STRAIGHT LINE

1. Equation of Straight Line

A relation between x and y which is satisfied by co-ordinates of every point lying on a line is called the equation of Straight Line. Every linear equation in two variable x and y always represents a straight line.

eg. $3x + 4y = 5$, $-4x + 9y = 3$ etc.

General form of straight line is given by

$ax + by + c = 0$.

2. Equation of Straight line Parallel to Axes

(i) Equation of x axis $\Rightarrow y = 0$.

Equation a line parallel to x axis (or perpendicular to y-axis) at a distance 'a' from it

$\Rightarrow y = a$.

(ii) Equation of y axis $\Rightarrow x = 0$.

Equation of a line parallel to y-axis (or perpendicular to x axis) at a distance 'a' from it

$\Rightarrow x = a$. eg.

Equation of a line which is parallel to x-axis and at a distance of 4 units in the negative direction is $y = -4$.

3. Slope of a Line

If θ is the angle made by a line with the positive direction of x axis in anticlockwise sense, then the value of tanθ is called the Slope (also called gradient) of the line and is denoted by m or slope

$\Rightarrow m = \tan\theta$

eg. A line which is making an angle of 45º with the x-axis then its slope is $m = \tan 45^\circ = 1$.

Note :

(i) Slope of x axis or a line parallel to x-axis is $\tan 0^\circ = 0$.

(ii) Slope of y axis or a line parallel to y-axis is $\tan 90^\circ = \infty$.

(iii) The slope of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

eg. Slope of a line joining two points (3, 5) and (7, 9) is $= \frac{9-5}{7-3} = \frac{4}{4} = 1$.

4. Different forms of the Equation of Straight line

4.1 Slope - Intercept Form :

The equation of a line with slope m and making an intercept c on y–axis is $y = mx + c$. If the line passes through the origin, then $c = 0$. Thus the equation of a line with slope m and passing through the origin $y = mx$.

4.2 Slope Point Form :

The equation of a line with slope m and passing through a point (x_1, y_1) is

$y - y_1 = m(x - x_1)$

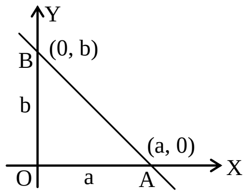
4.3 Two Point Form :

The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is -

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

4.4 Intercept Form :

The equation of a line which makes intercept a and b on the x-axis and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$. Here, the length of intercept between the co-ordinates axis = $\sqrt{a^2 + b^2}$



Area of $\Delta OAB = \frac{1}{2} OA. OB = \frac{1}{2}$ a.b.

4.5 Normal (Perpendicular) Form of a Line :

If p is the length of perpendicular on a line from the origin and α is the inclination of perpendicular with x- axis then equation on this line is $x\cos\alpha + y\sin\alpha = p$

4.6 Parametric Form (Distance Form) :

If θ be the angle made by a straight line with x-axis which is passing through the point (x_1, y_1) and r be the distance of any point (x, y) on the line from the point (x_1, y_1) then its equation.

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

5. Reduction of general form of Equations into Standard forms

General Form of equation ax + by + c = 0 then its-

(i) Slope Intercept Form is

$y = -\frac{a}{b}x - \frac{c}{b}$, here slope $m = -\frac{a}{b}$, Intercept $C = \frac{c}{b}$

(ii) Intercept Form is

$\frac{x}{-c/a} + \frac{y}{-c/b} = 1$, here x intercept is

$= -c/a$, y intercept is $= -c/b$

(iii) Normal Form is to change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$ like

$$-\frac{ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}},$$

here $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$ and $p = \frac{c}{\sqrt{a^2+b^2}}$

6. Position of a point relative to a line

(i) The point (x_1, y_1) lies on the line $ax + by + c = 0$ if, $ax_1 + by_1 + c = 0$

(ii) If $P(x_1, y_1)$ and $Q(x_2, y_2)$ do not lie on the line $ax + by + c = 0$ then they are on the same side of the line, if $ax_1+by_1+ c$ and $ax_2 + by_2 +c$ are of the same sign and they lie on the opposite sides of line if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the opposite sign.

(iii) (x_1, y_1) is on origin or non origin sides of the line $ax + by + c = 0$ if $ax_1 + by_1 + c = 0$ and c are of the same or opposite signs.

7. Angle between two Straight lines

The angle between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$

Note :

(i) If any one line is parallel to y axis then the angle between two straight line is given by

$$\tan\theta = \pm\frac{1}{m}$$

Where m is the slope of other straight line

(ii) If the equation of lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then above formula would be

$$\tan\theta = \left|\frac{a_1b_2 - b_1a_2}{a_1a_2 + b_1b_2}\right|$$

(iii) Here two angles between two lines, but generally we consider the acute angle as the angle between them, so in all the above formula we take only positive value of $\tan\theta$.

