TRIGONOMETRIC EQUATIONS

1. Trigonometric Equation

An equation involving one or more trigonometric ratios of an unknown angle is called a trigonometric equation.

2. Solution of Trigonometric Equation

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. if $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots\ldots$

Thus, the trigonometric equation may have infinite number of solutions (because of their periodic nature) and can be classified as :

(i) Principal solution

Numerically least value of the unknown angle is called as principal solution.

(ii) General solution

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called General solution.

3. General solution of some Standard Trigonometric equations

- $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha,\ n \in I$
- $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha,\ n \in I$
- $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha,\ n \in I$
- $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha,\ n \in I$
- $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha,\ n \in I$
- $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha,\ n \in I$

4. Some Important deductions

- $\sin\theta = 0 \Rightarrow \theta = n\pi,\ n \in I$
- $\sin\theta = 1 \Rightarrow \theta = (4n + 1)\frac{\pi}{2},\ n \in I$
- $\sin\theta = -1 \Rightarrow \theta = (4n - 1)\frac{\pi}{2},\ n \in I$
- $\cos\theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2},\ n \in I$
- $\cos\theta = 1 \Rightarrow \theta = 2n\pi,\ n \in I$
- $\cos\theta = -1 \Rightarrow \theta = (2n + 1)\pi,\ n \in I$
- $\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in I$

5. General solution of Trigonometrical equation : $a\cos\theta + b\sin\theta = c$

Working methods -

(i) Put $a = r\cos\alpha$ and $b = r\sin\alpha$, where

$r = \sqrt{a^2 + b^2}$ and $\tan\alpha = \frac{b}{a}$ i.e. $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

(ii) Using the substitution in (i), the equation reduces

$r \cos(\theta - \alpha) = c$

$\Rightarrow \cos(\theta - \alpha) = \frac{c}{r} \Rightarrow \cos(\theta - \alpha) = \cos\beta$ (say)

Solve the equation by using the formula.

6. General solution of Trigonometrical equation : acos θ + bsin θ = c

If two equations are given then find the common values of θ between 0 and 2π and then add $2n\pi$ to this common value.

7. Important point to remember to solve a trigonometric equation

(a) Free style method

Any trigonometric equation can be solved without using any formulae. Find all angles in $[0, 2\pi]$ which satisfy the equation and then add $2n\pi$ to each.

(b) Do not divide any equation by a common factor, just take it common from all terms of equation.

For example $2 \sin x \cdot \cos x = \sin x$

$\sin x (2 \cos x - 1) = 0$ is the way to do.

(c) While equating one of the factors to zero, take care of the other factor that it should not become infinite.

(d) Avoid squaring.

(e) Always verify your results.

SOLVED EXAMPLES

Ex.1 The number of solutions of equation, sin 5x cos 3x = sin 6x cos 2x, in the interval [0, π] are

(A) 3 (B) 4 (C) 5 (D) 6

Sol. The given equation can be written as

$\frac{1}{2}(\sin 8x + \sin 2x) = \frac{1}{2}(\sin 8x + \sin 4x)$

or, $\sin 2x - \sin 4x \Rightarrow -2 \sin x \cos 3x = 0$

Hence sin x = 0 or cos 3x = 0.

That is, $x = n\pi$ ($n \in I$), or $3x = k\pi + \frac{\pi}{2}$ ($k \in I$). Therefore,

since $x \in [0, \pi]$, the given equation is satisfied if $x = 0, \pi, \frac{\pi}{6}$, or $\frac{5\pi}{6}$. **Ans.[C]**

Ex.2 If the equation 2 cos x + cos 2λx = 3 has only one solution then λ is

(A) 1 (B) A rational number

(C) An irrational number (D) None of these

Sol. As max. cos θ = 1, 2 cos x + cos 2λx = 3 is possible only when cos x = 1 and cos 2λx = 1,

i.e., cos x = 1 and sin λx = 0

Clearly, if λ is rational, say p/q , then x = 2qπ,

$q \in I$, satisfies both the equations. Therefore, for exactly one solution, x = 0, λ should be irrational. **Ans.[C]**

Ex.3 The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ $(0 \le x \le 2\pi)$ is -

(A) 0 (B) 1 (C) 2 (D) 3

Sol. If cot x > 0

then $\frac{1}{\sin x} = 0$ (impossible)

Now, if cot x < 0, then

$-\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow \frac{2\cos x + 1}{\sin x} = 0 \Rightarrow \cos x = -\frac{1}{2}$

$\Rightarrow \cos x = \cos\left(\frac{2\pi}{3}\right)$

$x = 2n\pi \pm \frac{2\pi}{3}; n \in I$

and $0 \le x \le 2\pi$ then $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

But $\cot x < 0 \therefore x = \frac{2\pi}{3}$ **Ans.[B]**

Ex.4 Let n be positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then

(A) $6 \le n \le 8$ (B) $4 < n \le 8$

(C) $6 < n < 8$ (D) $4 < n < 8$

Sol. $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \sqrt{2}\sin\left(\frac{\pi}{2n}+\frac{\pi}{4}\right)$

or, $\sin\left(\frac{\pi}{2n}+\frac{\pi}{4}\right) = \frac{\sqrt{n}}{2\sqrt{2}}$

since $\frac{\pi}{4} < \frac{\pi}{2n} + \frac{\pi}{4} < \frac{3\pi}{4}$ for $n > 1$

or, $\frac{1}{\sqrt{2}} < \frac{\sqrt{n}}{2\sqrt{2}} \le 1$

or, $2 < \sqrt{n} \le 2\sqrt{2}$ or, $4 < n \le 8$.

