RELATIONS AND FUNCTIONS

1. Set Theory

Collection of well defined objects which are distinct and distinguishable. A collection is said to be well defined if each and every element of the collection has some definition.

1.1 Notation of a set : Sets are denoted by capital letters like A, B, C or { } and the entries within the bracket are known as elements of set.

1.2 Cardinal number of a set : Cardinal number of a set X is the number of elements of a set X and it is denoted by n(X) e.g. $X = [x_1, x_2, x_3] \therefore n(X) = 3$

2. Representation of Sets

2.1 Set Listing Method (Roster Method) :

In this method a set is described by listing all the elements, separated by commas, within braces { }

2.2 Set builder Method (Set Rule Method) :

In this method, a set is described by characterizing property P(x) of its elements x. In such case the set is described by {x : P(x) holds} or {x | P(x) holds}, which is read as the set of all x such that P(x) holds. The symbol '|' or '**:**' is read as such that.

3. Type of Sets

3.1 Finite set :

A set X is called a finite set if its element can be listed by counting or labeling with the help of natural numbers and the process terminates at a certain natural number n. i.e. n(X) = finite no. eg (a) A set of English Alphabets (b) Set of soldiers in Indian Army

3.2 Infinite set :

A set whose elements cannot be listed counted by the natural numbers (1, 2, 3.......n) for any number n, is called a infinite set. e.g.

(a) A set of all points in a plane

(b) $X = \{x : x \in R, 0 < x < 0.0001\}$

(c) $X = \{x : x \in Q, 0 \le x \le 0.0001\}$

3.3 Singleton set :

A set consisting of a single element is called a singleton set. i.e. n (X) = 1,

e.g. $\{x : x \in N, 1 < x < 3\}$, {{}} : Set of null set, {ϕ} is a set containing alphabet ϕ.

3.4 Null set :

A set is said to be empty, void or null set if it has no element in it, and it is denoted by ϕ. i.e. X is a null set if n(X) = 0. e.g. : $\{x : x \in R \text{ and } x^2 + 2 = 0\}, \{x : x > 1 \text{ but } x < 1/2\}, \{x : x \in R, x^2 < 0\}$

3.5 Equivalent Set :

Two finite sets A and B are equivalent if their cardinal numbers are same i.e. n(A) = n(B).

3.6 Equal Set :

Two sets A and B are said to be equal if every element of A is a member of B and every element of B is a member of A. i.e. A = B, if A and B are equal and $A \neq B$, if they are not equal.

4. Universal Set

It is a set which includes all the sets under considerations i.e. it is a super set of each of the given set. Thus, a set that contains all sets in a given context is called the universal set. It is denoted by U. e.g. If A = {1, 2, 3}, B = {2, 4, 5, 6} and C = {1, 3, 5, 7}, then U = {1, 2, 3, 4, 5, 6, 7} can be taken as the universal set.

5. Disjoint Set

Sets A and B are said to be disjoint iff A and B have no common element or $A \cap B = \phi$. If $A \cap B \neq \phi$ then A and B are said to be intersecting or overlapping sets.

e.g. : (i) If A = {1, 2, 3}, B = {4, 5, 6} and C = {4, 7, 9} then A and B are disjoint set where B and C are intersecting sets.
(ii) Set of even natural numbers and odd natural numbers are disjoint sets.

6. Complementary Set

Complementary set of a set A is a set containing all those elements of universal set which are not in A. It is denoted by $\overline{A}$, A^C or A′. So $A^C = \{x : x \in U \text{ but } x \notin A\}$. e.g. If set A = {1, 2, 3, 4, 5} and universal set U = {1, 2, 3, 4,.......50} then $\overline{A}$ = {6, 7, ...50}

NOTE : All disjoint sets are not complementary sets but all complementary sets are disjoint.

7. Subset

A set A is said to be a subset of B if all the elements of A are present in B and is denoted by $A \subset B$ (read as A is subset of B) and symbolically written as : $x \in A \Rightarrow x \in B \Leftrightarrow A \subset B$

7.1 Number of subsets :

Consider a set X containing n elements as $\{x_1, x_2, ..., x_n\}$ then the total number of subsets of X = 2^n

Proof : Number of subsets of above set is equal to the number of selections of elements taking any number of them at a time out of the total n elements and it is equal to 2^n

$\because \quad {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

7.2 Types of Subsets :

A set A is said to be a **proper subset** of a set B if every element of A is an element of B and B has at least one element which is not an element of A and is denoted by $A \subset B$.

