\right\}\left[2a+2\left(\frac{c-a}{b-a}\right)(b-a)\right]$$

$$= \frac{1}{2}\left\{2\left(\frac{c-a}{b-a}\right)+1\right\}\{2c\} = \frac{2c(c-a)}{b-a} + c$$

Ans.[B]

Ex.2 If $a_1, a_2, a_3.....a_n$ are in AP where $a_i > 0 \ \forall$ i then the value of $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ...+ \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$ =

(A) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$

(C) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

Sol. Let d be the common difference of the A.P. Now on rationalising each term, we get

$$= \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} +.....+ \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}$$

$$= -\left(\frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+.....\sqrt{a_n}}{d}\right)$$

$$= -\frac{\left(\sqrt{a_1}-\sqrt{a_n}\right)}{d} = \frac{1}{d}\,\frac{(a_n-a_1)}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{(n-1)d}{d[\sqrt{a_n}+\sqrt{a_1}]} \qquad [\because a_n = a_1 + (n-1)d]$$

$= \dfrac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$

Ans.[D]

Ex.3 If the AM and GM between two numbers are in the ratio m : n then the number are in the ratio–

(A) $m+\sqrt{m^2-n^2} : m-\sqrt{m^2-n^2}$

(B) $m-\sqrt{m^2-n^2} : m+\sqrt{m^2-n^2}$

(C) $n+\sqrt{n^2-m^2} : n-\sqrt{n^2-m^2}$

(D) $n-\sqrt{n^2-m^2} : n+\sqrt{n^2-m^2}$

Sol. $\dfrac{a+b}{2\sqrt{ab}} = \dfrac{m}{n}$

$\Rightarrow \dfrac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \dfrac{m+n}{m-n}$

$\left(\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right) = \dfrac{\sqrt{m+n}}{\sqrt{m-n}}$

$\dfrac{2\sqrt{a}}{2\sqrt{b}} = \dfrac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

$\dfrac{a}{b} = \dfrac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

Ans.[A]

Ex.4 $1+ 2.2 + 3.2^2 + 4.2^3 + ...+ 100.2^{99}$ equals-

(A) 99.2^{100} (B) 100.2^{100}

(C) $1 + 99.2^{100}$ (D) None of these

Sol. Let

$S = 1 + 2.2 + 3.2^2 + 4.2^3 + ...+100.2^{99}$ ...(1)

$\Rightarrow 2S = 2 + 2.2^2 + 3.2^3 +...+ 99.2^{99} +100.2^{100}$...(2)

Subtracting (2) from (1), we get

$– S = (1+ 2+ 2^2 + 2^3 + ...+ 2^{99}) – 100.2^{100}$

$\Rightarrow S = 100.2^{100} – \dfrac{2^{100}-1}{2-1}$

$= 100.2^{100} – 2^{100} + 1$

$= 1 + 99.2^{100}$

Ans.[C]

Ex.5 If 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in A.P., then

(A) $z^3 = x$ (B) $x = y^{-1}$

(C) $z^{-3} = y$ (D) $x = y^{-1} = z^3$

Sol. Let d be the common difference. Then

$\log_y x = 1 + d \Rightarrow x = y^{1+d}$

$\log_z y = 1 + 2d \Rightarrow y = z^{1+2d}$

and $-15 \log_x z = 1 + 3d \Rightarrow z = x^{-(1+3d)/15}$

$\therefore x = y^{1+d} = z^{(1+2d)(1+d)}$

$= x^{-(1+d)(1+2d)(1+3d)/15}$

$\Rightarrow (1 + d)(1 + 2d)(1 + 3d) = -15$

$\Rightarrow 6d^3 + 11d^2 + 6d + 16 = 0$

$\Rightarrow (d + 2)(6d^2 - d + 8) = 0$

$\Rightarrow d = -2$

[Note that $6d^2 - d + 8 = 0$ has complex roots]

