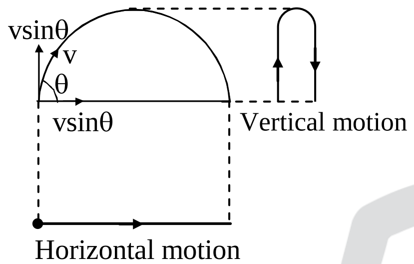
PROJECTILE MOTION

1. Basic concept

(i) Any particle which is thrown into space or air such that it moves under the effect of gravity and not propelled by any engine is called as projectile. The motion of this particle is referred as projectile motion.

(ii) If F is constant, then a = constant and when force is in oblique direction with initial velocity, the result is the parabolic path.



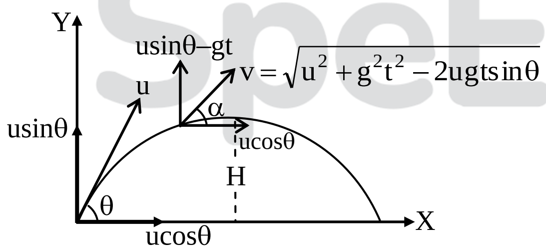
Parabolic motion = vertical motion + horizontal motion

(iii) Projectile motion is considered as two simultaneous motion in mutually perpendicular directions which are completely independent from each other i.e. horizontal motion and vertical motion.

2. Projectile thrown at an angle with horizontal

(i) Consider a projectile thrown with a velocity u making an angle θ with the horizontal from the ground.

(ii) Initial velocity u is resolved in horizontal and vertical direction i.e. $u_x = u\cos\theta$, $u_y = u\sin\theta$.



(iii) Again this projectile motion can be thought of as the combination of horizontal and vertical motion. So,

In horizontal direction	In vertical direction
(a) Initial velocity $u_x = u\cos\theta$	Initial velocity $u_y = u\sin\theta$
(b) Acceleration $a_x = 0$ (in absence of external force)	Acceleration $a_y = -g$
(c) Velocity at time t, $v_x = u\cos\theta$	Velocity at time t, $v_y = u\sin\theta - gt$

2.1 Trajectory equation

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$y = px - qx^2$ (path is parabolic)

$p = \tan\theta$ and $q = \dfrac{g}{2u^2\cos^2\theta}$

2.2 Time of flight: Time taken by projectile in air.

$$T = \frac{2u\sin\theta}{g}$$

2.3 Horizontal range: Maximum horizontal distance covered by projectile.

$$R = u_x . T$$

$$R = \frac{u^2\sin 2\theta}{g}$$

2.4 Maximum height: Maximum vertical distance achieved by projectile.

$$H = \frac{u^2\sin^2\theta}{2g}$$

2.5 Resultant velocity at time 't':

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

where

$$|v| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

and $\tan\theta = \dfrac{v_x}{v_y} = \dfrac{\text{velocity along y} - \text{axis}}{\text{velocity along x} - \text{axis}}$

2.6 General results:

(i) For maximum range $\theta = 45^\circ$

$$R_{max} = \frac{u^2}{g}$$

(ii) We get the same range for two angles of projections α and $(90 - \alpha)$ but in both cases, maximum height attained by the particle is different.

(iii) If $R = H$

i.e. $\dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2\sin^2\theta}{2g}$

$\tan\theta = 4$

$\theta = \tan^{-1}4 = 76^\circ$

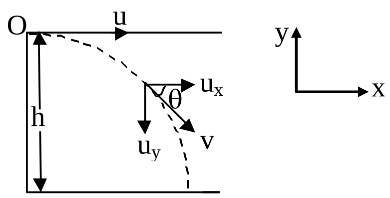
(iv) Range can also be expressed as

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u \sin\theta . u\cos\theta}{g} = \frac{2u_x u_y}{g}$$

3. Projectile thrown parallel to the horizontal

Consider a projectile thrown from point O at some height h from the ground with a velocity u.

Now we shall deal the characteristics of projectile motion with the help of horizontal and vertical direction i.e.



Horizontal direction

(i) Initial velocity $u_x = u$

(ii) Acceleration $a_x = 0$

Vertical direction

(i) Initial velocity $u_y = 0$

(ii) Acceleration $a_y = -g$

3.1 Trajectory Equation:

$$y = \frac{-1}{2} g \frac{x^2}{u^2}$$

This is called trajectory equation

3.2 Velocity at a general point P (x, y)

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\therefore \quad v = \sqrt{u^2 + g^2 t^2}$$

3.3 Displacement: The displacement of the particle is expressed as

$$S = x\hat{i} + y\hat{j} \qquad \text{where } |S| = \sqrt{x^2 + y^2}$$

$$= (ut)\hat{i} - (½ gt^2)\hat{j}$$

3.4 Time of flight: $t = \sqrt{\frac{2h}{g}}$

3.5 Horizontal range:

$$R = u_x.t$$

$$R = u\sqrt{\frac{2h}{g}}$$

3.6 Velocity at height h:

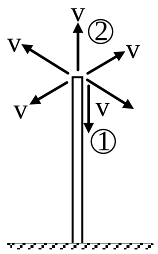
$$v_y = \sqrt{2gh}$$

NOTE:

(i) If a projectile is projected with initial horizontal velocity u and another particle is dropped from same height at the same time, both the projectile would strike the ground with same vertical velocity. Both will have same vertical components of velocity but their net velocities would be different.

(ii) Relative motion of one horizontal projectile w.r.t. motion of particle dropped from same height at the same time would be in straight line joining them.

(iii) All the particles thrown with same initial speed would strike the ground with same speed at different times irrespective of their initial direction of velocities.