The set A itself and the empty set is known as **improper subset** and is denoted as $A \subseteq B$.

e.g. If X = $\{x_1, x_2, ...,x_n\}$ then total number of proper sets = $2^n - 2$ (excluding itself and the null set). The statement $A \subset B$ can be written as $B \supset A$, then B is called the **super set** of A and is written as $B \supset A$.

8. Power Sets

The collection of all subsets of set A is called the power set of A and is denoted by P(A)

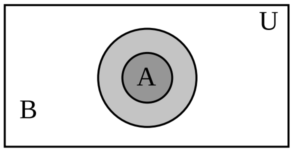
i.e. P(A) = {x : x is a subset of A}. If

$X = \{x_1, x_2, x_3, ...x_n\}$ then $n(P(X)) = 2^n$; $n(P(P(x))) = 2^{2^n}$.

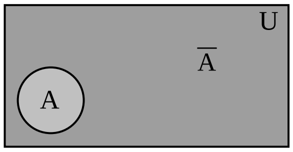
9. Venn(Euler) Diagrams

The diagrams drawn to represent sets are called Venn diagram or Euler-Venn diagrams. Here we represents the universal U as set of all points within rectangle and the subset A of the set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B by two intersecting circles.

e.g. If A is subset of B then it is represented diagrammatically in fig.



e.g. If A is a set then the complement of A is represented in fig.



10. Operations on Sets

10.1 Union of sets :

If A and B are two sets then union ($\cup$) of A and B is the set of all those elements which belong either to A or to B or to both A and B. It is also defined as $A \cup B = \{x : x \in A \text{ or } x \in B\}$. It is represented through Venn diagram in fig.1 & fig.2

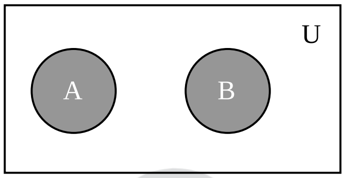


Fig. (1)

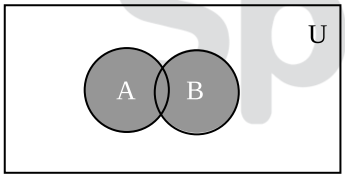
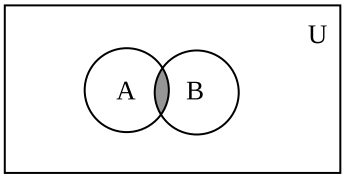


Fig. (2)

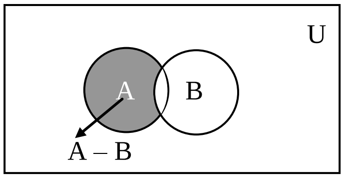
10.2 Intersection of sets :

If A and B are two sets then intersection ($\cap$) of A and B is the set of all those elements which belong to both A and B. It is also defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$ represented in Venn diagram (see fig.)



10.3 Difference of two sets :

If A and B are two sets then the difference of A and B, is the set of all those elements of A which do not belong to B.



Thus, A – B = {x : x ∈ A and x ∉ B}

or A – B = {x ∈ A ; x ∉ B}

Clearly x ∈ A – B ⇔ x ∈ A and x ∉ B

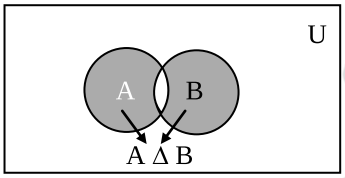
It is represented through the Venn diagrams.

10.4 Symmetric difference of two sets :

Set of those elements which are obtained by taking the union of the difference of A & Bis (A – B) & the difference of B & A is (B – A), is known as the symmetric difference of two sets A & B and it is denoted by (A Δ B).

Thus A Δ B = (A – B) ∪ (B – A)

Representation through the venn diagram is given in the fig.



11. Number of Elements in Different Sets

If A, B & C are finite sets and U be the finite universal set, then

(i) n(A∪B) = n(A) + n(B) – n(A ∩ B)

(ii) n(A∪B) = n(A) + n(B) (if A & B are disjoint sets)

(iii) n(A – B) = n(A) – n(A ∩ B)

(iv) n(A Δ B) = n[(A – B) ∪ (B – A)]

= n(A) + n(B) – 2n(A ∩ B)

(v) n(A ∪ B ∪ C) = n(A) + n(B) + n(C) – n(A ∩ B)

– n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

(vi) n(A′ ∪ B′) = n(A ∩ B)′ = n(U) – n(A ∩ B)

(vii) n(A′ ∩ B′) = n(A ∪ B)′ = n(U) – n(A ∪ B)

12. Cartesian Product of two Sets

Cartesian product of A to B is a set containing the elements in the form of ordered pair (a, b) such that a ∈ A and b ∈ B. It is denoted by A × B.