7.1 Parallel Lines :

Two lines are parallel, then angle between them is 0

$$\Rightarrow \frac{m_1 - m_2}{1+m_1m_2} = \tan 0^\circ = 0$$

$$\Rightarrow m_1 = m_2$$

Note : Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

7.2 Perpendicular Lines :

Two lines are perpendicular, then angle between them is 90º

$$\Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \tan 90^\circ = \infty$$

$\Rightarrow m_1 m_2 = -1$

Note : Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 = 0$

7.3 Coincident Lines :

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident only and only if $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

8. Equation of Parallel & Perpendicular lines

(i) Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + k = 0$

(ii) Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$

The value of k in both cases is obtained with the help of additional information given in the problem.

9. Length of Perpendicular

The length P of the perpendicular from the point (x_1, y_1) on the line $ax + by + c = 0$ is given by

$$P = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Note :

(i) Length of perpendicular from origin on the line $ax + by + c = 0$ is $c / \sqrt{a^2 + b^2}$

(ii) Length of perpendicular from the point (x_1, y_1) on the line $x \cos\alpha + y \sin\alpha = p$ is -

$x_1\cos\alpha + y_1 \sin\alpha = p$

9.1 Distance between Two Parallel Lines :

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Note :

(i) Distance between two parallel lines $ax + by + c_1 = 0$ and $kax + kby + c_2 = 0$ is

$$\frac{\left|c_1 - \frac{c_2}{k}\right|}{\sqrt{a^2 + b^2}}$$

(ii) Distance between two non parallel lines is always zero.

SOLVED EXAMPLES

Ex.1 The equation of the line which passes through the point (3, 4) and the sum of its intercept on the axes is 14, is -

(A) $4x - 3y = 24,\ x - y = 7$ (B) $4x + 3y = 24,\ x + y = 7$

(C) $4x + 3y + 24 = 0,\ x + y + 7 = 0$ (D) $4x - 3y + 24 = 0,\ x - y + 7 = 0$

Sol. Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ ...(1)

This passes through (3, 4), therefore

$\frac{3}{a} + \frac{4}{b} = 1$ ...(2)

It is given that $a + b = 14 \Rightarrow b = 14 - a$. Putting $b = 14 - a$ in (2), we get

$\frac{3}{a} + \frac{4}{b} = 1 \quad \Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow (a - 7)(a - 6) = 0 \Rightarrow a = 7, 6$

For a = 7, b = 14 – 7 = 7 and for a = 6, b = 14 – 6 = 8.

Putting the values of a and b in (1), we get the equations of the lines

$\frac{x}{7} + \frac{y}{7} = 1$ and $\frac{x}{6} + \frac{y}{8} = 1$

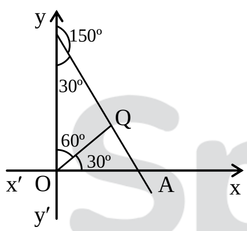
or $x + y = 7$ and $4x + 3y = 24$ **Ans. [B]**

Ex.2 The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction of y-axis. The equation of the line is –

(A) $\sqrt{3}\,x + y = 14$ (B) $\sqrt{3}\,x - y = 14$

(C) $\sqrt{3}\,x + y + 14 = 0$ (D) $\sqrt{3}\,x - y + 14 = 0$

Sol. Here p = 7 and $\alpha = 30^\circ$



$\therefore$ Equation of the required line is

$x \cos 30^\circ + y \sin 30^\circ = 7$

or $x\frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 7$

or $\sqrt{3}\,x + y = 14$ **Ans. [A]**

Ex.3 If the intercept made by the line between the axes is bisected at the point (x_1, y_1), then its equation is -

(A) $\frac{x}{x_1} + \frac{y}{y_1} = 2$ (B) $\frac{x}{x_1} + \frac{y}{y_1} = 1$

(C) $\frac{x}{x_1} + \frac{y}{y_1} = \frac{1}{2}$ (D) None of these

Sol. Let the equations of the line be $\frac{x}{a} + \frac{y}{b} = 1$, then the coordinates of point of intersection of this line and x-axis and y-axis are respectively (a, 0). (0, b). Hence mid point of the intercept is (a/2, b/2).

$\therefore$ $a/2 = x_1 \Rightarrow a = 2x_1$ and $b/2 = y_1$

$\Rightarrow b = 2y_1$

Hence required equation of the line is

$$\frac{x}{2x_1} + \frac{y}{2y_1} = 1$$

$$\Rightarrow \quad \frac{x}{x_1} + \frac{y}{y_1} = 2$$ **Ans. [A]**

Ex.4 The distance of the point (2, 3) from the line 2x –3y + 9 = 0 measured along a line x – y + 1 = 0, is -

(A) $\sqrt{2}$ (B) $4\sqrt{2}$

(C) $\sqrt{8}$ (D) $3\sqrt{2}$

Sol. The slope of the line x – y + 1 = 0 is 1. So it makes an angle of 45º with x-axis.