(a) Time would be least for the particle thrown with velocity v downward i.e. particle (1)

(b) Time would be maximum for the particle thrown with velocity v vertically upwards i.e. particle (2)

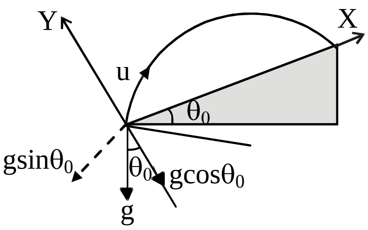
4. Projectile thrown from an inclined plane

Consider a particle thrown from the base of an inclined plane with a velocity u at an angle θ from the horizontal. The angle of inclination is θ_0.

We assume X axis along the inclined plane and Y axis perpendicular to it.

$a_x = -g \sin\theta_0$ $\quad$ $a_y = -g \cos\theta_0$

$u_x = u \cos(\theta - \theta_0)$ $\quad$ $u_y = u \sin(\theta - \theta_0)$



For vertical direction, using II[nd] eqn. of motion

$$0 = u_y T + \frac{1}{2} a_y . T^2$$

$$0 = u \sin(\theta - \theta_0) T - \frac{1}{2} g \cos\theta_0 T^2$$

$$T = \frac{2u\sin(\theta - \theta_0)}{g\cos\theta_0}$$

For horizontal direction,

$$R = u\cos(\theta - \theta_0) T - \frac{1}{2} g \sin\theta_0 . T^2$$

$$= u\cos(\theta - \theta_0) \frac{2u\sin(\theta-\theta_0)}{g\cos\theta_0} - \frac{1}{2} g \sin\theta \left[\frac{2u\sin(\theta-\theta_0)}{g\cos\theta_0}\right]^2$$

$$R = \frac{2u^2 \sin(\theta - \theta_0)\cos\theta}{g\cos^2\theta_0}$$

NOTE:

(i) Range is maximum when,

$$\theta = \pi/4 + \theta_0/2$$

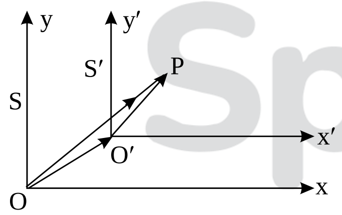
(ii) When a projectile is thrown from base towards the upper side of inclined plane then,

$$R_{max} = \frac{u^2}{g(1+\sin\theta_0)}$$

(iii) When R = R_{max}, projectile hits the inclined plane at right angles.

5. Relative motion in two dimension

I. (A) Consider two frames of reference S and S′ and suppose a particle P is observed from both the frames.



(B) $\vec{r}_{P,S'} = \vec{r}_{P,S} - \vec{r}_{S',S}$

Where $\vec{r}_{P,S'}$ = position vector of particle P w.r.t. S′ - frame

$\vec{r}_{P,S}$ = position vector of particle P w.r.t. S - frame

$\vec{r}_{S',S}$ = position vector of the origin of S′ w.r.t. the origin of S

(C) $\vec{v}_{P,S'} = \vec{v}_{P,S} - \vec{v}_{S',S}$

$\vec{v}_{P,S'}$ = velocity of particle P w.r.t. S′ - frame

$\vec{v}_{P,S}$ = velocity of particle P w.r.t. S – frame

$\vec{v}_{S',S}$ = velocity of S′ - frame w.r.t. S - frame

(D) $\boxed{\vec{a}_{P,S'} = \vec{a}_{P,S} - \vec{a}_{S',S}}$

$\vec{a}_{P,S'}$ = acceleration of particle P w.r.t. S' - frame

$\vec{a}_{P,S}$ = acceleration of particle P w.r.t. S - frame

$\vec{a}_{S',S}$ = acceleration of S' - frame w.r.t. S – frame

(E) In case of relative motion, modified newton's equations of motion for two body A and B are

$$\vec{v}_{AB} = \vec{u}_{AB} + \vec{a}_{AB}\,t$$

$$\vec{S}_{AB} = \vec{u}_{AB}t + \frac{1}{2}\vec{a}_{AB}\,t^2$$

$$v_{AB}^2 = u_{AB}^2 + 2\,\vec{a}_{AB}\cdot\vec{S}_{AB}$$

II. The river and the swimmer problem:

If $\vec{V}$ = swimming velocity of a man w.r.t. stationary river

$\vec{V}_R$ = velocity of the river w.r.t. ground

$\vec{V}_m$ = velocity of the man w.r.t. ground

then $\vec{V} = \vec{V}_m - \vec{V}_R$

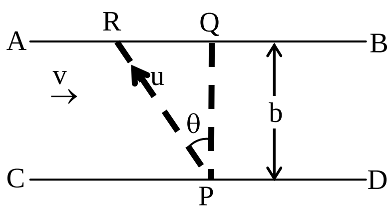
$\Rightarrow \vec{V}_m = \vec{V} + \vec{V}_R$

A useful hint for solving problems regarding the motion of object across a river:

AB and CD $\rightarrow$ two banks of river, v $\rightarrow$ velocity of river, b $\rightarrow$ width of river

A swimmer wants to cross the river starting from a point P to reach a point directly opposite to P on the bank CD in a given time t, then

(a) $t = \dfrac{b}{u\cos\theta} \Rightarrow t = \dfrac{b}{\sqrt{u^2 - v^2}}$



(b) Resultant velocity of swimmer

$V = \sqrt{u^2 - v^2}$

(c) Distance covered in the direction of flow

$= \left(\dfrac{b}{u}\right)v = b\left(\dfrac{v}{u}\right)$

III. The man and the rain problem:

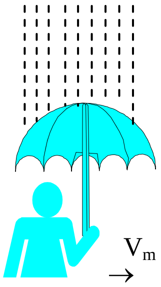
If rain is falling vertically with a velocity $\vec{V}_R$ and an observer is moving horizontally with speed $\vec{V}_m$, the velocity of rain relative to observer:

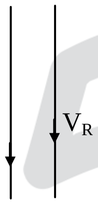
$$\vec{V}_{Rm} = \vec{V}_R - \vec{V}_m ,$$

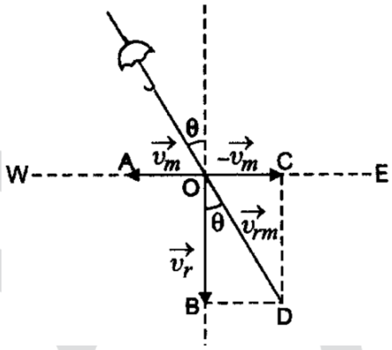
which by law of vector addition has magnitude

$V_{RM} = \sqrt{V_R^2 + V_m^2}$ and direction

$\theta = \tan^{-1}\left(\frac{V_m}{V_R}\right)$ with vertical.