If n = 1, L.H.S. = 1, R.H.S. = 1/2

Similarly for n = 8, $\sin\left(\frac{\pi}{16}+\frac{\pi}{4}\right) \ne 1$

$\therefore$ $4 < n < 8$ **Ans.[D]**

Ex.5 The number of solutions of the equation $5\sec\theta - 13 = 12\tan\theta$ in $[0, 2\pi]$ is

(A) 2 (B) 1 (C) 4 (D) 0

Sol. $5\sec\theta - 13 = 12\tan\theta$

or, $13\cos\theta + 12\sin\theta = 5$

or, $\frac{13}{\sqrt{13^2+12^2}}\cos\theta + \frac{12}{\sqrt{13^2+12^2}}\sin\theta = \frac{5}{\sqrt{13^2+12^2}}$

or, $\cos(\theta - \alpha) = \frac{5}{\sqrt{313}}$, where $\cos\alpha = \frac{13}{\sqrt{313}}$

$\therefore$ $\theta = 2n\pi \pm \cos^{-1}\frac{5}{\sqrt{313}} + \alpha$

$= 2n\pi \pm \cos^{-1}\frac{5}{\sqrt{313}} + \cos^{-1}\frac{13}{\sqrt{313}}$

As $\cos^{-1}\frac{5}{\sqrt{313}} > \cos^{-1}\frac{13}{\sqrt{313}}$, then

$\theta \in [0, 2\pi]$, when n = 0 (One value, taking positive sign) and when n = 1 (One value, taking negative sign.)

Ans.[A]

Ex.6 The number of solution of equation $x^3 + x^2 + 4x + 2\sin x = 0$ in $0 \le x \le 2\pi$ is

(A) Zero (B) One (C) Two (D) Four

Sol. Here, $x^3 + (x + 2)^2 + 2\sin x = 4$. Clearly, x = 0 satisfies the equation.

If $0 < x \le \pi$, $x^3 + (x + 2)^2 + 2\sin x > 4$.

If $\pi < x \le 2\pi$, $x^3 + (x + 2)^2 + 2\sin x > 27 + 25 - 2$.

so, x = 0 is the only solution. **Ans.[B]**

Ex.7 Find value of θ for $\sin 2\theta = \cos 3\theta$, where $0 \le \theta \le 2\pi$; Use the above equation to find the value of sin 18º.

Sol. The given equation is $\sin 2\theta = \cos 3\theta$

or, $\cos 3\theta = \sin 2\theta$

or, $\cos 3\theta = \cos((\pi/2) - 2\theta)$

or, $3\theta = 2n\pi \pm ((\pi/2) - 2\theta)$ where $n \in I$

Taking + sign, $3\theta = 2n\pi + (\pi/2 - 2\theta)$

or, $5\theta = (4n + 1)(\pi/2)$

or, $\theta = (4n+1)(\pi/10)$, where $n \in I$ ....(1)

Again taking –sign, $3\theta = 2n\pi - ((\pi/2) - 2\theta)$

or, $\theta = (4n + 1)(\pi/2)$ ....(2)

Putting n = 0,1, 2, 3, ..., in (1) the values of θ in the interval $0 \le \theta \le 2\pi$ are given by

$\theta = \pi/10, 5\pi/10, 9\pi/10, 13\pi/10, 17\pi/10$.

or 18º, 90º, 162º, 234º, 346º.

Again putting n = 0, ± 1, ± 2, ..., in (2) the value of θ in the interval $0 \le \theta \le 2\pi$ is $3\pi/2$ i.e. 270º only.

Hence the required values of θ in $0 \le \theta \le 2\pi$ are 18º, 90º, 162º, 234º, 270º, 306º.

IInd Part : For the value of sin18º

Let $\theta = 18^\circ$

or, $5\theta = 90^\circ \Rightarrow 2\theta = 90^\circ - 3\theta$

or, $\sin 2\theta = \sin(90^\circ - 3\theta) = \cos 3\theta$

or, $2\sin\theta\cos\theta - (4\cos^3\theta - 3\cos\theta) = 0$

or, $\cos\theta(2\sin\theta - 4\cos^2\theta + 3) = 0$

or, $\cos\theta(2\sin\theta - 4(1 - \sin^2\theta) + 3) = 0$

or, $\cos\theta(4\sin^2\theta + 2\sin\theta - 1) = 0$

so either $\cos\theta = 0$ or $4\sin^2\theta + 2\sin\theta - 1 = 0$

But $\theta = 18^\circ \therefore \cos\theta \neq 0$

so, $\cos\theta = 0$ is rejected.

$\therefore 4\sin^2\theta + 2\sin\theta - 1 = 0$

or, $\sin\theta = \dfrac{-2 \pm \sqrt{4+6}}{2(4)} = \dfrac{-1 \pm \sqrt{5}}{4}$

or, $\sin\theta = (-1 + \sqrt{5})/4 = \sin 18^\circ$

[$\because$ sin 18º can not be negative]

Ex.8 Solve for x, $2\sin^3 x = \cos x$.

Sol. Here $2\sin^2 x = \dfrac{\cos x}{\sin x}$

{$\because \sin x \neq 0$ because $\sin x = 0$ does not satisfy the given equation}

or, $\dfrac{2}{\operatorname{cosec}^2 x} = \cot x$ or, $2 = \cot x.(1 + \cot^2 x)$

or, $\cot^3 x + \cot x - 2 = 0$

By trial, cot x = 1 satisfies the equation.