The equation of a line passing through (2, 3) and making an angle of 45º is

$$\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ} = r$$

$$\left[\text{Using } \frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r\right]$$

co-ordinats of any point on this line are

$(2 + r\cos45^\circ, 3 + r\sin45^\circ)$ or $\left(2+\frac{r}{\sqrt{2}}, 3+\frac{r}{\sqrt{2}}\right)$

If this point lies on the line 2x – 3y + 9 = 0,

then $4 + r\sqrt{2} - 9 - \frac{3r}{\sqrt{2}} + 9 = 0$

$\Rightarrow r = 4\sqrt{2}$.

So the required distance = $4\sqrt{2}$. **Ans. [B]**

Ex.5 If x + 2y = 3 is a line and A(–1, 3); B(2, –3); C(4, 9) are three points, then -

(A) A is on one side and B, C are on other side of the line

(B) A, B are on one side and C is on other side of the line

(C) A, C on one side and B is no other side of the line

(D) All three points are on one side of the line

Sol. Substituting the coordinates of points A, B and C in the expression x + 2y – 3, we get

The value of expression for A is

= –1 + 6 – 3 = 2 > 0

The value of expression for B is

= 2 – 6 – 3 = – 7 < 0

The value of expression for C is

= 4 + 18 – 3 = 19 > 0

∵ Signs of expressions for A, C are same while for B, the sign of expression is different

∴ A, C are on one side and B is on other side of the line **Ans. [C]**

Ex.6 The equation of two equal sides of an isosceles triangle are 7x – y + 3 = 0 and x + y – 3 = 0 and its third side is passes through the point (1, –10). The equation of the third side is

(A) x – 3y – 31 = 0 but not 3x + y + 7 = 0

(B) neither 3x + y + 7 = 0 nor x – 3y – 31 = 0

(C) 3x = y + 7 = 0 or x – 3y – 31 = 0

(D) 3x + y + 7 = 0 but not x – 3y – 31 = 0

Sol. Third side passes through (1, –10) so let its equation be y + 10 = m(x –1)

If it makes equal angle, say θ with given two sides, then

$$\tan\theta = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)} \Rightarrow m = -3 \text{ or } 1/3$$

Hence possible equations of third side are

$$y + 10 = -3(x-1) \text{ and } y + 10 = \frac{1}{3}(x-1)$$

or $3x + y + 7 = 0$ and $x - 3y - 31 = 0$ **Ans.[C]**

Ex.7 Triangle formed by lines $x + y = 0$, $3x + y = 4$ and $x + 3y = 4$ is -

(A) equilateral (B) right angled

(C) isosceles (D) None of these

Sol. Slope of the given lines are $-1, -3, -\frac{1}{3}$ respectively

Let $m_1 = -\frac{1}{3}$, $m_2 = -1$, $m_3 = -3$

$$\therefore \quad \tan A = \frac{-\frac{1}{3}+1}{1+\frac{1}{3}.1} \Rightarrow A = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\tan B = \frac{-1+3}{1+1.3} \Rightarrow B = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\text{and } \tan C = \frac{-\frac{1}{3}+1}{1+3.\frac{1}{3}} \Rightarrow C = \tan^{-1}\left(-\frac{4}{3}\right)$$

$\because \angle A = \angle B$, hence triangle is isosceles triangle. **Ans.[C]**

Ex.8 If A(–2,1), B(2,3) and C(–2,–4) are three points, then the angle between BA and BC is -

(A) $\tan^{-1}\left(\frac{3}{2}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\tan^{-1}\left(\frac{7}{4}\right)$ (D) None of these

Sol. Let m_1 and m_2 be the slopes of BA and BC respectively. Then

$$m_1 = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2} \quad \text{and} \quad m_2 = \frac{-4-3}{-2-2} = \frac{7}{4}$$

Let θ be the angle between BA and BC. Then

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \left|\frac{\frac{7}{4}-\frac{1}{2}}{1+\frac{7}{4}\times\frac{1}{2}}\right| = \left|\frac{\frac{10}{8}}{\frac{15}{8}}\right| = \pm\frac{2}{3}$$

$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$ **Ans. [B]**

Ex.9 The area of the parallelogram formed by the lines $4y - 3x = 1$, $4y - 3x - 3 = 0$, $3y - 4x + 1 = 0$, $3y - 4x + 2 = 0$ is -

(A) 3/8 (B) 2/7

(C) 1/6 (D) None of these

Sol. Let the equation of sides AB, BC, CD and DA of parallelogram ABCD are respectively

$y = \frac{3}{4}x + \frac{1}{4}$ ...(1); $y = \frac{3}{4}x + \frac{3}{4}$ ...(2)

$y = \frac{4}{3}x - \frac{1}{3}$ ...(3); $y = \frac{4}{3}x - \frac{2}{3}$ ...(4)