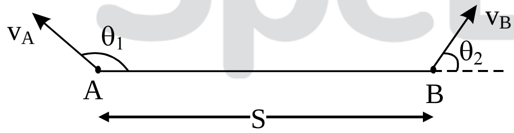




Note: In the above problem if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain with respect to man i.e. the umbrella should be held making an angle $\left(\theta = \tan^{-1}\frac{v_m}{v_r}\right)$ west of vertical.

IV. If two particles A and B are separated by a distance S at time t and their velocities are as shown in figure.

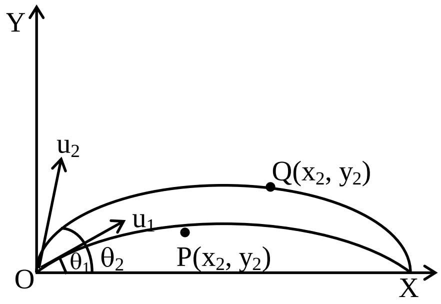
$$+\frac{ds}{dt} = v_B \cos\theta_2 - v_A \cos\theta_1$$



SOLVED EXAMPLES

Ex.1 Show that the motion of one projectile as seen from another projectile will always be a straight-line motion.

Sol. As clearly shown in figure, assume two projectiles to be thrown from the origin O of the XY–plane with velocities u_1 and u_2, making angles θ_1 and θ_2 respectively with X–axis.



After time t, let the two projectile occupy positions P (x_1, y_1) and Q (x_2, y_2). Then,

$x_1 = u_1\cos\theta_1.t$

and $y_1 = u_1\sin\theta_1.t - ½\, gt^2$

Also, $x_2 = u_2\cos\theta_2.t$

and $y_2 = u_2\sin\theta_2.t - ½\, gt^2$

∴ $x_2 - x_1 = (u_2\cos\theta_2 - u_1\cos\theta_1).t$

$y_2 - y_1 = (u_2\sin\theta_2 - u_1\sin\theta_1).t$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{u^2 \sin\theta_2 - u_1 \sin\theta_1}{u_2\cos\theta_2 - u_1\cos\theta_1} = m \text{ (a cont.)}$$

If (x,y) be the coordinates of point Q relative to the point P, then

$x_2 - x_1 = x$ and $y_2 - y_1 = y$

∴ $\frac{y}{x} = m$ or $y = mx$

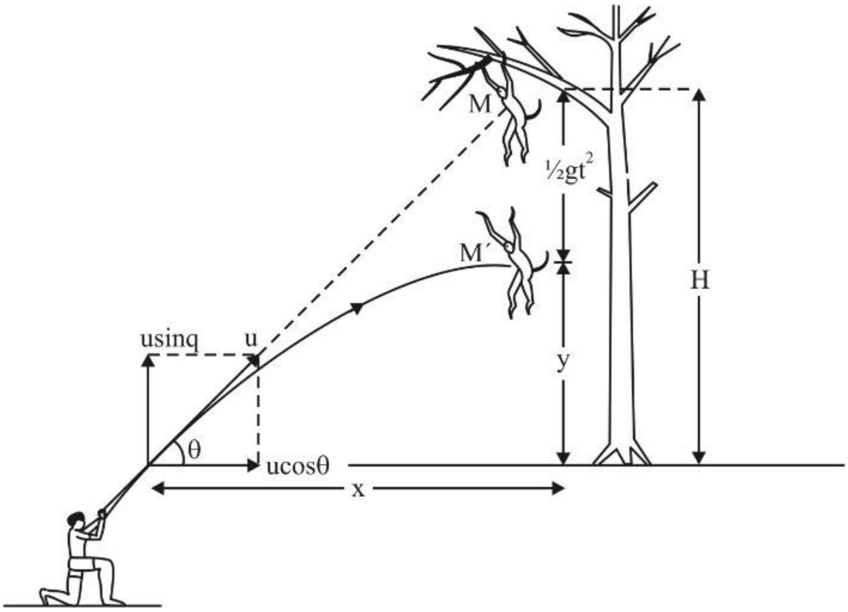
This is the equation of a straight line. Hence the motion of a projectile as seen from another projectile is a straight-line motion.

Ex.2 A hunter aims his gun and fires a bullet directly towards a monkey sitting at a distant tree. At the instant the bullet leaves the barrel of the gun, the monkey drops from the tree freely. Will the bullet hit the monkey?

Sol. Suppose the horizontal distance to the tree be x from the hunter and original height of the monkey be H. The angle of projection θ will be given by $\tan\theta = (H/x)$. If there was no gravity the bullet would reach height H in the time t taken for it to travel the horizontal distance x.

i.e., $H = (u \sin\theta) \times t$

and $t = (x/u \cos\theta)$



However, because of gravity, the bullet has an acceleration 'g' vertically downwards: so in time t the bullet will reach a height

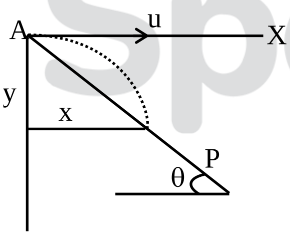
$y = (u \sin\theta) \times t - (½)\, gt^2 = H - ½\, gt^2$,

This is lower than H by $½gt^2$ which is exactly the amount the monkey falls in this time.