$\therefore$ $(\cot x - 1)(\cot^2 x + \cot x + 2) = 0$

or, $\cot x = 1$ or $\cot^2 x + \cot x + 2 = 0$

When $\cot x = 1$, $\tan x = 1$ $\therefore$ $x = n\pi + \frac{\pi}{4}$, $n \in I$.

When $\cot^2 x + \cot x + 2 = 0$,

cot x has no real value because $D = 1 - 8 < 0$

$\therefore$ Solutions are $x = n\pi + \frac{\pi}{4}$, $n \in I$.

Ex.9 Solve the equation $\cos^2[\frac{\pi}{4}(\sin x + \sqrt{2}\cos^2 x)] - \tan^2(x + \frac{\pi}{4}\tan^2 x) = 1$.

Sol. Given

$\cos^2[\frac{\pi}{4}(\sin x + \sqrt{2}\cos^2 x)] - \tan^2(x + \frac{\pi}{4}\tan^2 x) = 1$

or $\sin^2\{\frac{\pi}{4}(\sin x + \sqrt{2}\cos^2 x)\} + \tan^2(x + \frac{\pi}{4}\tan^2 x) = 0$

It is possible only when

$\sin^2\{\frac{\pi}{4}(\sin x + \sqrt{2}\cos^2 x)\} = 0$ ......(1)

and $\tan^2(x + \frac{\pi}{4}\tan^2 x) = 0$ ......(2)

from equation (1)

$\sin^2\{\frac{\pi}{4}(\sin x + \sqrt{2}\cos^2 x)\} = 0$

$\therefore$ $\frac{\pi}{4}(\sin x + \sqrt{2}\cos^2 x) = n\pi$, $n \in I$.

or, $\sin x + \sqrt{2}\cos^2 x = 4n$

$\therefore$ $|\sin x + \sqrt{2}\cos^2 x| \le |\sin x| + \sqrt{2}\,|\cos x|^2$

$\le 1 + \sqrt{2} < 4$

$\therefore$ The equation has no solution for $n \ne 0$

we consider $n = 0$

$\therefore$ $\sin x + \sqrt{2}\cos^2 x = 0$

i.e., $\sqrt{2}\sin^2 x - \sin x - \sqrt{2} = 0$

or, $(\sin x - \sqrt{2})(\sqrt{2}\sin x + 1) = 0$

$\because$ $\sin x \ne \sqrt{2}$ $\therefore$ $\sin x = -\frac{1}{\sqrt{2}}$

$\therefore$ The value of x satisfy the equation (2), then general solution of given equation is

$x = k\pi + (-1)^{k+1}\frac{\pi}{4}$, $k \in I$

Ex.10 Solve the equation, 2(sinx + sin y) – 2 cos (x – y) = 3, for smallest positive values of x and y.

Sol. Given equation is,

$2(\sin x + \sin y) - 2\cos(x - y) = 3$

or $2.2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$-2\left[2\cos^2\left(\frac{x-y}{2}\right)-1\right] = 3$

or $4\sin\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right) - 4\cos^2\left(\frac{x-y}{2}\right) = 1$

or $4\cos\left(\frac{x-y}{2}\right).\left[\sin\left(\frac{x+y}{2}\right) - \cos\left(\frac{x-y}{2}\right)\right] = 1$

or $4\cos\left(\frac{x-y}{2}\right).\left[\sin\left(\frac{x+y}{2}\right) - \sin\left(\frac{\pi}{2} - \frac{x-y}{2}\right)\right] = 1$

or $4\cos\left(\frac{x-y}{2}\right).2\cos\left(\frac{\pi+2y}{4}\right).\sin\left(\frac{2x-\pi}{4}\right) = 1$

or $\cos\left(\frac{x-y}{2}\right)\cos\left(\frac{\pi}{4}+\frac{y}{2}\right)\sin\left(\frac{x}{2}-\frac{\pi}{4}\right) = \frac{1}{8} = \left(\frac{1}{2}\right)^3$

If x and y be positive and smallest, then

$\cos\left(\frac{x-y}{2}\right) = \cos\left(\frac{\pi}{4}+\frac{y}{2}\right) = \sin\left(\frac{x}{2}-\frac{\pi}{4}\right) = \frac{1}{2}$

$\because \frac{\pi}{4} + \frac{y}{2} = \frac{\pi}{3}, \frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{6}$

$\Rightarrow \frac{y}{2} = \frac{\pi}{12}, \frac{x}{2} = \frac{5\pi}{12}, \therefore y = \frac{\pi}{6}, x = \frac{5\pi}{6}$

which satisfy $\cos\left(\frac{x-y}{2}\right) = \frac{1}{2}$

Hence required solution is

$x = \frac{5\pi}{6}$ and $y = \frac{\pi}{6}$.