i.e. A × B = {(a, b) : a ∈ A and b ∈ B}

= $\{(a_1, b_1),(a_1, b_2),(a_2, b_1),(a_2, b_2),(a_3, b_1),(a_3, b_2)\}$

If set A = $\{a_1, a_2, a_3\}$ and B = $\{b_1, b_2\}$ then

A × B and B × A can be written as :

A × B = {(a, b) : a ∈ A and b ∈ B} and

B × A = {(b, a) ; b ∈ B and a ∈ A}

Clearly A × B ≠ B × A until A and B are equal

Note :

1. If number of elements in A : n(A) = m and n(B) = n then number of elements in (A × B) = m × n
2. Since A × B contains all such ordered pairs of the type (a, b) such that $a \in A$ & $b \in B$, that means it includes all possibilities in which the elements of set A can be related with the elements of set B. Therefore, A × B is termed as largest possible relation defined from set A to set B, also known as universal relation from A to B.

13. Algebraic Operations on Sets

13.1 Idempotent operation :

For any set A, we have (i) $A \cup A = A$ and (ii) $A \cap A = A$

Proof :

(i) $A \cup A = \{x : x \in A \text{ or } x \in A\} = \{x : x \in A\} = A$

(ii) $A \cap A = \{x : x \in A \;\&\; x \in A\} = \{x : x \in A\} = A$

13.2 Identity operation :

For any set A, we have

(i) $A \cup \phi = A$ and

(ii) $A \cap U = A$ i.e. ϕ and U are identity elements for union and intersection respectively

Proof :

(i) $A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$

$= \{x : x \in A\} = A$

(ii) $A \cap U = \{x : x \in A \text{ and } x \in U\}$

$= \{x : x \in A\} = A$

13.3 Commutative operation :

For any set A and B, we have

(i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

13.4 Associative operation :

If A, B and C are any three sets then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

13.5 Distributive operation :

If A, B and C are any three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

13.6 De-Morgan's Principle :

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof : (i) Let x be an arbitrary element of $(A \cup B)'$. Then $x \in (A \cup B)'$ $\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$ $\Rightarrow x \in A' \cap B'$

Again let y be an arbitrary element of $A' \cap B'$. Then $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$ $\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$ $\Rightarrow y \in (A \cup B)'$

$\therefore$ $A' \cap B' \subseteq (A \cup B)'$.

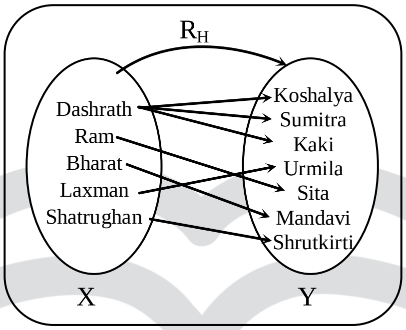
Hence $(A \cup B)' = A' \cap B'$

Similarly (ii) can be proved.

14. Relation

A relation R from set X to Y ($R : X \to Y$) is a correspondence between set X to set Y by which some or more elements of X are associated with some or more elements of Y. Therefore a relation (or binary relation) R, from a non-empty set X to another non-empty set Y, is a subset of $X \times Y$. i.e. $R_H : X \to Y$ is nothing but subset of $A \times B$.

e.g. Consider a set X and Y as set of all males and females members of a royal family of the kingdom Ayodhya X = {Dashrath, Ram, Bharat, Laxman, shatrughan} and Y = {Koshaliya, Kakai, sumitra, Sita, Mandavi, Urmila, Shrutkirti} and a relation R is defined as "was husband of "from set X to set Y.



Then R_H = {(Dashrath, Koshaliya), (Ram, sita), (Bharat, Mandavi), (Laxman, Urmila), (Shatrughan, Shrutkirti), (Dashrath, Kakai), (Dashrath, Sumitra)}

Note :

(i) If a is related to b then symbolically it is written as a R b where a is pre-image and b is image

(ii) If a is not related to b then symbolically it is written as $a \not R\, b$.

14.1 Domain, Co-domain & Range of Relation :

Domain : of relation is collection of elements of the first set which are participating in the correspondence i.e. it is set of all pre-images under the relation R. e.g. Domain of R_H : {Dashrath, Ram, Bharat, Laxman, Shatrughan}

Co-Domain : All elements of set Y irrespective of whether they are related with any element of X or not constitute co-domain. e.g. Y = {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti} is co-domain of R_H.

Range : of relation is a set of those elements of set Y which are participating in correspondence i.e. set of all images. Range of R_H : {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti}.