So the bullet will hit the monkey regardless of the initial velocity of the bullet so long as it is great enough to travel the horizontal distance to the tree before hitting the ground. However, for large u lesser will be the time of motion: so the monkey is hit near its initial position and for small u it is hit just before it reaches the floor.

Ex.3 A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane.

Sol. Suppose that the particle strikes the plane at a point P with coordinates (x,y). Consider the motion between points A and P by resolving them into two directions



Motion in x – direction

Initial velocity = u,

acceleration = 0

$\therefore \quad x = ut$ ...(i)

Motion in y – direction

Initial velocity = 0,

acceleration = g,

$y = (½)\, gt^2$ ...(ii)

Eliminating t from (i) & (ii)

$$y = \frac{1}{2} g \frac{x^2}{u^2}$$

Also, $y = x \tan\theta$

Therefore $\frac{gx^2}{2u^2} = x \tan\theta$

or $\frac{gx^2}{2u^2} - x\tan\theta = 0$

or $x\left(\frac{gx}{2u^2} - \tan\theta\right) = 0$

or $x = \frac{2u^2 \tan\theta}{g}$ ...(iii)

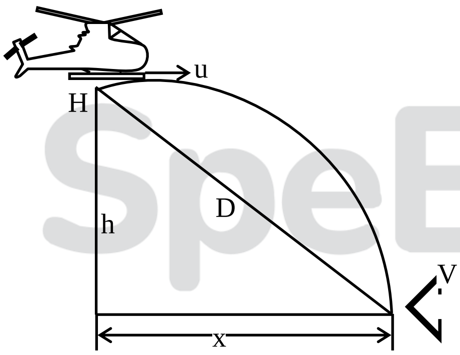
Thus, $y = x\tan\theta = \frac{2u^2 \tan^2\theta}{g}$ ...(iv)

$\therefore$ Distance $AP = \sqrt{[x^2 + y^2]}$

$$= \frac{2u^2}{g}\tan\theta = \sqrt{(1+\tan^2\theta)} = \frac{2u^2}{g}\tan\theta \sec\theta,$$

Ex.4 A helicopter on a flood relief mission flying horizontally with a speed u at an altitude h, has to drop a food packet for a victim standing on the ground. At what distance from the victim should the food packet be dropped?

Sol. In figure H, represents positions of the helicopter and V that of the victim. For vertical motion of the packet



$S = ut + ½at^2$

$h = 0 + \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2h}{g}}$

Horizontal distance covered by the food packet in time t, $x = ut = u\sqrt{\frac{2h}{g}}$

Now the distance of the point of projection from the food packet is

$$D = \sqrt{h^2 + x^2} = \sqrt{h^2 + \frac{2u^2 h}{g}}$$

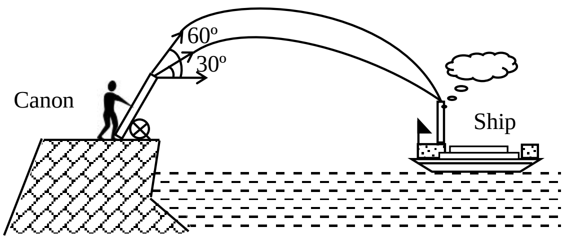
Ex.5 At a harbor, enemy ship is at a distance $180\sqrt{3}$m from the security cannon having a muzzle velocity of 60m/s.

(a) At what angle must the cannon be elevated to hit the ship?

(b) What is the time of flight?

(c) How for should the ship be moved away from its initial position so that it becomes beyond the range of the cannon? (take g = 10m/s^2)

Sol. (a) For hitting the ship, the range of cannon must be equal to the distance of ship from cannon i.e.,



Range $= 180\sqrt{3}$

$$\frac{u^2 \sin 2\theta}{g} = 180\sqrt{3}$$

or $\sin 2\theta = \frac{180\sqrt{3} \times 10}{60 \times 60} = \frac{\sqrt{3}}{2}$

i.e., $2\theta = 60^\circ$ or 120°

or $\theta = 30^\circ$ or 60°

(a) The cannon must be elevated at an angle of 30º or 60º

(b) As T = (2u sinθ/g), depending on θ there are two times of flight which are given by

$$t_1 = \frac{2 \times 60}{10} \times \sin 30 = 6\text{s}$$

and $t_2 = \frac{2 \times 60}{10} \times \sin 60 = 6\sqrt{3} = 10.4\text{s}$

(c) The maximum range of cannon

(when θ = 45º)

$$R_{max} = \frac{u^2}{g} = \frac{60 \times 60}{10} = 360\text{m}$$

And as initially the ship is 180√3m, so to become out of maximum range of cannon, the ship should be moved away from the harbour from its initial position by at least 360 – 180√3 = 48.6m.

Ex.6 Two persons simultaneously aim their guns at a bird sitting on a tree. The first person fires his shot with a speed of 100m/s at an angle of projection 30 degrees. The second person is ahead of the first by a distance of 50m and fires his shot at a speed of 80m/s. How must he aim his gun so that both the shots hit the bird simultaneously? Calculate the distance of the foot of the tree from the two persons and the height of the tree. When do the two shots hit the bird?

Sol. The situation of the problem is shown in the fig.

For first shot:

Let horizontal displacement = x_1

Vertical displacement = h

For second shot :

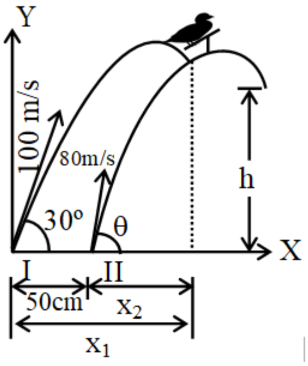
Horizontal displacement = x_2

Vertical displacement = h

Now it is given $x_1 = x_2 + 50$

For first shot, $h = (100 \sin 30)t - ½ gt^2$

For second shot $h = (80 \sin\theta)t - ½gt^2$



or $\sin\theta = \frac{100 \sin 30}{80} = \frac{50}{80} = 0.625$

$\therefore \quad \theta = \sin^{-1}0.625 = 38°68'$

Now, $x_1 = (100 \cos 30°)t$

and $x_2 = (80 \cos 38°68')t$

$\therefore$ (100 cos 30°)t = 80 cos(38°68´)t + 50 or t [100(√3/2)– 80 × 0.7806] = 50

or $t = \frac{50}{24.1526} = 2.070$ sec.