Ex.11 Solve the equation : $\cos(\pi 3^x) - 2\cos^2(\pi 3^x) + 2\cos(4\pi 3^x) - \cos(7\pi 3^x) = \sin(\pi 3^x) + 2\sin^2(\pi 3^x) - 2\sin(4\pi 3^x)$ $+ 2\sin(\pi 3^{x+1}) - \sin(7\pi 3^x)$

Sol. Denote $\pi 3^x$ by y to get

$\cos y - 2\cos^2 y + 2\cos 4y - \cos 7y$

$= \sin y + 2\sin^2 y - 2\sin 4y + 2\sin 3y - \sin 7y$

Transposing all terms to the left side. We have

or $(\cos y - \cos 7y) + (\sin 7y - \sin y)$

$+ 2(\cos 4y + \sin 4y) - 2(\sin 3y + 1) = 0$

or, $2\sin 4y\sin 3y + 2\cos 4y\sin 3y$

$+2(\cos 4y + \sin 4y) - 2(\sin 3y + 1) = 0$

[use C & D formulae]

or, 2sin 3y(sin4y + cos4y) + 2(cos 4y + sin 4y)

– 2 (sin3y +1) = 0

or, (sin 3y + 1)(sin 4y + cos 4y–1) = 0

This enables us to write down three groups of solutions :

$$y_1 = -\frac{\pi}{6} + \frac{2k\pi}{3}, \quad y_2 = \frac{n\pi}{2}, \quad y_3 = \frac{\pi}{8} + \frac{m\pi}{2}$$

where k, n and m are arbitary integers. Recalling that $y = \pi 3^x$ we obtain an infinity of equations for determining the roots of the original equations:

$$3^x = -\frac{1}{6} + \frac{2k}{3}, \quad k = 0, \pm 1, \pm 2, ...$$

$$3^x = \frac{n}{2}, n = 0, \pm 1, \pm 2, ...$$

$$3^x = \frac{1}{8} + \frac{m}{2}, \quad m = 0, \pm 1, \pm 2, ...$$

The equation $3^x = a$ has a (unique) root only for positive a and it is given by the formula $x = \log_3 a$. Therefore, the equation (1) has solution only for those (integral) values of k, n, m for which the corresponding right members of the relations (1) are positive.

It is easy to see that of the first equation of (1) is positive for integer k > 0; the right side of second equation of (1) is positive for integral n > 0; and the right side of the third equation of (1) is positive for m ≥ 0. Thus we have to solve (1) only for the indicated values of k, m, n. The resulting values of x are then the roots of the original equation :

$$x = \log_3\left(-\frac{1}{6} + \frac{2k}{6}\right); \qquad k = 1, 2, ...$$

$$x = \log_3\left(\frac{n}{2}\right), \qquad n = 1, 2, ...$$

$$x = \log_3\left(\frac{1}{8} + \frac{1}{m}\right) \quad m = 0, 1, 2$$

Ex.12 Solve the equation : $\sqrt{\left(\frac{1}{16} + \cos^4 x - \frac{1}{2}\cos^2 x\right)} + \sqrt{\left(\frac{9}{16} + \cos^4 x - \frac{3}{2}\cos^2 x\right)} = \frac{1}{2}$

Sol. The given equation is

$$\sqrt{\left(\frac{1}{16} + \cos^4 x - \frac{1}{2}\cos^2 x\right)} + \sqrt{\left(\frac{9}{16} + \cos^4 x - \frac{3}{2}\cos^2 x\right)} = \frac{1}{2}$$

$$\text{or, } \left|\cos^2 x - \frac{1}{4}\right| + \left|\cos^2 x - \frac{3}{4}\right| = \frac{1}{2}$$

Case I: If $\cos^2 x \geq \frac{3}{4}$

Then $\cos^2 x - \frac{1}{4} + \cos^2 x - \frac{3}{4} = \frac{1}{2}$

or $2\cos^2 x - 1 = \frac{1}{2}$

or $\cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$ $\quad \therefore 2x = 2n\pi \pm \frac{\pi}{3}$

$\therefore\ x = n\pi \pm \frac{\pi}{6}$, n = 0, ± 1, ± 2, ...(1)

Case II: If $\frac{1}{4} \le \cos^2 x < \frac{3}{4}$

then $\cos^2 x - \frac{1}{4} + \frac{3}{4} - \cos^2 x = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2}$

$\therefore\ \frac{1}{4} \le \cos^2 x < \frac{3}{4}$

or, $\frac{1}{2} \le 2\cos^2 x < \frac{3}{2}$ $\quad$ or, $-\frac{1}{2} \le \cos 2x < \frac{1}{2}$

or, $\cos\frac{2\pi}{3} \le \cos 2x < \cos\frac{\pi}{3}$

or, $2n\pi \pm \frac{2\pi}{3} \ge 2x > 2n\pi \pm \frac{\pi}{3}$

$\therefore\ n\pi + \frac{\pi}{6} < x \le n\pi + \frac{\pi}{3}$ and

$n\pi - \frac{\pi}{3} \ge x > n\pi - \frac{\pi}{6}$ ....(2)

Hence solution from (1) & (2) is

$n\pi + \frac{\pi}{6} \le x \le n\pi + \frac{\pi}{3}$ and

$n\pi - \frac{\pi}{3} \le x \le n\pi - \frac{\pi}{6}$ where $n \in I$.

Ex.13 Find the general solution of the equation $\sin^4 x + \cos^4 x = \sin x \cos x$.