Here $m = \frac{3}{4}$, $n = \frac{4}{3}$, $a = \frac{1}{4}$, $b = \frac{3}{4}$, $c = -\frac{1}{3}$, $d = -\frac{2}{3}$

$\therefore$ Area of parallelogram ABCD

$$= \left|\frac{(a-b)(c-d)}{m-n}\right| = \left|\frac{\left(\frac{1}{4}-\frac{3}{4}\right)\left(-\frac{1}{3}+\frac{2}{3}\right)}{\frac{3}{4}-\frac{4}{3}}\right|$$

$$= \left|\frac{-\frac{1}{2}\times\frac{1}{3}}{-\frac{7}{12}}\right| = \frac{2}{7}$$

Ans. [B]

Ex.10 The equation of a line parallel to $ax + by + c' = 0$ and passing through the point (c, d) is -

(A) $a(x + c) - b(y + d) = 0$

(B) $a(x + c) + b(y + d) = 0$

(C) $a(x - c) + b(y - d) = 0$

(D) None of these

Sol. Equation of a line parallel to $ax + by + c = 0$ is written as

$ax + by + k = 0$ ...(1)

f it passes through (c, d), then

$ac + bd + k = 0$ ...(2)

Subtracting (2) and (1), we get

$a(x - c) + b(y - d) = 0$

Which is the required equation of the line. **Ans.[C]**

Ex.11 A straight line L perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and co-ordinates axes is 5, then the equation of line, is -

(A) $x + 5y = \pm 5$ (B) $x + 5y = \pm\sqrt{2}$

(C) $x + 5y = \pm 5\sqrt{2}$ (D) None of these

Sol. Let the line L cut the axes at A and B say. OA = a, OB = b

$\therefore$ Area Δ OAB $= \frac{1}{2}ab = 5$ ...(1)

Now equation of line perpendicular to lines $5x - y = 1$ is $x + 5y = k$

Putting x = 0, y = = b, y = 0, x = k = a

$\therefore \frac{1}{2}k.\ k/5 = 5$ from ... (1)

$k^2 = 50 \Rightarrow k = 5\sqrt{2}$

Hence the required line is $x + 5y = \pm 5\sqrt{2}$ **Ans.[C]**

Note : Trace the line approximately and try to make use of given material as per the question.

Ex.12 The sides AB, BC, CD and DA of a quadrilateral have the equations x + 2y = 3, x = 1, x – 3y = 4, 5x + y + 12 = 0 respectively, then the angle between the diagonals AC and BD is -

(A) 60º (B) 45º

(C) 90º (D) None of these

Sol. Solving for A,

$x + 2y - 3 = 0$

$5x + y + 12 = 0$

$\Rightarrow \frac{x}{+24+3} = \frac{y}{-15-12} = \frac{1}{-9}$

$\therefore$ A (–3, 3)

Similarly B(1,1), C(1, –1), D(–2, –2)

Now m_1 = slope of AC = – 1

m_2 = slope of BD = 1

$m_1 m_2 = -1$ $\therefore$ the angle required is 90º **Ans. [C]**

Ex.13 The vertices of ΔOBC are respectively (0, 0), (–3, –1) and (–1, –3). The equation of line parallel to BC and at a distance 1/2 from O which intersects OB and OC is -

(A) $2x + 2y + \sqrt{2} = 0$ (B) $2x - 2y + \sqrt{2} = 0$

(C) $2x + 2y - \sqrt{2} = 0$ (D) None of these

Sol. Slope of BC = $\frac{-3+1}{-1+3} = -1$

Now equation of line parallel to BC is

$y = -x + k \Rightarrow y + x = k$

Now length of perpendicular from O on this line

$= \pm \frac{k}{\sqrt{2}} = \frac{1}{2} \Rightarrow k = -\frac{\sqrt{2}}{2}$

$\therefore$ Equation of required line is

$2x + 2y + \sqrt{2} = 0$ **Ans. [A]**

Ex.14 The equation of a line through the point of intersection of the lines x – 3y + 1 = 0 and 2x + 5y – 9 = 0 and whose distance from the origin is $\sqrt{5}$, is -

(A) 2x + y – 5 = 0 (B) 2x – y + 5 = 0

(C) 2x + y – 10 = 0 (D) 2x – y – 10 = 0

Sol. Let the required line by method P + λQ = 0 be $(x - 3y + 1) + \lambda(2x + 5y - 9) = 0$

$\therefore$ perpendicular from (0, 0) = $\sqrt{5}$ gives

$$\frac{1-9\lambda}{\sqrt{(1-2\lambda)^2 + (5-3\lambda)^2}} = \sqrt{5},$$

squaring and simplifying $(8\lambda - 7)^2 = 0$

$\Rightarrow \lambda = 7/8$

Hence the line required is

$(x - 3y + 1) + 7/8\,(2x + 5y - 9) = 0$

or $22x + 11y - 55 = 0 \Rightarrow 2x + y - 5 = 0$ **Ans.[A]**

Note: Here to find the point of intersection is not necessary.