$\therefore x = y^{1+d} = y^{-1}$, $y = z^{1-4} = z^{-3}$

$\therefore x = (z^{-3})^{-1} = z^3$. Also $x = y^{-1} = z^3$

Ans.[A, B, C, D]

Ex.6 If log2, $\log (2^x - 1)$ and $\log (2^x + 3)$ are in A.P., then x is equal to

(A) 5/2 (B) $\log_2 5$ (C) $\log_3 2$ (D) 3/2

Sol. As log2, $\log (2^x - 1)$ and $\log (2^x + 3)$ are in A.P.,

$2 \log (2^x - 1) = \log 2 + \log (2^x + 3)$

$\Rightarrow (2^x - 1)^2 = 2(2^x + 3)$

$\Rightarrow 2^{2x} - 4 \times 2^x - 5 = 0$

$\Rightarrow (2^x - 5)(2^x + 1) = 0$

As 2^x cannot be negative, we get

$2^x - 5 = 0$

$\Rightarrow 2^x = 5$ or $x = \log_2 5$

Ans.[B]

Ex.7 If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P., then x is equal to

(A) $\log_a (\log_b a)$

(B) $\log_a (\log_e a) - \log_a (\log_e b)$

(C) $-\log_a (\log_a b)$

(D) $\log_a (\log_e b) - \log_a (\log_e a)$

where $e > 0$ and $e \neq 1$

Sol. As $\log_x a$, $a^{x/2}$, $\log_b x$ are in G.P.,

$(a^{x/2})^2 = \log_x a \cdot \log_b x$

$$\Rightarrow a^x = \frac{\log a}{\log x} \cdot \frac{\log x}{\log b} = \frac{\log a}{\log b} = \log_b a$$

$$\Rightarrow x = \log_a(\log_b a) = \log_a(\log_e a) - \log_a(\log_e b)$$

Ans.[A, B]

Ex.8 If $\sum_{r=1}^{n} r(r+1)(2r+3) = an^4 + bn^3 + cn^2 + dn + e$, then

(A) a = 1/2 (B) b = 8/3

(C) c = 9/2 (D) e = 0

Sol. We have

$$\sum_{r=1}^{n} r(r+1)(2r+3) = \sum_{r=1}^{n}(2r^3 + 5r^2 + 3r)$$

$$= 2\sum_{r=1}^{n} r^3 + 5\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r$$

$$= 2\left[\frac{1}{4}n^2(n+1)^2\right] + 5\left[\frac{1}{6}n(n+1)(2n+1)\right] + 3\left[\frac{1}{2}n(n+1)\right]$$

$$= \frac{1}{6}n(n+1)[3n(n+1) + 5(2n+1) + 9]$$

$$= \frac{1}{6}n(n+1)(3n^2 + 13n + 14)$$

$$= \frac{1}{6}[3n^4 + 16n^3 + 27n^2 + 14n]$$

$$= \frac{1}{2}n^4 + \frac{8}{3}n^3 + \frac{9}{2}n^2 + \frac{7}{3}n$$

Ans.[A, B, C, D]

Ex.9 The interior angles of a polygon are in arithmetic progression. The Smallest angle is 120º and the common difference is 5º. Find the number of sides of the polygon.

Sol. Let n = number of sides of the polygon

sum of all the interior angles of a polygon of

n sides = (2n – 4) × 90

Here the interior angles form an A.P. with

a = 120º and d = 5º

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \frac{n}{2}[2(120^\circ) + (n-1)5] = 2(n-2) \times 90$$