Sol. The given equation can be written as

$4\sin^4 x + 4\cos^4 x = 4\sin x \cos x$

or, $(1 - \cos 2x)^2 + (1 + \cos 2x)^2 = 2\sin 2x$

or, $2(1 + \cos^2 2x) = 2\sin 2x$

$\Rightarrow 1 + \cos^2 2x = \sin 2x$

or, $1 + 1 - \sin^2 2x = \sin 2x$

$\Rightarrow \sin^2 2x + \sin 2x = 2$

This relation is possible if and only if $\sin 2x = 1$

or $2x = 2n\pi + \frac{\pi}{2}$

$\Rightarrow x = n\pi + \frac{\pi}{4} = \frac{(4n+1)\pi}{4}$ $(n \in I)$

Ex.14 Solve the equation: $\sin^6 x = 1 + \cos^4 3x$

Sol. L.H.S. = $\sin^6 x \le 1$

R.H.S = $1 + \cos^4 3x \ge 1$

Hence $\sin^6 x = 1 + \cos^4 3x$ is possible only

When L.H.S. = R.H.S. = 1

$\Rightarrow \sin^6 x = 1$ and $1 + \cos^4 3x = 1$

$\Rightarrow \sin^2 x = 1$ and $\cos^4 3x = 0$

$\Rightarrow \cos^2 x = 0$ and $\cos 3x = 0$

$\Rightarrow \cos x = 0$ and $\cos 3x = 0$

$\Rightarrow x = (2m + 1)\dfrac{\pi}{2}$

and $3x = (2x + 1)\dfrac{\pi}{2}$ where $m, n \in I$

$\Rightarrow x = (2m + 1)\dfrac{\pi}{2}$ and $x = (2x + 1)\dfrac{\pi}{6}$.

common values of x is $(2n + 1)\dfrac{\pi}{2}$, where $n \in I$

The required solution $x = (2n + 1)\dfrac{\pi}{2}$, $n \in I$

Ex.15 Solve for x and y : $4^{\sin x} + 3^{\frac{1}{\cos y}} = 11$ & $5.\ 16^{\sin x} - 2.\ 3^{\sec y} = 2$

Sol. Let, $4^{\sin x} = \lambda$, $3^{\frac{1}{\cos y}} = \mu$

Then the equation becomes

$\lambda + \mu = 11$ .....(1)

$5\lambda^2 - 2\mu = 2$ .....(2)

Now apply operation, $2 \times (1) + (2)$

or, $2\lambda + 5\lambda^2 = 24$

or $5\lambda^2 + 2\lambda - 24 = 0$ or $5\lambda^2 + 12\lambda - 10\lambda - 24 = 0$

or, $\lambda(5\lambda + 12) - 2(5\lambda + 12) = 0$

or, $(5\lambda + 12)(\lambda - 2) = 0$ So $\lambda = 2, -\dfrac{12}{5}$.

If $\lambda = 2$, $4^{\sin x} = 2$; $\therefore 2^{2\sin x} = 2$;

$\therefore 2\sin x = 1$; $\therefore \sin x = \dfrac{1}{2}$

If $\lambda = -\dfrac{12}{5}$ then $4^{\sin x} = -\dfrac{12}{5}$ which is impossible for $4^{\sin x} > 0$

When $\lambda = 2$, we get $\mu = 11 - 2 = 9$

$\therefore 3^{\frac{1}{\cos y}} = 9 = 3^2$

$\therefore \dfrac{1}{\cos y} = 2; \quad \cos y = \dfrac{1}{2}$

When $\lambda = -12/5$, x has no solution. So y has no solution.

Thus we have $\sin x = \dfrac{1}{2}$, $\cos y = \dfrac{1}{2}$

$\therefore x = n\pi + (-1)^n \dfrac{\pi}{6}$ and $y = 2m\pi \pm \dfrac{\pi}{3}$

where $m, n \in I$.

Ex.16 Solve the equation : $\sqrt{17\sec^2 x + 16\left(\dfrac{1}{2}\tan x \sec x - 1\right)} = 2\tan x\,(1 + 4\sin x)$

Sol. The given equation is

$$\sqrt{17\sec^2 x + 16\left(\dfrac{1}{2}\tan x \sec x - 1\right)}$$

$= 2\tan x\,(1 + 4\sin x)$ .....(1)

or, $\sqrt{16\tan^2 x + 8\tan x \sec x + \sec^2 x}$

$= 2\tan x(1 + 4\sin x)$

or, $4\tan x + \sec x = 2\tan x\,(1 + 4\sin x)$

or, $8\sin^2 x - 2\sin x - 1 = 0$

$\therefore \sin x = \dfrac{1}{2}$ and $\sin x = -\dfrac{1}{4}$

or, $\sin x = \sin \pi/6$ and $x = \sin^{-1}\left(-\dfrac{1}{4}\right)$

$\therefore$ Solution of (1) are

$x = n\pi + (-1)^n \dfrac{\pi}{6}$ and

$x = n\pi + (-1)^{n+1} \sin^{-1}\left(\dfrac{1}{4}\right)$ where $n \in I$.