Further, $x_1 = 100 \cos 30° \times 2.070 = 179.27$m

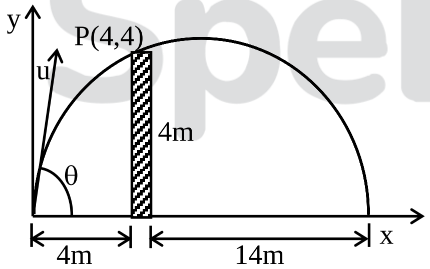
and $x_2 = 179.27 - 50 = 129.27$ m

or,$h = (100 \sin 30°)(2.07) - (½) \times 9.8 \times (2.07)^2 = 82.504$m

Ex.7 A ball is thrown from ground level so as to just clear a wall 4m high at a distance of 4m and falls at a distance of 14m from the wall. Find the magnitude and direction of the velocity.

Sol. The ball passes through the point P(4,4). So its range = 4 + 14 = 18m.

The trajectory of the ball is,



$$y = x \tan\theta - \frac{1}{2}\frac{g}{u^2 \cos^2\theta} x^2$$

$$= x \tan\theta \left[1 - \frac{gx}{2u^2 \cos^2\theta \tan\theta}\right]$$

$$= x \tan\theta \left[1 - \frac{gx}{2u^2 \sin\theta \cos\theta}\right]$$

$$= x \tan\theta \left[1 - \frac{gx}{u^2 \sin 2\theta}.x\right] = x \tan\theta \left[1 - \frac{x}{R}\right]$$

Now x = 4m, y = 4m and R = 18 m

$\therefore \quad 4 = 4\tan\theta\left[1-\frac{4}{18}\right] = 4\tan\theta.\ \frac{7}{9}$

or $\tan\theta = \frac{9}{7}$, $\sin\theta = \frac{9}{\sqrt{130}}$, $\cos\theta = \frac{7}{\sqrt{130}}$

Hence $\quad R = \frac{2u^2\sin\theta\cos\theta}{g}$

or $\quad 18 = \frac{2}{9.8} \times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}}$

or $\quad u^2 = \frac{18\times9.8\times130}{2\times9\times7} = 182$

or $\quad u = \sqrt{182} = 13.5\ ms^{-1}$,

Also $\theta \;= \tan^{-1}(9/7) = 52.1^\circ$

Ex.8 A gun kept on a straight horizontal road is used to hit a car travelling along the same road away from the gun with a uniform speed of 72 km h^{-1}. The car is at a distance of 500 m from the gun, when the gun is fired at an angle of 45º with the horizontal. Find

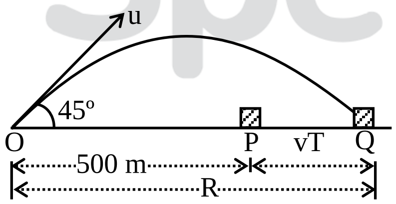
(a) the speed of projection of the shell from the gun.

(b) the distance of the car from the gun, when the shell hits it.

Sol. Consider that at the instant, the shell is fired from the gun at point O, the car is at point P The shell will move along parabolic path for T its time of flight and will then strike the ground at point Q at a distance R, equal to the horizontal range. Therefore, the shell will hit the car, if in the time T (time of flight), the car moves the distance PQ= v T, where v is velocity of the car. Since the car is initially at a distance of 500 m from the gun, it follows that

R = 500 + v T .....(1)

(a) Let u be the speed of projection of shell.



Now, $\quad R = \frac{u^2\sin2\theta}{g}$

Here $\theta = 45^\circ$

$R = \frac{u^2\sin2\times45^\circ}{9.8} = \frac{u^2\sin90^\circ}{9.8} = \frac{u^2}{9.8}$

Also, $\quad T = \frac{2u\sin\theta}{g} = \frac{2u\sin45^\circ}{9.8}$

$= \frac{2u}{9.8} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}u}{9.8}$

Further, velocity of the car, $v = 72\ km\ h^{-1} = 72 \times (1000m) \times (60 \times 60\ s)^{-1} = 20\ m\ s^{-1}$

Substituting for R, T and v in equation (1), we have $\frac{u^2}{9.8} = 500 + 20 \times \frac{\sqrt{2}u}{9.8}$

or $u^2 - \sqrt{2}\, u - 500 \times 9.8 = 0$

On solving for u we have

$u = 10\,(\sqrt{2} + \sqrt{51})$

$u = 10\,(1.414 + 7.141)$

$= 10 \times 8.555 = 85.55\ m\ s^{-1}$

(b) When the shell hits the car, it will be at distance equal to the horizontal range of the shell.