$\therefore \frac{n}{2}[240 + (n-1)5] = 180(n-2)$

$\therefore n[240 + (n-1)5] = 360(n-2)$

$\therefore 240n + n(n-1)5 = 360(n-2)$

$\therefore 48n + n(n-1) = 72(n-2)$

$\therefore 48n + n^2 - n = 72n - 144$

$\therefore n^2 + 48n - n - 72n + 144 = 0$

$\therefore n^2 - 25n + 144 = 0$

$\therefore (n-9)(n-16) = 0$

$\therefore n = 9$ or $n = 16$

But note that if n = 16 then greatest angle

$= a + (n-1)d$

$= 120 + (16-1)5$

$= 120 + 75 = 195.$

Greatest angle is 195º and common difference is 5º

∴ One of the angles would be 180º which is not possible in a polygon

Hence this is to be omitted

∴ The only possible value of n is n = 9. **Ans.**

Ex.10 If the sum of m terms of an A.P. is equal to the sum of the next n terms and the next p terms, prove that

$$(m+n)\left(\frac{1}{m}-\frac{1}{p}\right) = (m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$$

Sol. Let a denote the first term and d the common difference of the A.P. Then a_k, the k^{th} term of the A.P., is given by $a_k = a + (k-1)d$.

Suppose $a_1 + a_2 + ... + a_m = a_{m+1} + a_{m+2} + ... + a_{m+n}$

$\Rightarrow 2S_m = S_{m+n}$

where $S_k = a_1 + a_2 + ... + a_k$

$$\Rightarrow 2.\frac{m}{2}[2a+(m-1)d] = \frac{m+n}{2}[2a+(m+n-1)d]$$

$$\frac{2m}{m+n} = \frac{2a+(m+n-1)d}{2a+(m-1)d} = 1 + \frac{nd}{2a+(m-1)d}$$

$$\Rightarrow \frac{nd}{2a+(m-1)d} = \frac{m-n}{m+n} \quad ...(i)$$

$$\text{Similarly, } \frac{pd}{2a+(m-1)d} = \frac{m-p}{m+p} \quad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{n}{p} = \frac{m-n}{m+n} \cdot \frac{m-p}{m+p}$$

$$\Rightarrow \frac{(m+n)(m-p)}{p} = \frac{(m+p)(m-n)}{n}$$

$$\Rightarrow \frac{(m+n)(m-p)}{mp} = \frac{(m+p)(m-n)}{mn}$$

$$\Rightarrow (m + n)\left(\frac{1}{m} - \frac{1}{p}\right) = (m + p)\left(\frac{1}{m} - \frac{1}{n}\right)$$ **Ans.**

Ex.11 If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then prove that $x^{y+z}\, y^{z+x}\, z^{x+y} = 1$ Also prove that $x^{y+z} + y^{z+x} + z^{x+y} \geq 3$.

Sol. For logx, logy, logz to defined

$x > 0, y > 0, z > 0$

Now $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

$x = 10^{k(y-z)}$, $y = 10^{k(z-x)}$, $z = 10^{k(x-y)}$

$\Rightarrow (x)^{y+z} \cdot (y)^{z+x} \cdot (z)^{x+y}$

$= 10^{k(y^2+z^2+x^2-x^2-y^2-z^2)} = 10^0 = 1$

i.e. $x^{y+z} \cdot y^{z+x} \cdot z^{x+y} = 1$ ...(i)

Again take the positive number x^{y+z}, y^{z+x} and z^{x+y}

A.M. $\geq$ G.M.

$$\frac{x^{y+z} + y^{z+x} + z^{x+y}}{3} \geq \sqrt[3]{x^{y+z} y^{z+x} z^{x+y}} = 1$$

$\therefore x^{y+z} + y^{z+x} + z^{x+y} \geq 3$ Hence proved. **Ans.**

EXERCISE

Q.1 If the ratio of the sum of n terms of two AP's is 2n : (n+1), then ratio of their 8^{th} terms is-

(A) 15 : 8 (B) 8 : 13

(C) n : (n– 1) (D) 5 : 17

Q.2 The sum of n terms of an AP is $3n^2 + 5n$. The number of term which equals 164 is-