Ex.17 For all $\theta \in \left[0, \dfrac{\pi}{2}\right]$, show that $\cos(\sin\theta) > \sin(\cos\theta)$

Sol. $\cos(\sin\theta) - \sin(\cos\theta)$

$= \cos(\sin\theta) - \cos\left[\dfrac{\pi}{2} - \cos\theta\right]$

$= 2\sin\dfrac{1}{2}\left(\dfrac{\pi}{2} + \sin\theta - \cos\theta\right).\sin\dfrac{1}{2}\left(\dfrac{\pi}{2} - \cos\theta - \sin\theta\right)$

$= 2\sin\left[\dfrac{\pi}{4} + \dfrac{\sin\theta - \cos\theta}{2}\right].\sin\left[\dfrac{\pi}{4} - \dfrac{\sin\theta + \cos\theta}{2}\right]$

$$= 2\sin\left[\frac{\pi}{4} + \frac{\sin\left(\theta - \frac{\pi}{4}\right)}{\sqrt{2}}\right].\sin\left[\frac{\pi}{4} - \frac{\sin\left(\theta + \frac{\pi}{4}\right)}{\sqrt{2}}\right]$$

...(1)

Now, minimum value of

$$\frac{\sin\left(\theta - \frac{\pi}{4}\right)}{\sqrt{2}} = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{2}} = -\frac{1}{2} \text{ for } 0 \le \theta \le \frac{\pi}{2}$$

Maximum value of

$$\frac{\sin\left(\theta - \frac{\pi}{4}\right)}{\sqrt{2}} = \frac{1}{2}, \text{ for } 0 \le \theta \le \frac{\pi}{2}$$

$$\therefore \frac{\pi}{4} - \frac{1}{2} \le \frac{\pi}{4} + \frac{\sin\left(\theta - \frac{\pi}{4}\right)}{\sqrt{2}} \le \frac{\pi}{4} + \frac{1}{2}$$

Also minimum value of

$$\frac{\sin\left(\theta + \frac{\pi}{4}\right)}{\sqrt{2}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}, \text{ for } 0 \le \theta \le \frac{\pi}{2}$$

Maximum value of

$$\frac{\sin\left(\theta + \frac{\pi}{4}\right)}{\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ for } 0 \le \theta \le \frac{\pi}{2}$$

$$\frac{\pi}{4} - \frac{1}{\sqrt{2}} \le \frac{\pi}{4} - \frac{\sin\left(\theta + \frac{\pi}{4}\right)}{\sqrt{2}} \le \frac{\pi}{4} - \frac{1}{2}$$

Clearly, $\left(\frac{\pi}{4} - \frac{1}{2}\right)$; $\left(\frac{\pi}{4} + \frac{1}{2}\right)$; $\left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ radians are all angles between 0 and π.

Hence both the factors in (1) are positive.

Hence cos (sinθ) – sin (cosθ) > 0

cos (sinθ) > sin (cosθ)

Ex.18 Let A, B, C are three angles such that A = π/4 and tan B.tan C = p then find all possible value of p so that A, B, C are three angles of a triangle.

Sol. In ΔABC, $\because A + B + C = \pi$

$\Rightarrow B + C = 3\pi/4$ $[\because A = \pi/4]$

$\therefore 0 < B, C < \frac{3\pi}{4}$

Now, tan B tan C = p $\Rightarrow \frac{\sin B \sin C}{\cos B \cos C} = \frac{p}{1}$

Using componendo & dividendo

$$\frac{\cos B\cos C+\sin B\sin C}{\cos B\cos C-\sin B\sin C}=\frac{1+p}{1-p}$$

$$\frac{\cos(B-C)}{\cos(B+C)}=\frac{1+p}{1-p}$$

$$\cos(B-C)=\left(\frac{1+p}{p-1}\right)\times\frac{1}{\sqrt{2}} \qquad ....(1)$$

$$\therefore\ 0 < B, C < \frac{3\pi}{4} \Rightarrow -\frac{1}{\sqrt{2}} < \cos(B-C)\le 1$$

using (1) we get

$$-\frac{1}{\sqrt{2}} < \frac{p+1}{\sqrt{2}(p-1)} \le 1$$

$$\Rightarrow -\frac{1}{\sqrt{2}} < \frac{p+1}{\sqrt{2}(p-1)} \ \&\ \frac{p+1}{\sqrt{2}(p-1)} \le 1$$

$$\frac{2p}{p-1} > 0 \ \&\ \frac{p+1-\sqrt{2}(p-1)}{\sqrt{2}(p-1)} \le 0$$

$$p < 0,\ p > 1 \qquad ...(2) \qquad \& \quad \frac{\left(p-\left(\sqrt{2}+1\right)^2\right)}{(p-1)}$$

$p \ge (\sqrt{2}+1)^2$, $p < 1$ ...(3)

from (2) & (3)

$p \in (-\infty, 0) \cup [(\sqrt{2}+1)^2, \infty)$

Ex.19 For $x \in (-\pi, \pi)$ find the value of x for which the given equation $\left(\sqrt{3}\sin x+\cos x\right)^{\sqrt{\left(\sqrt{3}\sin 2x-\cos 2x+2\right)}} = 4$.

Sol. The given equation is

$$\left(\sqrt{3}\sin x+\cos x\right)^{\sqrt{\left(\sqrt{3}\sin 2x-\cos 2x+2\right)}} = 4$$

$$\text{or}\ \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\sqrt{(3\sin^2 x+\cos^2 x+2\sqrt{3}\sin x\cos x)}} = 4$$

$$\text{or}\ \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\left|\sqrt{3}\sin x+\cos x\right|} = 4$$

$$\text{or}\ \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\left|2\sin\left(x+\frac{\pi}{6}\right)\right|} = 4$$

Hence, $2\sin\left(x+\frac{\pi}{6}\right) = \pm 2$ or, $\sin\left(x+\frac{\pi}{6}\right) = \pm 1$

or $x+\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{2}$

or $x = 2n\pi \pm \frac{\pi}{2} - \frac{\pi}{6}$

at n = 0, 1,

$\therefore\ x = \frac{\pi}{3}, -\frac{5\pi}{3}$ and $x = -\frac{2\pi}{3}$ are the solutions of the given equation.