Therefore, required distance,

$$R = \frac{u^2}{9.8} = \frac{(85.55)^2}{9.8} = 746.82\ m$$

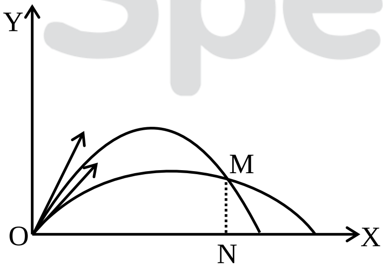
Ex.9 Two particles are projected from a point simultaneously with velocities whose horizontal and vertical components are u_1, v_1 and u_2, v_2 respectively. Prove that the interval between their passing through the other common point of their path is, $\frac{2(v_1u_2 - v_2u_1)}{g(u_1 + u_2)}$

Sol. We have analyzed the problem with the help of figure. Suppose the two particles pass through the other common point at M. Let the two particles starting from 1st common point O take time t_1 and t_2 respectively to pass through the other common point M. As distances covered by two particles in horizontal directions up to point M will be same, hence

$u_1t_1 = u_2t_2$ ....(i)

Similarly, in vertical directions also

$v_1t_1 - \frac{1}{2}gt_1^2 = MN = v_2t_2 - \frac{1}{2}gt_2^2$ ....(ii)



From eqn. (ii) we get

$$t_1^2 - t_2^2 = \frac{2(v_1t_1 - v_2t_2)}{g} \quad ....(iii)$$

but from equation (i)

$$\frac{u_1}{u_2} = \frac{t_2}{t_1} \text{ or } \frac{u_1 + u_2}{u_2} = \frac{t_2 + t_1}{t_1}$$

or $t_1 + t_2 = \frac{(u_1 + u_2)}{u_2} \times t_1$ ....(iv)

From eqn. (iii) we get

$$(t_1 + t_2)(t_1 - t_2) = \frac{2(v_1t_1 - v_2t_2)}{g}$$

or $$(t_1 - t_2) \times \frac{(u_1 - u_2)}{u_2} \times t_1 = \frac{2}{g}\left(v_1t_1 - v_2\frac{u_1}{u_2} \times t_1\right)$$

or $$(t_1 - t_2)\times \frac{(u_1 + u_2)}{u_2} = \frac{2}{g}\left(v_1 - v_2\frac{u_1}{u_2}\right)$$

or $$(t_1 - t_2) = \frac{2}{g} \times \frac{u_2}{(u_1 + u_2)} \times \frac{(v_1u_2 - v_2u_1)}{u_2}$$

or $$(t_1 - t_2) = \frac{2}{g}\frac{(v_1u_2 - v_2u_1)}{(u_1 + u_2)}$$

Ex.10 If the horizontal range of a projectile is a and the maximum height attained by it is b, then prove that the velocity of projection is, $\left[2g\left(b + \frac{a^2}{16b}\right)\right]^{1/2}$.

Sol. Suppose u = velocity of projection and

α = angle of projection

Now α = horizontal range $= \frac{u^2 \sin 2\alpha}{g} = \frac{u^2 2\sin\alpha\cos\alpha}{g}$

or $2 \sin\alpha \cos\alpha = \frac{ag}{u^2}$ ...(i)

On squaring eqn. (i), we get

$4 \sin^2\alpha \cos^2\alpha = \frac{a^2g^2}{u^4}$ ...(ii)

From the problem

b = maximum height attained $= \frac{u^2 \sin^2\alpha}{2g}$

or $\sin^2\alpha = \frac{2bg}{u^2}$ ...(iii)

From eq. (ii) & (iii)

or $4 \sin^2\alpha (1 - \sin^2\alpha) = \frac{a^2g^2}{u^4}$

or $4. \frac{2bg}{u^2}\left(1 - \frac{2bg}{u^2}\right) = \frac{a^2g^2}{u^4}$

or $8bg - \frac{16b^2g^2}{u^2} = \frac{a^2g^2}{u^2}$

or $8 bgu^2 = a^2g^2 + 16b^2g^2$,

or $u^2 = \frac{a^2g^2 + 16b^2g^2}{8bg} = 2bg + \frac{a^2g}{8b}$ $\quad\therefore\quad u = \left[2g\left(b + \frac{a^2}{16b}\right)\right]^{1/2}$

EXERCISE - 1

Two-dimensional motion: General study

Q.1 A bullet is fired in a horizontal direction from a tower while a stone is simultaneously dropped from the same point then –

(A) The bullet and the stone will reach the ground simultaneously

(B) The stone will reach earlier

(C) The bullet will reach earlier

(D) Nothing can be predicted

Projectile motion : Projectile hitting ground at same horizontal level

Q.2 If T be the total time of flight of a current of water and H be the maximum height attained by it from the point of projection, then H/T will be - (u = projection velocity, θ = projection angle)

(A) (1/2) u sin θ   (B) (1/4) u sin θ

(C) u sin θ   (D) 2u sin θ

Q.3 If a baseball player can throw a ball at maximum distance d over a ground, the maximum vertical height to which he can throw it, will be (Ball have same initial speed in each case) -

(A) d/2   (B) d

(C) 2d   (D) d/4

Q.4 What is the average velocity of a projectile between the instants is crosses half the maximum height. It is projected with a speed u at an angle θ with the horizontal-

(A) u sinθ   (B) u cosθ

(C) u tanθ   (D) u

Q.5 An artillery piece which consistently shoots its shell with the same muzzle speed has a maximum range of R. To hit a target which is R/2 from the gun and on the same level, at what elevation angle should the gun be pointed(height of gun from ground in neglected)-

(A) 30º   (B) 45º

(C) 60º   (D) 75º

Q.6 A large number of bullets are fired in all directions with the same speed v from ground. What is the maximum area on the ground on which these bullets will spread (height of gun from ground assume negligible)-

(A) $\frac{\pi v^2}{g}$ (B) $\frac{\pi v^4}{g^2}$

(C) $\frac{\pi^2 v^4}{g^2}$ (D) $\frac{\pi^2 v^2}{g^2}$

Q.7 A cannon ball has a range R on a horizontal plane. If h and h' are the greatest heights in the two paths for which this is possible, then–

(A) $R = 4\sqrt{hh'}$ (B) $R = \frac{4h}{h'}$ (C) R = 4 h h'   (D) $R = \sqrt{hh'}$

Q.8 Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\pi/3$ and its maximum height is y_1 then the maximum height of the other will be –

(A) $3y_1$ (B) $2y_1$

(C) $y_1/2$ (D) $y_1/3$

Q.9 An object is thrown at an angle α to the horizontal ($0^\circ < \alpha < 90^\circ$) with a certain velocity on horizontal ground. Then during ascent (ignoring air drag) the acceleration –