15. Types of Relations

15.1 Reflexive Relation

$R : X \to Y$ is said to be reflexive iff $x R x \ \forall\ x \in X$. i.e. every element in set X, must be a related to itself therefore $\forall\ x \in X$; $(x, x) \in R$ then relation R is called as reflexive relation.

15.2 Identity Relation :

Let X be a set. Then the relation $I_x = \{(x, x) : x \in X\}$ on X is called the identity relation on X. i.e. a relation I_x on X is identity relation if every element of X related to itself only. e.g. y = x

Note : All identity relations are reflexive but all reflexive relations are not identity.

15.3 Symmetric Relation

$R : X \to Y$ is said to be symmetric iff $(x, y) \in R$

$\Rightarrow (y, x) \in R$ for all $(x, y) \in R$ i.e. $x R y$

$\Rightarrow$ y R x for all (x, y) $\in$ R. e.g. perpendicularity of lines in a plane is symmetric relation.

15.4 Transitive Relation

R : X $\to$ Y is transitive iff (x, y) $\in$ R and (y, z) $\in$ R

$\Rightarrow$ (x, z) $\in$ R for all (x, y) and (y, z) $\in$ R. i.e. x R y and y R z $\Rightarrow$ x R z. e.g. The relation "being sister of " among the members of a family is always transitive.

Note :

(i) Every null relation is a symmetric and transitive relation.

(ii) Every singleton relation is a transitive relation.

(iii) Universal and identity relations are reflexive, symmetric as well as transitive.

15.5 Anti-symmetric Relation

Let A be any set. A relation R on set A is said to be an antisymmetric relation iff (a, b) $\in$ R and (b, a) $\in$ R $\Rightarrow$ a = b for all a, b $\in$ A e.g. Relations "being subset of "; "is greater than or equal to" and "identity relation on any set A" are antisymmetric relations.

15.6 Equivalence Relation

A relation R from a set X to set Y (R : X $\to$ Y) is said to be an equivalence relation iff it is reflexive, symmetric as well as transitive. The equivalence relation is denoted by ~ e.g. Relation "is equal to" Equality, Similarity and congruence of triangles, parallelism of lines are equivalence relation.

16. Inverse of a Relation

Let A, B be two sets and let R be a relation from a set A to B. Then the inverse of R, denoted by

R^{-1} , is a relation from B to A and is defined by

$R^{-1} = \{(b, a) : (a, b) \in R\}$, Clearly,

$(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$ Also,

Dom of R = Range of R^{-1} and

Range of R = Dom of R^{-1}

17. Interval

The set of the numbers between any two real numbers is called **interval**.

(a) Close Interval :

$[a,b] = \{ x, a \le x \le b \}$

(b) Open Interval:

(a, b) or]a, b[= $\{ x, a < x < b \}$

(c) Semi open or semi close interval:

[a,b[or [a,b) = $\{x; a \le x < b\}$

]a,b] or (a,b]= $\{x ; a < x \le b\}$

18. Function

Let A and B be two given sets and if each element a $\in$ A is associated with a unique element b $\in$ B under a rule f , then this relation is called **function**.

Here b, is called the image of a and a is called the pre- image of b under f.

Note :

(i) Every element of A should be associated with B but vice-versa is not essential.

(ii) Every element of A should be associated with a unique (one and only one) element of but any element of B can have two or more relations in A.

18.1 Representation of Function :

It can be done by three methods :

(a) By Mapping

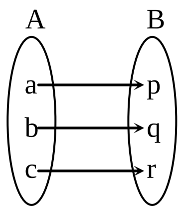
(b) By Algebraic Method

(c) In the form of Ordered pairs

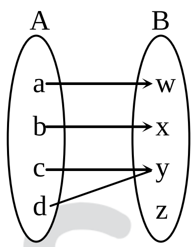
(A) Mapping :

It shows the graphical aspect of the relation of the elements of A with the elements of B .

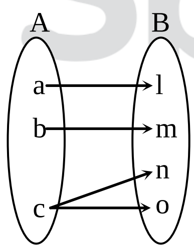
Ex. f_1**:**



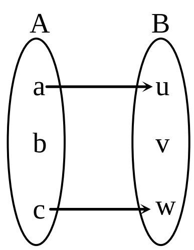
f_2 **:**



f_3 **:**



f_4 **:**



In the above given mappings rule f_1 and f_2

shows a function because each element of A is associated with a unique element of B. Whereas f_3 and f_4 are not function because in f_3, element c is associated with two elements of B, and in f_4 , b is not associated with any element of B, which do not follow the definition of function. In f_2, c and d are associated with same element, still it obeys the rule of definition of function because it does not tell that every element of A should be associated with different elements of B.

(B) Algebraic Method :

It shows the relation between the elements of two sets in the form of two variables x and y where x is independent variable and y is dependent variable.