(A) 13 (B) 21

(C) 27 (D) None of these

Q.3 If a, b, c be the I^{st}, 3^{rd} and n^{th} terms respectively of an A.P., then sum to n terms is –

(A) $\frac{c+a}{2} + \frac{c^2 - a^2}{b-a}$ (B) $\frac{c+a}{2} - \frac{c^2 - a^2}{b-a}$

(C) $\frac{c+a}{2} + \frac{c^2 + a^2}{b-a}$ (D) $\frac{c+a}{2} + \frac{c^2 + a^2}{b+a}$

Q.4 If a_1, a_2, a_3, ...is an A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ then $a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to-

(A) 909 (B) 75

(C) 750 (D) 900

Q.5 The sum of all even positive integers less then 200 which are not divisible by 6 is –

(A) 6534 (B) 6354

(C) 6543 (D) 6454

Q.6 If x, y, z are in AP, a is AM between x and y and b is AM between y and z; then AM between a and b will be-

(A) $\frac{1}{3}(x + y + z)$ (B) z

(C) x (D) y

Q.7 If n AM's are inserted between 1 and 31 and ratio of 7^{th} and $(n-1)^{th}$ A.M. is 5 : 9, then n equals-

(A) 12 (B) 13 (C) 14 (D) None

Q.8 If the angles of a quadrilateral are in A.P. whose common difference is 10º, then the angles of the quadrilateral are-

(A) 65º, 85º, 95º, 105º (B) 75º, 85º, 95º, 105º

(C) 65º, 75º, 85º, 95º (D) 65º, 95º, 105º, 115º

Q.9 Divide 20 into four parts which are in A.P., such that the product of the first and fourth is to the product of the second and third is 2 : 3 -

(A) 2, 4, 6, 8 (B) 3, 5, 7, 9

(C) 4, 6, 8, 10 (D) None of these

Q.10 If $a^2(b+c)$, $b^2(c+a)$, $c^2(a+b)$ are in A.P., then-

(A) a,b,c are in A.P (B) ab + bc + ca = 0.

(C) a,b,c are in G.P (D) ab – bc – ca = 0

Q.11 The sum of the series

$1.3^2 + 2.5^2 + 3.7^2 + ...$ upto 20 terms is-

(A) 188090 (B) 180890

(C) 189820 (D) None of these

Q.12 The sum to infinity of the series

$\frac{1}{2.4} + \frac{1}{4.6} + \frac{1}{6.8} + \frac{1}{8.10} + ...$is-

(A) 1/4 (B) 1/8 (C) 1/2 (D) 1/16

Q.13 A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying odd places, the common ratio will be equal to-

(A) 2 (B) 3 (C) 4 (D) 5

Q.14 If in a geometric progression $\{a_n\}, a_1 = 3$, $a_n = 96$ and $S_n = 189$, then the value of n is-

(A) 5 (B) 6

(C) 7 (D) 8

Q.15 In any G.P. the first term is 2 and last term is 512 and common ratio is 2, then 5^{th} term from end is-

(A) 16 (B) 32

(C) 64 (D) None of these

Q.16 If the sum of an infinite G.P. be 3 and the sum of the squares of its term is also 3, then its first term and common ratio are-

(A) 3/2, 1/2 (B) 1/2, 3/2

(C) 1, 1/2 (D) None of these

Q.17 Sum $\frac{1}{5} + \frac{1}{7} + \frac{1}{5^2} + \frac{1}{7^2} + ...$ to ∞ =

(A) 5/12 (B) 3/4 (C) 7/12 (D) 3/49

Q.18 The sum of infinite number of terms of a decreasing G.P. is 4 and the sum of the squares of its terms to infinity is $\frac{16}{3}$, then the G.P is –