(A) With which the object moves is $\vec{g}$ at all points

(B) Tangential to the path decreases

(C) Normal to the path increases, becoming equal to g at the highest point

(D) All of the above

Q.10 A projectile is thrown with a velocity of 20 m/s, at an angle of 60º with the horizontal. After how much time the velocity vector will make an angle of 45º with the horizontal (in upward direction) is (take $g = 10 m/s^2$)-

(A) $\sqrt{3}$ sec (B) $1/\sqrt{3}$ sec

(C) $(\sqrt{3} - 1)$ sec (D) None of these

Projectile thrown from some height above ground, Projection on inclined plane

Q.11 An aeroplane was flying horizontally with a velocity of 720 km/h at an altitude of 490 m. When it is just vertically above the target a bomb is dropped from it. How far horizontally it missed the target ?

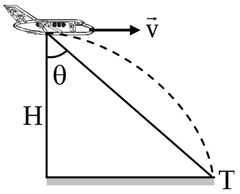
(A) 1000 m (B) 2000 m

(C) 100 m (D) 200 m

Q.12 From the top of a tower of height h a body of mass m is projected in the horizontal direction with a velocity v, it falls on the ground at a distance x from the tower. If a body of mass 2m is projected from the top of another tower of height 2h in the horizontal direction so that it falls on the ground at a distance 2x from the tower, the horizontal velocity of the second body is -

(A) 2v (B) $\sqrt{2}$ V (C) $\frac{V}{2}$ (D) $\frac{V}{\sqrt{2}}$

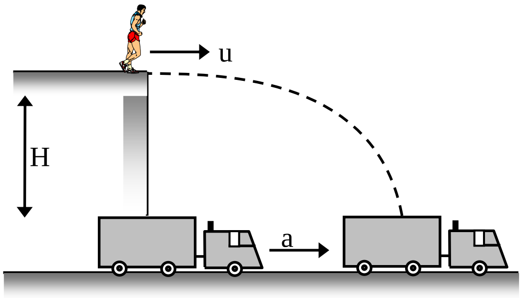
Q.13 A bomber is moving with a velocity v (m/s) above H meter from the ground. The bomber releases a bomb to hit a target T as shown in figure Then the relation between θ, H and v is-



(A) $\theta = \tan^{-1} v\sqrt{2Hg}$ (B) $\theta = \tan^{-1} v\sqrt{2/gH}$

(C) $\theta = \tan^{-1} v\sqrt{H/2g}$ (D) None of the above

Q.14 A stunt performer is to run and dive off a tall platform and land in a net in the back of a truck below. Originally the truck is directly under the platform, it starts forward with a constant acceleration a at the same instant the performer leaves the platform. If the platform is H above the net in the truck, then the horizontal velocity u that the performer must have as he leaves the platform is –



(A) $a\sqrt{2H/g}$ (B) $a\sqrt{H/2g}$

(C) $\sqrt{g/2H}$ (D) None of these

Two dimensional : Relative motion

Q.15 To an observer moving along East, the wind appears to blow from North. If he doubles his speed , the air would appear to come from -

(A) North (B) East

(C) North-East (D) North-West

Q.16 A car A is going north-east at 80km/hr. and another car B is going south-east at 60km/hr. Then the direction of the velocity of A relative to B makes with the north an angle α such that $\tan\alpha$ is –

(A) 1/7 (B) 3/4

(C) 4/3 (D) 3/5

Q.17 A boat man could row his boat with a speed 10m/sec. He wants to take his boat from P to a point Q just opposite on the other bank of the river flowing at a speed 4m/sec. He should row his boat –

(A) at right angle to the stream

(B) at an angle of $\sin^{-1}$ (2/5) with PQ up the stream

(C) at an angle of $\sin^{-1}$ (2/5) with PQ down the stream

(D) at an angle $\cos^{-1}$ (2/5) with PQ down the stream

Q.18 A bus moves over a straight level road with an acceleration a . A boy in the bus drops a ball outside. The acceleration of the ball with respect to the bus and the earth are respectively -

(A) a and g (B) a + g and g – a

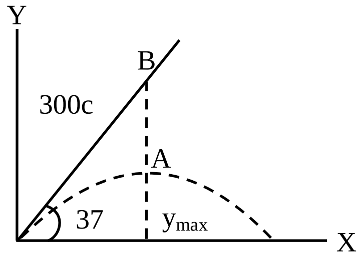
(C) $\sqrt{a^2+g^2}$ and g (D) $\sqrt{a^2+g^2}$ and a

Q.19 A man standing on a road has to hold his umbrella at 30º with the vertical to keep the rain away. He thrown the umbrella and starts running at 10 km/h. He finds that rain drop are hitting his head vertically. Find the speed of rain w.r.t. road-

(A) 10 km/s (B) 20 km/h (C) $10\sqrt{3}$ km/s (D) $20\sqrt{3}$ km/h

Condition for collision of two particles

Q.20 A ball A is projected from origin with an initial velocity v_0 = 700 cm/s, in a direction 37º above the horizontal as shown in fig. Another ball B 300 cm from origin on a line 37º above the horizontal is released from rest at the instant A starts. Then how far will B have fallen when it is hit by A –



(A) 90 cm (B) 80 cm

(D) 70 cm (D) 60 cm

EXERCISE - 2

Part – A: (Only single correct answer type questions)

Q.1 A ball is projected upwards from the top of the tower with a velocity 50 m/s making an angle 30^0 with the horizontal. The height of the tower is 70m. After how many seconds from the instant of throwing will the ball reach the ground?