If A and B be two given sets

A = { 1,2,3 }, B = {5,7,9}

then $f : A \to B$, $y = f(x) = 2x + 3$.

(C) In the form of ordered pairs:

A function $f : A \to B$ can be expressed as a set of ordered pairs in which first element of every ordered pair is a member of A and second element is the member of B. So f is a set of ordered pairs (a, b) such that :

(i) a is an element of A

(ii) b is an element of B

(iii) Two ordered pairs should not have the same first element.

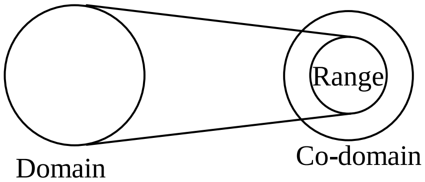
18.2 Domain, Co-domain and Range:

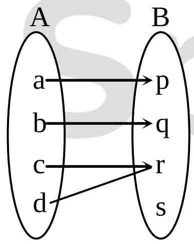
If a function f is defined from a set of A to set B then for $f: A \to B$ set A is called the **domain** of function f and set B is called the **co-domain** of function f. The set of the f- images of the elements of A is called the **range** of function f.

In other words, we can say

Domain = All possible values of x for which f(x) exists.

Range = For all values of x, all possible values of f(x).





Domain = {a,b,c,d} = A

Co-domain = {p,q,r,s} = B

Range = {p,q,r}

18.3 Algebra of functions:

Let f and g be two given functions and their domain are D_f and D_g respectively, then the sum, difference, product and quotient functions are defined as :

(a) $(f + g)(x) = f(x) + g(x),\ \forall\ x \in D_f \cap D_g$

(b) $(f - g)(x) = f(x) - g(x),\ \forall x \in D_f \cap D_g$

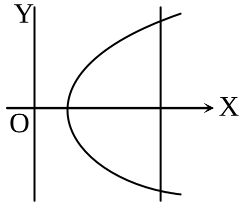
(c) $(f . g)(x) = f(x) . g(x),\ \forall\ x \in D_f \cap D_g$

(d) $(f/g)(x) = \dfrac{f(x)}{g(x)}$; $g(x) \neq 0,\ \forall\ x \in D_f \cap D_g$

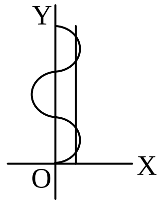
18.4 Testing for a function: A relation f : A B is a function or not, it can be checked by following methods.

(a) See Article 3 (a) & 3 (b)

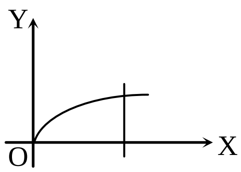
(b) Vertical Line Test: If we are given a graph of the relation then we can check whether the given relation is function or not. If it is possible to draw a vertical line which cuts the given curve at more than one point then given relation is not a function and when this vertical line means line parallel to Y - axis cuts the curve at only one point then it is a function.



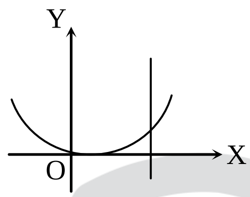
(i)



(ii)



(iii)



(iv)

fig. (iii) and (iv) represents a function.

SOLVED EXAMPLES

Ex.1 If a set A = {a, b, c} then find the number of subsets of the set A and also mention the set of all the subsets of A.

Sol. Since n(A) = 3

$\therefore$ number of subsets of A is $2^3 = 8$

and set of all those subsets is P(A) named as power set

P(A):{ϕ,{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c}}

Ex.2 Show that n {P[P(ϕ)]} = 4

Sol. We have P(ϕ) = {ϕ} $\therefore$ P(P(ϕ)) = {ϕ, {ϕ}}

$\Rightarrow$ P[P(P(ϕ))] = { ϕ, {ϕ}, {{ϕ}}, {ϕ, {ϕ}}}.

Hence, n{P[P(ϕ)]} = 4

Ex.3 If A = {x : x = 2n + 1, n ∈ Z and B = {x : x = 2n, n ∈ Z}, then find A ∪ B.

Sol. A ⋃ B = {x : x is an odd integer} ∪ {x : x is an even integer} = {x : x is an integer} = Z

Ex.4 If A = {x : x = 3n, n ∈ Z} and

B = {x : x = 4n, n ∈ Z} then find A ∩ B.

Sol. We have,

x ∈ A ∩ B ⇔ x = 3n, n ∈ Z and x = 4n, n ∈ Z

⇔ x is a multiple of 3 and x is a multiple of 4

⇔ x is a multiple of 3 and 4 both

⇔ x is a multiple of 12 ⇔ x = 12n,

n ∈ Z

Hence A ∩ B = {x : x = 12n, n ∈ Z}

Ex.5 If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in A ∪ B? Find also, the maximum number of elements in A ∪ B.