Ex.20 Solve the equation: $\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$.

Sol. $\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$

or, $\left(\frac{1-\cos 2x}{2}\right)^5 + \left(\frac{1+\cos 2x}{2}\right)^5 = \frac{29}{16}\cos^4 2x$

Let $\cos 2x = t$

Then $\left(\frac{1-t}{2}\right)^5 + \left(\frac{1+t}{2}\right)^5 = \frac{29}{16} t^4$

or, $24t^4 - 10t^2 - 1 = 0$

or, $(2t^2 - 1)(12t^2 + 1) = 0$

or, $12t^2 + 1 \neq 0$

$\therefore$ so, $2t^2 - 1 = 0$ then $t^2 = \frac{1}{2}$

Put the value of t

so, $\cos^2 2x = \frac{1}{2}$ or, $2\cos^2 2x - 1 = 0$

or, $\cos 4x = 0$

or, $4x = n\pi + \frac{\pi}{2}$ or, $x = \frac{n\pi}{4} + \frac{\pi}{8}, n \in I$

EXERCISE

Q.1 If $\frac{\tan 2\theta + \tan\theta}{1 - \tan\theta\ \tan 2\theta}$ = 0, then the general value of θ is

(A) $n\pi$; $n \in I$
(B) $\frac{n\pi}{3}$; $n \in I$
(C) $\frac{n\pi}{4}$; $n \in I$
(D) $\frac{n\pi}{6}$; $n \in I$

Q.2 Find the general solution of x, $\cot\left(\frac{\pi}{4} - \frac{x}{3}\right) = \frac{\sqrt{3}}{3}$

(A) $\frac{\pi}{2}(12n - 1)$; $n \in I$
(B) $\frac{\pi}{2}(12n + 1)$; $n \in I$
(C) $\frac{-\pi}{4}(12n + 1)$; $n \in I$
(D) $\frac{\pi}{4}(12n + 1)$; $n \in I$

Q.3 If $\tan a\theta - \tan b\theta = 0$, then the values of θ for a series in –

(A) A.P.
(B) G.P.
(C) H.P.
(D) None of these

Q.4 Find the general solution of x when $4\tan^2\theta = 3\sec^2\theta$

(A) $n\pi \pm \frac{\pi}{4}$, $n \in I$
(B) $n\pi \pm \frac{\pi}{6}$, $n \in I$
(C) $n\pi \pm \frac{\pi}{3}$, $n \in I$
(D) None of these

Q.5 Find the general solution of x when 2cosx . cos2x = cosx

(A) $n\pi + \frac{\pi}{2}$ & $k\pi \pm \frac{\pi}{6}$, n, $k \in I$
(B) $n\pi \pm \pi$ & $k\pi \pm \frac{\pi}{3}$, n, $k \in I$
(C) $(2n + 1)\pi$ & $k\pi \pm \frac{\pi}{2}$, n, $k \in I$
(D) None of these

Q.6 The solution set of (2 cos x – 1)(3 + 2 cos x) = 0 in the interval $0 \le x \le 2\pi$ is -

(A) {π/3}
(B) {π/3, 5π/3}
(C) {π/3, 5π/3, $\cos^{-1}(-3/2)$}
(D) None of these

Q.7 Find the general solution of 2 sinx + tanx = 0

(A) $n\pi$, $(3k \pm 1)\frac{2\pi}{3}$; $k \in I$
(B) $2n\pi$, $(3k + 1)\frac{2\pi}{3}$; $k \in I$
(C) $2n\pi$, $(3k \pm 1)\frac{2\pi}{3}$; $k \in I$
(D) None of these

Q.8 The general solution of the equation $\tan^2\theta + 2\sqrt{3}\tan\theta = 1$ is given by -

(A) $\theta = \frac{\pi}{2}$

(B) $\left(n + \frac{1}{12}\right)\pi \,;\, n \in I$

(C) $(6n + 1)\frac{\pi}{12}$

(D) $\frac{n\pi}{12}$

Q.9 Find the general solution of θ when $\tan^2\theta - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 0$

(A) $n\pi + \pi/3$ & $n\pi + \pi/4$, $n \in I$

(B) $n\pi + \pi/2$ & $n\pi + \pi/4$, $n \in I$

(C) $(2n + 1)\frac{\pi}{6}$ & $n\pi + \pi/6$, $n \in I$

(D) None of these

Q.10 If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation, $\sin 7x + \sin 4x + \sin x = 0$ is-

(A) 3

(B) 5

(C) 6

(D) None

Q.11 $\sin x$, $\sin 2x$, $\sin 3x$ are in A.P if $x =$

(A) $n\pi/2$, $n \in I$

(B) $n\pi$, $n \in I$

(C) $2n\pi$, $n \in I$

(D) $(2n + 1)\pi$, $n \in I$

Q.12 The general solution of $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$

(A) $(2n + 1)\frac{\pi}{6}, (2n + 1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{3}, n \in I$

(B) $(2n + 1)\frac{\pi}{4}, (2n + 1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I$

(C) $(2n + 1)\frac{\pi}{3}, (2n + 1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{4}, n \in I$

(D) None of these

Q.13 The general solution of $4 \sin x . \sin 2x . \sin 4x = \sin 3x$

(A) $2n\pi, \frac{n\pi}{3} \pm \frac{\pi}{6}, n \in I$

(B) $n\pi, \frac{n\pi}{6} \pm \frac{\pi}{9}, n \in I$

(C) $n\pi, \frac{n\pi}{3} \pm \frac{\pi}{9}, n \in I$

(D) None of these

Q.14 If $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \tan 4\theta \tan 7\theta$, then $\theta =$