Ex.15 A variable line passes through the fixed point P. If the algebraic sum of perpendicular distances of the points (2, 0); (0, 2) and (1, 1) from the line is zero, then P is -

(A) (1,1) (B) (1, –1)

(C) (2, 2) (D) None of these

Sol. Let equation of variable line is

$ax + by + c = 0$ ...(1)

Now sum of perpendicular distance

$$\frac{2a+c}{\sqrt{a^2+b^2}} + \frac{2b+c}{\sqrt{a^2+b^2}} + \frac{a+b+c}{\sqrt{a^2+b^2}} = 0$$

$\Rightarrow$ $a + b + c = 0$ … (2)

on subtracting (2) from (1), we get

$a(x - 1) + b(y - 1) = 0$

Which obviously passes through a fixed point P(1, 1). **Ans. [A]**

Ex.16 If the sides of triangle are $x + y - 5 = 0$, $x - y + 1 = 0$ and $y - 1 = 0$, then its circumcentre is -

(A) (2, 1) (B) (2, –2)

(C) (1, 2) (D) (1, –2)

Sol. Here the sides $x + y - 5 = 0$ and $x - y + 1$ are perpendicular to each other, therefore $y = 1$ will be hypotenuse of the triangle. Now its middle point will be the circumcentre.

Now solving the pair of equations

$x + y - 5 = 0$, $y - 1 = 0$

and $x - y + 1 = 0$, $y - 1 = 0$, we get

$P \equiv (4, 1)$, $Q \equiv (0, 1)$

Mid point of PQ or circumcentre = (2, 1) **Ans. [A]**

EXERCISE

Q.1 The angle made by the line joining the points (1, 0) and $(-2, \sqrt{3})$ with x axis is -

(A) 120º (B) 60º (C) 150º (D) 135º

Q.2 If A(2,3), B(3,1) and C(5,3) are three points, then the slope of the line passing through A and bisecting BC is -

(A) 1/2 (B) –2 (C) –1/2 (D) 2

Q.3 If the vertices of a triangle have integral coordinates, then the triangle is -

(A) Isosceles (B) Never equilateral

(C) Equilateral (D) None of these

Q.4 The equation of a line passing through the point (–3, 2) and parallel to x-axis is -

(A) $x - 3 = 0$ (B) $x + 3 = 0$

(C) $y - 2 = 0$ (D) $y + 2 = 0$

Q.5 If the slope of a line is 2 and it cuts an intercept – 4 on y-axis, then its equation will be -

(A) $y - 2x = 4$ (B) $x = 2y - 4$

(C) $y = 2x - 4$ (D) None of these

Q.6 The equation of the line cutting of an intercept –3 from the y-axis and inclined at an angle $\tan^{-1} 3/5$ to the x axis is -

(A) $5y - 3x + 15 = 0$ (B) $5y - 3x = 15$

(C) $3y - 5x + 15 = 0$ (D) None of these

Q.7 If the line $y = mx + c$ passes through the points (2, 4) and (3, –5), then -

(A) m = –9, c = –22 (B) m = 9, c = 22

(C) m = –9, c = 22 (D) m = 9, c = –22

Q.8 The equation of the line inclined at an angle of 60º with x-axis and cutting y-axis at the point (0, –2) is -

(A) $\sqrt{3}\,y = x - 2\sqrt{3}$ (B) $y = \sqrt{3}\,x - 2$

(C) $\sqrt{3}\,y = x + 2\sqrt{3}$ (D) $y = \sqrt{3}\,x + 2$

Q.9 The equation of a line passing through the origin and the point (a cosθ, a sin θ) is-

(A) $y = x \sin\theta$ (B) $y = x \tan\theta$

(C) $y = x \cos\theta$ (D) $y = x \cot\theta$

Q.10 Slope of a line which cuts intercepts of equal lengths on the axes is -

(A) –1 (B) 2 (C) 0 (D) $\sqrt{3}$

Q.11 The angle between the lines $y - x + 5 = 0$ and $\sqrt{3}\,x - y + 7 = 0$ is -

(A) 15º (B) 60º

(C) 45º (D) 75º

Q.12 The angle between the lines $2x + 3y = 5$ and $3x - 2y = 7$ is -

(A) 45º (B) 30º
(C) 60º (D) 90º

Q.13 The angle between the lines $2x - y + 5 = 0$ and $3x + y + 4 = 0$ is-
(A) 30º (B) 90º
(C) 45º (D) 60º

Q.14 The obtuse angle between the line $y = -2$ and $y = x + 2$ is -
(A) 120º (B) 135º
(C) 150º (D) 160º