Sol. We have, n(A ∪ B) = n(A) + n(B) – n(A ∩ B).

This shows that n(A ∪ B) is minimum or maximum according as n(A ∩ B) is maximum or minimum respectively.

Case-I

When n(A ∩ B) is minimum, i.e., n(A ∩ B) = 0

This is possible only when A ∩ B = ϕ.

In this case,

n(A∪B) = n(A)+ n(B) – 0=n(A)+n(B)=3+6= 9.

So, maximum number of elements in A∪ B is 9.

Case-II

When n(A ∩ B) is maximum.

This is possible only when A ⊆ B. In this case, n(A ∩ B) = 3

$\therefore$ n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

= (3 + 6 – 3) = 6

So, minimum number of elements in A ∪ B is 6.

Ex.6 If A = {2, 3, 4, 5, 6, 7} and B = {3, 5, 7, 9, 11, 13} then find A – B and B – A.

Sol. A – B = {2, 4, 6} & B – A = {9, 11, 13)

Ex.7 If the number of elements in A is m and number of element in B is n then find

(i) The number of elements in the power set of A × B.

(ii) number of relation defined from A to B

Sol. **(i)** Since n(A) = m; n(B) = n

then n(A × B)= mn

So number of subsets of A × B = 2^{mn}

$\Rightarrow$ n (P(A × B)) = 2^{mn}

(ii) number of relation defined from A to B = 2^{mn}

Any relation which can be defined from set A to set B will be subset of A × B

$\because$ A × B is largest possible relation A $\rightarrow$ B

$\therefore$ no. of relation from A $\rightarrow$ B = no. of subsets of set (A × B)

Ex.8 Let A and B be two non-empty sets having elements in common, then prove that A × B and B × A have n^2 elements in common.

Sol. We have (A × B) $\cap$ (C × D) = (A $\cap$ C)× (B $\cap$ D)

On replacing C by B and D by A, we get

$\Rightarrow$ (A × B) $\cap$ (B × A) = (A $\cap$ B) × (B $\cap$ A)

It is given that AB has n elements so

(A $\cap$ B) × (B $\cap$ A) has n^2 elements

But (A × B) $\cap$ (B × A) = (A $\cap$ B) × (B $\cap$ A)

$\therefore$ (A × B) $\cap$ (B × A) has n^2 elements

Hence A × B and B × A have n^2 elements in common.

Ex.9 Let R be the relation on the set N of natural numbers defined by

R : {(x, y)} : x + 3y = 12 x $\in$ N, y $\in$ N} Find

(i) R **(ii)** Domain of R

(iii) Range of R

Sol. **(i)** We have, x + 3y =12 $\Rightarrow$ x = 12 – 3y

Putting y = 1, 2, 3, we get x = 9, 6, 3 respectively

For y = 4, we get x = 0 $\notin$ N. Also for y > 4, x $\notin$ N

$\therefore$ R = {(9, 1), (6, 2), (3, 3)}

(ii) Domain of R = {9, 6, 3}

(iii) Range of R = {1, 2, 3}

Ex.10 If X = $\{x_1, x_2, x_3\}$ and y = $(x_1, x_2, x_3, x_4, x_5\}$ then find which is a reflexive relation of the following :

(a) $R_1 : \{(x_1, x_1), (x_2, x_2)$

(b) $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)$

(c) $R_3 : \{(x_1,x_1), (x_2, x_2),(x_3, x_3),(x_1, x_3),(x_2, x_4)$

(d) $R_3 : \{(x_1, x_1), (x_2, x_2),(x_3, x_3),(x_4, x_4)$

Sol. (a) non-reflexive because $(x_3, x_3) \notin R_1$

(b) Reflexive

(c) Reflexive

(d) non-reflexive because $x_4 \notin X$

Ex.11 Which of the following is a function?

(A) {(2,1), (2,2), (2,3), (2,4)}

(B) {(1,4), (2,5), (1,6) , (3,9)}

(C) {(1,2), (3,3), (2,3), (1,4)}

(D) { (1,2), (2,2), (3,2), (4,2)}

Sol. We know that for a relation to be function every element of first set should be associated with one and only one element of second set but elements of first set can have same f-image in second set which is given in (D).