(A) 2, 1, 1/2, 1/4,…. (B)1/2, 1/4, 1/8,……

(C) 2, 4, 8, …. (D) None of these

Q.19 The sum of 10 terms of the series

.7 + .77 + .777 + ...is-

(A) $\frac{7}{9}\left(89+\frac{1}{10^{10}}\right)$ (B) $\frac{7}{81}\left(89+\frac{1}{10^{10}}\right)$

(C) $\frac{7}{81}\left(89+\frac{1}{10^{9}}\right)$ (D) None of these

Q.20 If 0 < x, y, a, b < 1, then the sum of the infinite terms of the series $\sqrt{x}\,(\sqrt{a}+\sqrt{x}) + \sqrt{x}\,(\sqrt{ab}+\sqrt{xy}) + \sqrt{x}\,(b\sqrt{a}+y\sqrt{x}) + \ldots$ is-

(A) $\frac{\sqrt{ax}}{1+\sqrt{b}}+\frac{x}{1+\sqrt{y}}$ (B) $\frac{\sqrt{x}}{1+\sqrt{b}}+\frac{\sqrt{x}}{1+\sqrt{y}}$

(C) $\frac{\sqrt{x}}{1-\sqrt{b}}+\frac{\sqrt{x}}{1-\sqrt{y}}$ (D) $\frac{\sqrt{ax}}{1-\sqrt{b}}+\frac{x}{1-\sqrt{y}}$

Q.21 The sum to n terms of the series

$\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+\ldots$ is equal to-

(A) $1-(1/3)^n$ (B) $2-\frac{1}{2}(2/3)^n$

(C) $n-n(1/3)^n$ (D) None of these

Q.22 The product of three geometric means between 4 and 1/4 will be -

(A) 4 (B) 2

(C) – 1 (D) 1

Q.23 If the A.M. is twice the G.M. of the numbers a and b, then a : b will be-

(A) $\frac{\sqrt{3}+2}{\sqrt{3}-2}$ (B) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

(C) $\frac{\sqrt{3}-2}{\sqrt{3}+2}$ (D) None of these

Q.24 If one A.M. i.e., A and two G.M.'s i.e., p and q are to be inserted between two given numbers, then $\frac{p^2}{q}+\frac{q^2}{p}=$

(A) A (B) 2A (C) A/2 (D) A^2

Q.25 If the product of three numbers in GP is 3375 and their sum is 65, then the smallest of these numbers is-

(A) 3 (B) 5 (C) 4 (D) 6

Q.26 Three numbers whose sum is 15 are in A.P. If 1,4,19 be added to them respectively the resulting numbers are in G.P. Then the numbers are-

(A) 2, 5 , 8 (B) 26, 5, – 16

(C) 36, 5, –16 (D) None of these

Q.27 Four numbers are such that the first three are in A.P. while the last three are in G.P. If the first number is 6 and common ratio of G.P. is 1/2, then the numbers are -

(A) 6, 8, 4, 2 (B) 6, 10, 14, 7

(C) 6, 9, 12, 6 (D) 6, 4, 2, 1

Q.28 If x, y, z are in G.P. then $x^2 + y^2$, $xy + yz$, $y^2 + z^2$ are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.29 If a, b, c, d are in G.P. then a + b, b + c, c + d are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.30 If a, b, c are in G.P. then-

(A) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right) = a^3 + b^3 + c^3$

(B) $(a^2 - b^2)(b^2 + c^2) = (b^2 - c^2)(a^2+b^2)$

(C) $a^2b^2c^2\left(\frac{1}{a^3}-\frac{1}{b^3}-\frac{1}{c^3}\right) = a^3 + b^3 + c^3$

(D) $(a^2 + b^2)(b^2 + c^2) = (b^2 + c^2)(a^2+b^2)$

ANSWER KEY

EXERCISE

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	A	D	A	D	C	B	A	A,B	A	A	C	B	B	A	A	A	B	D
Que.	21	22	23	24	25	26	27	28	29	30										
Ans.	D	D	B	B	B	A	D	B	B	A,B										