Q.15 The acute angle between the lines $y = 3$ and $y = \sqrt{3}x + 9$ is -
(A) 30º (B) 60º
(C) 45º (D) 90º

Q.16 Orthocenter of the triangle whose sides are given by $4x - 7y + 10 = 0$, $x + y - 5 = 0$ & $7x + 4y - 15 = 0$ is -
(A) (–1, –2) (B) (1, –2)
(C) (–1, 2) (D) (1, 2)

Q.17 The angle between the lines $x - \sqrt{3}y + 5 = 0$ and y-axis is -
(A) 90º (B) 60º
(C) 30º (D) 45º

Q.18 If the lines $mx + 2y + 1 = 0$ and $2x + 3y + 5 = 0$ are perpendicular then the value of m is -
(A) –3 (B) 3 (C) –1/3 (D) 1/3

Q.19 If the line passing through the points (4, 3) and (2, λ) is perpendicular to the line $y = 2x + 3$, then λ is equal to -
(A) 4 (B) –4
(C) 1 (D) –1

Q.20 The equation of line passing through (2, 3) and perpendicular to the line adjoining the points (–5, 6) and (–6, 5) is -
(A) $x + y + 5 = 0$ (B) $x - y + 5 = 0$
(C) $x - y - 5 = 0$ (D) $x + y - 5 = 0$

Q.21 The equation of perpendicular bisector of the line segment joining the points (1, 2) and (–2, 0) is -
(A) $5x + 2y = 1$ (B) $4x + 6y = 1$
(C) $6x + 4y = 1$ (D) None of these

Q.22 If the foot of the perpendicular from the origin to a straight line is at the point (3, –4). Then the equation of the line is -
(A) $3x - 4y = 25$ (B) $3x - 4y + 25 = 0$
(C) $4x + 3y - 25 = 0$ (D) $4x - 3y + 25 = 0$

Q.23 Equation of the line passing through the point (1, –1) and perpendicular to the line $2x - 3y = 5$ is -
(A) $3x + 2y - 1 = 0$ (B) $2x + 3y + 1 = 0$
(C) $3x + 2y - 3 = 0$ (D) $3x + 2y + 5 = 0$

Q.24 The equation of the line passing through the point (c, d) and parallel to the line $ax + by + c = 0$ is -
(A) $a(x + c) + b(y + d) = 0$ (B) $a(x + c) - b(y + d) = 0$

(C) $a(x - c) + b(y - d) = 0$ (D) None of these

Q.25 The equation of a line passing through the point (a, b) and perpendicular to the line $ax + by + c = 0$ is -

(A) $bx - ay + (a^2 - b^2) = 0$

(B) $bx - ay - (a^2 - b^2) = 0$

(C) $bx - ay = 0$

(D) None of these

Q.26 The line passes through (1, –2) and perpendicular to y-axis is -

(A) $x + 1 = 0$ (B) $x - 1 = 0$

(C) $y - 2 = 0$ (D) $y + 2 = 0$

Q.27 The equation of a line passing through (a, b) and parallel to the line $x/a + y/b = 1$ is -

(A) $x/a + y/b = 0$ (B) $x/a + y/b = 2$

(C) $x/a + y/b = 3$ (D) $x/a + y/b + 2 = 0$

Q.28 The length of the perpendicular from the origin on the line $\sqrt{3}x - y + 2 = 0$ is -

(A) 3 (B) 1

(C) 2 (D) 2.5

Q.29 The length of perpendicular from (2, 1) on line $3x - 4y + 8 = 0$ is-

(A) 5 (B) 4 (C) 3 (D) 2

Q.30 The length of perpendicular from the origin on the line $x/a + y/b = 1$ is -

(A) $\frac{b}{\sqrt{a^2+b^2}}$ (B) $\frac{a}{\sqrt{a^2+b^2}}$

(C) $\frac{ab}{\sqrt{a^2+b^2}}$ (D) None of these

Q.31 The distance between the lines $5x + 12y + 13 = 0$ and $5x + 12y = 9$ is -

(A) 11/13 (B) 22/17

(C) 22/13 (D) 13/22

Q.32 The distance between the parallel lines $y = 2x + 4$ and $6x = 3y + 5$ is -

(A) $17/\sqrt{3}$ (B) 1

(C) $3/\sqrt{5}$ (D) $17\sqrt{5}/15$

Q.33 The foot of the perpendicular drawn from the point (7, 8) to the line $2x + 3y - 4 = 0$ is -

(A) $\left(\frac{23}{13}, \frac{2}{13}\right)$ (B) $\left(13, \frac{23}{13}\right)$

(C) $\left(-\frac{23}{13}, -\frac{2}{13}\right)$ (D) $\left(-\frac{2}{13}, \frac{23}{13}\right)$

Q.34 The coordinates of the point Q symmetric to the point P(–5, 13) with respect to the line $2x - 3y - 3 = 0$ are -

(A) (11, –11) (B) (5, –13)