Ans.[D]

Ex.12 The domain of $f(x) = \dfrac{1}{x^3 - x}$ is -

(A) R – {–1,0,1} (B) R

(C) R – {0,1} (D) None of these

Sol. Domain = $\{x;\ x \in R;\ x^3 - x \neq 0\}$

$= R - \{-1, 0, 1\}$ **Ans.[A]**

Ex.13 The range of $f(x) = \cos \dfrac{\pi[x]}{2}$ is -

(A) {0,1} (B) {–1,1}

(C) {–1,0,1} (D) [–1,1]

Sol. [x] is an integer, cos (–x) = cos x and

$\cos\left(\dfrac{\pi}{2}\right) = 0,\ \cos 2\left(\dfrac{\pi}{2}\right) = -1.$

$\cos 0\left(\dfrac{\pi}{2}\right) = 1,\ \cos 3\left(\dfrac{\pi}{2}\right) = 0, \ldots\ldots$

Hence range = {–1,0,1} **Ans.[C]**

Ex.14 The domain of function $f(x) = \sqrt{2^x - 3^x}$ is -

(A) $(-\infty, 0]$ (B) R

(C) $[0, \infty)$ (D) No value of x

Sol. Domain = $\{x\ ;\ 2^x - 3^x \geq 0\} = \{x\ ;\ (2/3)^x \geq 1\}$

$= x \in (-\infty, 0]$ **Ans.[A]**

Ex.15 The domain of the function

$f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is -

(A) [–2, 2] – (–1, 1)   (B) [–1,2] – {0}

(C) [1, 2]   (D) [–2,2] – {0}

Sol. We know that the domain of $\sin^{-1}x$ is [–1,1]. So for f(x) to be meaningful , we must have $-1 \le \log_2 \frac{x^2}{2} \le 1$

$\Rightarrow\ 2^{-1} \le x^2/2 \le 2 \quad x \ne 0$

$\Rightarrow\ 1 \le x^2 \le 4,\ x \ne 0$

$\Rightarrow\ x \in [-2, -1] \cup [1,2]$

$\Rightarrow\ x \in [-2, 2] - (-1, 1)$ **Ans.[A]**

Ex.16 The range of function $f(x) = \frac{x^2}{1+x^2}$ is -

(A) R – {1}   (B) $R^+ \cup \{0\}$

(C) [0, 1]   (D) None of these

Sol. Range is containing those real numbers y for which f(x) = y where x is real number.

Now $f(x) = y = \Rightarrow \frac{x^2}{1+x^2} = y$

$\Rightarrow\ x = \sqrt{\frac{y}{(1-y)}}$ ....(1)

by (1) clearly $y \ne 1$, and for x to be real

$\frac{y}{1-y} \ge 0 \Rightarrow$ $y \ge 0$ and $y < 1$.

($\because$ If y = 2 then $\frac{y}{1-y} = \frac{2}{1-2} = (-2)$ and

$\sqrt{\frac{y}{(1-y)}} = \sqrt{-2} \notin R$)

$\therefore\ 0 \le y < 1$

$\therefore$ Range of function = $(0 \le y < 1) = [0,1)$ **Ans.[D]**

EXERCISE

Q.1 Let A = {1, 2, 3, 4}, and let R = {(2, 2), (3, 3), (4, 4), (1, 2)} be a relation on A. Then R is-

(A) Reflexive (B) Symmetric

(C) Transitive (D) None of these

Q.2 The void relation on a set A is-

(A) Reflexive

(B) Symmetric and transitive

(C) Reflexive and symmetric

(D) Reflexive and transitive

Q.3 For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is -

(A) Reflexive (B) Symmetric

(C) Transitive (D) None of these

Q.4 Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is/are relations from X to Y-

(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$

(B) R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}

(C) R_3 = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}

(D) R_4 = {(1, 3), (2, 5), (2, 4), (7, 9)}

Q.5 Let R be a relation defined in the set of real numbers by $aRb \Leftrightarrow 1 + ab > 0$. Then R is-

(A) Equivalence relation

(B) Transitive

(C) Symmetric

(D) Anti-symmetric

Q.6 Which one of the following relations on R is equivalence relation-

(A) $xR_1y \Leftrightarrow |x| = |y|$ (B) $xR_2y \Leftrightarrow x \geq y$

(C) $xR_3y \Leftrightarrow x \mid y$ (D) $xR_4y \Leftrightarrow x < y$

Q.7 Let R be a relation in N defined by

$R = \{(1+ x, 1+ x^2): x \leq 5, x \in N\}$.