(A) 2 s (B) 5 s

(C) 7 s (D) 9 s

Q.2 A particle moves in the plane xy with velocity $\vec{v} = k_1\hat{i} + k_2 x\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes, and k_1 and k_2 are constants. At the initial moment of time the particle was located at the point x = y = 0 then the equation of the particle's trajectory y (x) is –

(A) $y = \frac{k_1}{2k_2}x^2$ (B) $y = \frac{k_2}{2k_1}x^2$

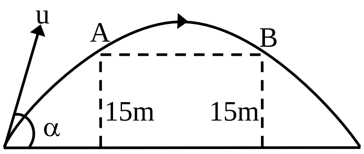
(C) $y = \frac{2k_1}{k_2}x^2$ (D) $y = \frac{2k_2}{k_1}x^2$

Q.3 A boy throws a ball with a velocity V_0 at an angle α to the horizontal. At the same instant he starts running with uniform velocity (minimum) to catch the ball before it hits the ground. To achieve this, he should run with a velocity of-

(A) $V_0 \cos\alpha$ (B) $V_0 \sin\alpha$

(C) $V_0 \tan\alpha$ (D) $\sqrt{V_0^2 \tan\alpha}$

Q.4 A golfer standing on level ground hits a ball with a velocity of u = 52 m/s at an angle α above the horizontal. If tanα = 5/12, then the time for which the ball is at least 15m above the ground (i.e. between A and B) will be (take g = 10 m/s^2) –



(A) 1 sec
(B) 2 sec
(C) 3 sec
(D) 4 sec

Q.5 A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30^\circ$, and the angle of the barrel to the horizontal $\beta = 60^\circ$. The initial velocity v of the shell is 21 m/sec. Then distance of point from the gun at which shell will fall –

(A) 10 m
(B) 20 m
(C) 30 m
(D) 40 m

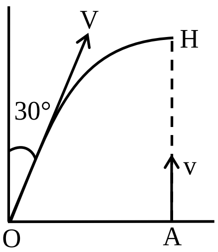
Q.6 An aircraft drives towards a stationary target which is at sea level and when it is at a height of 1390 m above sea level it launches a missile towards the target. The initial velocity of the missile is 410 m/s in a direction making an angle θ below the horizontal where $\tan\theta = 9/40$. Then the time of flight of the missile from the instant it was launched until it reaches sea level is nearly –

(A) 10 sec
(B) 15 sec
(C) 20 sec
(D) 25 sec

Q.7 A boat moves relative to water with a velocity which is 1/n times the river flow velocity. At what angle to the stream direction must be boat move to minimize drifting ?

(A) $\pi/2$
(B) $\sin^{-1}(1/n)$
(C) $\frac{\pi}{2} + \sin^{-1}(1/n)$
(D) $\frac{\pi}{2} - \sin^{-1}(1/n)$

Q.8 A particle is projected with a speed V from a point O making an angle of 30º with the vertical axis. At the same instant, a second particle is thrown vertically upwards from a point A (as shown in figure). The two particles reach H, the highest point on the parabolic path of particle one simultaneously. Then ratio $\frac{V}{v}$ is-



(A) $3\sqrt{2}$
(B) $2\sqrt{3}$
(C) $\frac{2}{\sqrt{3}}$
(D) $\frac{\sqrt{3}}{2}$

Q.9 A projectile can have the same range R for two angles of projection when projected with the same speed. If t_1 and t_2 be the times of flight in two cases, then the product of times of flight will be-

(A) $t_1t_2 \propto R$ (B) $t_1t_2 \propto R^2$

(C) $t_1t_2 \propto 1/R$ (D) $t_1t_2 \propto 1/R^2$

Q.10 The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6t$ meter where t is time in seconds. The velocity with which the projectile is projected is –

(A) 8 m/s (B) 6 m/s

(C) 10 /s (D) Cannot be determined

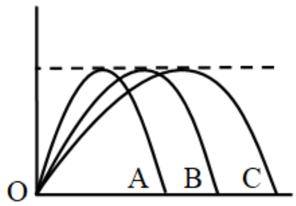
Q.11 Three projectile A, B and C are thrown from the same point in the same plane. Their trajectories are shown in the figure. Then which of the following statement is true –

(A) The time of flight is the same for all the three

(B) The launch speed is greatest for particle C

(C) The horizontal velocity component is greatest for particle C

(D) All of the above



Q.12 A particle is projected from a point O with a velocity u in a direction making an angle α upward with the horizontal. After some time at point P it is moving at right angle to its initial direction of projection. The time of flight from O to P is-

(A) $\frac{u \sin\alpha}{g}$ (B) $\frac{u\,\mathrm{cosec}\,\alpha}{g}$

(C) $\frac{u \tan\alpha}{g}$ (D) $\frac{u \sec\alpha}{g}$

Q.13 If R is the range of a projectile on a horizontal plane and h its maximum height, then maximum horizontal range with the same velocity of projection is-

(A) 2h (B) $\frac{R^2}{8h}$

(C) $2R + \frac{h^2}{8R}$ (D) $2h + \frac{R^2}{8h}$

Q.14 A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection then correct relation between (θ), (α) and (β)is-

(A) $\tan\alpha = \tan\theta + \tan\beta$ (B) $\tan\theta = \tan\alpha + \tan\beta$

(C) $\tan\theta = \tan\alpha - \tan\beta$ (D) $\tan\beta = \tan\theta + \tan\alpha$

Q.15 A particle is released from a certain height H = 400 m. Due to the wind the particle gathers the horizontal velocity $v_x = ay$ where $a = \sqrt{5}\ \text{sec}^{-1}$ and y is the vertical displacement of the particle from point of release, then the horizontal drift (displacement) of the particle when it strikes the ground is–

(A) 2.67 km (B) 8.67 m (C) 1.67 km (D) 5.1 km

ANSWER KEY

EXERCISE - 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	B	A	B	D	B	A	D	A	C	B	B	B	B	C
Q.No.	16	17	18	19	20										
Ans.	A	B	C	B	A										

EXERCISE – 2

PART – A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	A	B	C	A	C	C	A	C	D	B	D	B	A