(C) (7, –9) (D) (6, –3)

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 A square of side a lies above the x- axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \frac{\pi}{4})$ with the positive direction of x- axis. The equation of its diagonal not passing through the origin is- **[AIEEE 2003]**

(A) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

(B) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(D) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

Q.2 A ray of light along $x + \sqrt{3}\,y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is - **[JEE Main - 2013]**

(A) $y = \sqrt{3}\,x - \sqrt{3}$ (B) $\sqrt{3}\,y = x - 1$

(C) $y = x + \sqrt{3}$ (D) $\sqrt{3}\,y = x - \sqrt{3}$

Q.3 The equation of the straight line passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is – 1 is- **[AIEEE 2004]**

(A) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(B) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(C) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

(D) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

Q.4 The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is - **[AIEEE-2005]**

(A) below the x-axis at a distance of 3/2 from it

(B) below the x-axis at a distance of 2/3 from it

(C) above the x-axis at a distance of 3/2 from it

(D) above the x-axis at a distance of 2/3 from it

Q.5 If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point that point is – **[AIEEE-2005]**

(A) (–1, 2) (B) (–1, –2)

(C) (1, – 2) (D) $\left(1, -\frac{1}{2}\right)$

Q.6 A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is – **[AIEEE 2006]**

(A) $3x - 4y + 7 = 0$ (B) $4x + 3y = 24$

(C) $3x + 4y = 25$ (D) $x + y = 7$

Q.7 If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, x > 0 and y = 3x, x > 0, then a belongs to **[AIEEE 2006]**

(A) $(3, \infty)$ (B) $\left(\frac{1}{2}, 3\right)$

(C) $\left(-3, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{2}\right)$

Q.8 The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept-4. Then a possible value of k is -**[AIEEE 2008]**

(A) 2 (B) –2

(C) –4 (D) 1

Q.9 The line $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for **[AIEEE- 2009]**

(A) Exactly one value of p

(B) Exactly two values of p

(C) More than two values of p

(D) No value of p

Q.10 The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is - **[AIEEE- 2010]**

(A) $\frac{23}{\sqrt{15}}$ (B) $\sqrt{17}$

(C) $\frac{17}{\sqrt{15}}$ (D) $\frac{23}{\sqrt{17}}$

Q.11 The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval – **[AIEEE- 2011]**

(A) $(0, \infty)$ (B) $(1, \infty)$

(C) $(-1, \infty)$ (D) $(-1, 1]$

Q.12 A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE- 2012]**

(A) –4 (B) – 2

(C) $-\frac{1}{2}$ (D) $-\frac{1}{4}$

SECTION –B

Q.1 The orthocentre of the triangle formed by the lines xy = 0 and x + y = 1 is **[IIT 1995]**

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(C) (0, 0) (D) $\left(\frac{1}{4}, \frac{1}{4}\right)$

Q.2 The diagonals of parallelogram PQRS are along the lines $x + 3y = 4$ and $6x – 2y = 7$. Then PQRS must be a **[IIT 1998]**

(A) rectangle
(B) square
(C) cyclic quadrilateral
(D) rhombus

Q.3 Orthocentre of the triangle whose vertices are A (0, 0), B (3, 4) & C (4, 0) is : **[IIT Scr. 2003]**

(A) $\left(3, \frac{3}{4}\right)$ (B) $\left(3, \frac{5}{4}\right)$

(C) (3, 12) (D) (2, 0)

Q.4 Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1,–1) and parallel to PS is - **[IIT-Scr.-2000]**

(A) $2x – 9y – 7 = 0$ (B) $2x – 9y – 11 = 0$
(C) $2x + 9y – 11 = 0$ (D) $2x + 9y + 7 = 0$

Q.5 Find the number of integer value of m which makes the x coordinates of point of intersection of lines. $3x + 4y = 9$ and $y = mx + 1$ integer. **[IIT-Scr.-2001]**

(A) 2 (B) 0 (C) 4 (D) 1

Q.6 Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$, $y = nx + 1$ is **[IIT-Scr.-2001]**

(A) $|m + n| / (m – n)^2$ (B) $2 / |m + n|$
(C) $1 / |m + n|$ (D) $1 /|m – n|$

Q.7 A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at the points P and Q respectively. Then the point O divides the segment PQ in the ratio- **[IIT-Scr.-2002]**

(A) 1 : 2 (B) 3 : 4
(C) 2 : 1 (D) 4: 3

Q.8 A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the *x*-axis, then the equation of L is - **[IIT- 2011]**

(A) $y + \sqrt{3}x + 2 – 3\sqrt{3} = 0$

(B) $y – \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y – x + 3 + 2\sqrt{3} = 0$

(D) $\sqrt{3}y + x – 3 + 2\sqrt{3} = 0$

Q.9 For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then – **[JEE - Advance 2013]**