Which of the following is false -

(A) R = {(2, 2), (3, 5), (4, 10), (5, 17), (6, 25)}

(B) Domain of R = {2, 3, 4, 5, 6}

(C) Range of R = {2, 5, 10, 17, 26}

(D) None of these

Q.8 The relation R defined in A = {1, 2, 3} by aRb if $|a^2 - b^2| \leq 5$. Which of the following is false

(A) R = {(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)}

(B) $R^{-1} = R$

(C) Domain of R = {1, 2, 3}

(D) Range of R = {5}

Q.9 The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on the set A = {1, 2, 3} is -

(A) Reflexive but not symmetric
(B) Reflexive but not transitive
(C) Symmetric and transitive
(D) Neither symmetric nor transitive

Q.10 Let a relation R in the set N of natural numbers be defined as $(x, y) \in R$ if and only if $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. The relation R is -
(A) Reflexive (B) Symmetric
(C) Transitive (D) An equivalence relation

Q.11 Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is -
(A) Reflexive and transitive
(B) Reflexive and symmetric
(C) Reflexive and antisymmetric
(D) None of these

Q.12 If A = {2, 3} and B = {1, 2}, then A × B =
(A) {(2, 1), (2, 2), (3, 1), (3, 2)}
(B) {(1, 2), (1, 3), (2, 2), (2, 3)}
(C) {(2, 1), (3, 2)}
(D) {(1, 2), (2, 3)}

Q.13 Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d), then R is-
(A) Symmetric only
(B) Reflexive only
(C) Transitive only
(D) An equivalence relation

Q.14 If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x is greater than y'. The range of R is -
(A) {1, 4, 6, 9} (B) {4, 6, 9}
(C) {1} (D) None of these

Q.15 Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is -
[AIEEE-2004]
(A) transitive (B) not symmetric
(C) reflexive (D) a function

Q.16 Domain of the function $f(x) = \frac{1}{\sqrt{x+2}}$ is-
(A) R (B) $(-2, \infty)$
(C) $[2, \infty]$ (D) $[0, \infty]$

Q.17 The domain where function $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal, is-
(A) {1/2} (B) {2}

(C) {1/2, 2}   (D) {1/2, –2}

Q.18 The domain of the function log $\sqrt{\frac{3-x}{2}}$ is-

(A) $(3, \infty)$ (B) $(-\infty, 3)$

(C) (0, 3)   (D) (–3, 3)

Q.19 Domain of the function $\cos^{-1}(4x-1)$ is-

(A) (0,1/2)   (B) [0,1/2]

(C) [1/2,2]   (D) None of these

Q.20 Domain of the function log $|x^2 - 9|$ is-

(A) R   (B) R– [–3, 3]

(C) R – {–3, 3}   (D) None of these

Q.21 The domain of the function-

$f(x) = \sqrt{x-1} + \sqrt{6-x}$ is-

(A) (1, 6)   (B) [1, 6]

(C) $[1, \infty)$ (D) $(-\infty, 6]$

Q.22 The domain of the function

$f(x) = \sqrt{(2-2x-x^2)}$ is -

(A) $-\sqrt{3} \le x \le \sqrt{3}$ (B) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$

(C) $-2 \le x \le 2$ (D) $-2+\sqrt{3} \le x \le -2-\sqrt{3}$

Q.23 Domain of a function $f(x) = \sin^{-1} 5x$ is-

(A) $\left(-\frac{1}{5}, \frac{1}{5}\right)$ (B) $\left[-\frac{1}{5}, \frac{1}{5}\right]$

(C) R   (D) $\left(0, \frac{1}{5}\right)$

Q.24 The range of the function $f : R \to R$, $f(x) = \tan^{-1} x$ is-

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

(C) R   (D) None of these

Q.25 The range of $f(x) = \sin \frac{\pi}{2} [x]$ is -

(A) {–1, 1}   (B) {–1, 0, 1}

(C) {0, 1}   (D) [–1, 1]

Q.26 Domain and range of $f(x) = \frac{|x-3|}{x-3}$ are respectively-

(A) R, [–1, 1]   (B) R– {3}, {1, –1}

(C) R^+, R (D) None of these

Q.27 The domain of the function f(x) = sin 1/x is -

(A) R (B) R^+ (C) R_0 (D) R^-

Q.28 Range of the function f(x) = 9 – 7 sin x is-

(A) (2, 16) (B) [2, 16]

(C) [–1, 1] (D) (2, 16]

Q.29 For real values of x, range of function

$y = \frac{1}{2-\sin 3x}$ is -

(A) $\frac{1}{3} \le y \le 1$ (B) $-\frac{1}{3} \le y \le 1$

(C) $-\frac{1}{3} > y > -1$ (D) $\frac{1}{3} > y > 1$

ANSWER KEY

EXERCISE

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	A	A,B,C	C	A	A	D	A	A	B	A	D	C	B
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
Ans.	B	D	B	B	C	B	B	B	B	B	B	C	B	A	