(A) $a + b – c > 0$ (B) $a – b + c < 0$
(C) $a – b + c > 0$ (D) $a + b – c < 0$

Q.10 The locus of the orthocenter of the triangle formed by the lines **[IIT- 2009]**

$(1 + p)\, x - py + p\,(1 + p) = 0,$

$(1 + q)\, x - qy + q\,(1 + q) = 0,$

and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

ANSWER KEY

EXERCISE

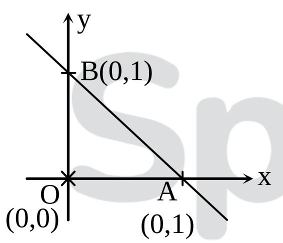
Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	B	C	C	A	C	B	B	A	A	D	C	B	B	D	B	A	A	D
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Ans.	C	A	A	C	C	D	B	B	D	C	C	D	A	A						

SECTION-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	
Ans.	A	D	D	A	C	B	B	C	A	D	B	B	

SECTION-B

1.[C]



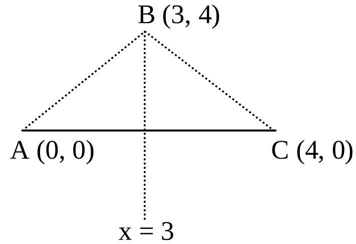
$xy = 0$ … (1)

$x + y = 0$ … (2)

$\because$ ΔOAB is a right angled triangle, so, right angle vertex will be the orthocentre, i.e.,(0, 0)

2.[D] As diagonals are perpendicular to each other so it must be rhombus.

3.[A]



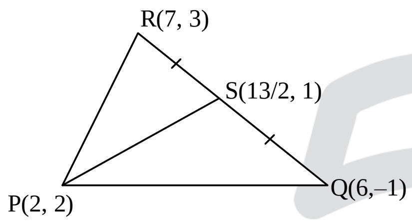
Slope BC = –4

Equation of altitude through A

$y = \frac{1}{4}x$

Therefore, orthocentre is $\left(3, \frac{3}{4}\right)$

4.[D]



Slope of PS = $\frac{2-1}{2-\frac{13}{2}} = \frac{1\times 2}{-9} = -\frac{2}{9}$

Equation of required line is

$y + 1 = \left(-\frac{2}{9}\right)(x - 1) \Rightarrow 2x + 9y + 7 = 0$

5.[A] $3x + 4y = 9$

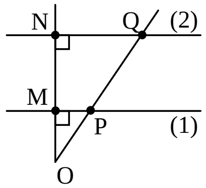
$mx - y = -1$

$x = \frac{5}{3+4m}$

$m = -1, -2$

6.[D] Area = $\left|\frac{(1-0)(1-0)}{m-n}\right| = \frac{1}{|m-n|}$

7.[B]



$4x + 2y = 9$ … (1)

$2x + y + 6 = 0$ … (2)

∵ ΔOPM & ΔOQN

are similar Δ's

Then, $\frac{OP}{OQ} = \frac{OM}{ON} = -\frac{(-9)}{12} = \frac{3}{4}$

8.[B] Let the slope of the line is m

$\tan 60^\circ = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|$

$\sqrt{3} = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|$

so $m + \sqrt{3} = \pm \sqrt{3}(1-\sqrt{3}m)$

$m+\sqrt{3} = \sqrt{3} - 3m$	$m+\sqrt{3} = -\sqrt{3}+3m$
m = 0	$m = \sqrt{3}$
hence line	hence line
y = –2	$y + 2 = \sqrt{3}(x-3)$
	$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

As line intersect x axis

So line will be $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

9.[A] ax + by + c = 0

bx + ay + c = 0

Intersection point

$\left(-\frac{c}{a+b}, -\frac{c}{a+b}\right)$

Distance

$\left(1+\frac{c}{a+b}\right)^2 + \left(1+\frac{c}{a+b}\right)^2 < 8$

$2(a + b + c)^2 < 8(a + b)^2$

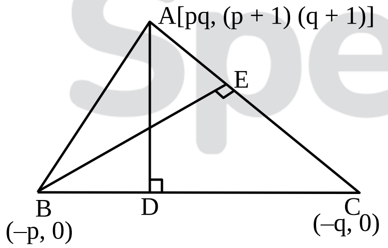
$(a + b + c)^2 < (2a + 2b)^2$

$(2a + 2b)^2 - (a + b + c)^2 > 0$

$(a + b - c)(3a + 3b + c) > 0$

so, $(a + b - c) > 0$

10.[D] Intersection points of given lines are (–p, 0), (–q, 0), [pq, (p + 1) (q + 1)] respectively



now equation of altitudes AD and BE are x = pq, and qx + (q + 1) y + pq = 0

Their point of intersection is (pq, – pq)

so h = pq, k = – pq

so locus is h = – k

h + k = 0

$\Rightarrow$ x + y = 0 which is a straight line