(A) $\frac{n\pi}{4}, n \in I$

(B) $\frac{n\pi}{7}, n \in I$

(C) $\frac{n\pi}{12}, n \in I$

(D) $n\pi$, $n \in I$

Q.15 $\sin x + \cos x = 1$ if -

(A) $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(B) $\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(C) $\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(D) $\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Q.16 The equation a sin x + b cos x = c, where $|c| > \sqrt{a^2 + b^2}$ has –

(A) A unique solution
(B) Infinite no. of solutions
(C) No solution
(D) None of these

Q.17 Find the general solution for x when $\sin x \left(\cos \frac{x}{4} - 2 \sin x\right) + \left(1 + \sin \frac{x}{4} - 2 \cos x\right) \cos x = 0$

(A) $2p\pi,\ p \in I$
(B) $(8p - 6)\pi,\ p \in I$
(C) $(3p - 2)\pi,\ p \in I$
(D) $(2p + 1)\pi, p \in I$

Q.18 Find the general solution for x when sin 3x + cos 2x = – 2

(A) $(4p - 3)\frac{\pi}{3}, p \in I$
(B) $(4p - 2)\frac{\pi}{3}, p \in I$
(C) $(4p + 1)\frac{\pi}{7}, p \in I$
(D) None of these

Q.19 If $\cos\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = -1$, then the most general value of θ which satisfies both the equations is

(A) $\theta = 2n\pi + \frac{\pi}{4}, n \in I$
(B) $\theta = 2n\pi + \frac{7\pi}{4}, n \in I$
(C) $\theta = n\pi + \frac{7\pi}{4}, n \in I$
(D) None of these

Q.20 The equation $\sin^6 x + \cos^6 x = \lambda$, has a solution if -

(A) $\lambda \in \left[\frac{1}{2}, 1\right]$
(B) $\lambda \in \left[\frac{1}{4}, 1\right]$
(C) $\lambda \in [-1, 1]$
(D) $\lambda \in \left[0, \frac{1}{2}\right]$

Q.21 Set of values of x lying in [0, 2π] satisfying the inequality $|\sin x| > 2\sin^2 x$ contains

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$
(B) $\left(0, \frac{7\pi}{6}\right)$
(C) π/6
(D) None of these

Q.22 Number of solutions of the equation $2^{\sin^2 x} + 5.2^{\cos^2 x} = 7$ in the interval [–π, π] is

(A) 4
(B) 2
(C) 6
(D) 0

Q.23 The general solution of the equation $7\cos^2 x + \sin x \cos x - 3 = 0$ is given by

(A) $n\pi + \frac{\pi}{2}$ (n ∈ I)
(B) $n\pi - \frac{\pi}{4}$ (n ∈ I)
(C) $n\pi + \tan^{-1}\frac{4}{3}$ (n ∈ I)
(D) $n\pi + \frac{3\pi}{4}, k\pi + \tan^{-1}\frac{4}{3}$ (n, k ∈ I)

Q.24 If $\cos x = \sqrt{1 - \sin 2x}$, 0 < x < π, then a value of x is-

(A) $\tan^{-1} 2$
(B) 0
(C) π
(D) None of these

Q.25 Which of the following can be a root of the equation; sin (x – 2) = sin (3x – 4) in (–π, π)

(A) $-\frac{7\pi}{2}+\frac{3}{2}$ (B) $-\frac{5\pi}{4}+\frac{3}{2}$

(C) $\frac{-3\pi}{4}+\frac{3}{2}$ (D) None of these

Q.26 $3^{\log \tan x}+3^{\log \cot x}=2$ then x is -

(A) $n\pi+\frac{\pi}{2}$ (B) $(4n+1)\frac{\pi}{4}$

(C) $n\pi-\frac{\pi}{4}$ (D) $n\pi$

Q.27 Find the number of solutions of the equation 30 |sin x| = x in $0 \le x \le 2\pi$

(A) 4 (B) 2 (C) 8 (D) 6

Q.28 The number of solutions of tan(5π cosα) = cot(5π sinα) for α in (0, 2π) is

(A) 7 (B) 14 (C) 21 (D) 28

Q.29 The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x}=1$ are in A.P. with common difference

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{8}$ (C) $\frac{3\pi}{8}$ (D) $\frac{5\pi}{8}$

Q.30 The solution of the inequality $\log_{1/2}\sin x > \log_{1/2}\cos x$ in (0, 2π) is

(A) $x \in \left(\frac{5\pi}{4}, 2\pi\right)$ (B) $x \in \left(0, \frac{\pi}{4}\right)$

(C) $x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ (D) None of these

Q.31 Find the general solution of x when $3\cos^2 x - 10\cos x + 3 = 0$

(A) $2n\pi \pm \cos^{-1}(1/3), n \in I$ (B) $2n\pi \pm \cos^{-1}(1/4), n \in I$

(C) $2n\pi \pm \cos^{-1}(1/5), n \in I$ (D) None of these

Q.32 All value of θ, between 0 & π, which satisfy the equation; $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}$ is

(A) $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$ (B) $\frac{\pi}{8}, \frac{\pi}{3}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

(C) $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$ (D) None of these

Q.33 The number of solutions of the equation $\sin\left(\frac{\pi x}{2\sqrt{3}}\right) = x^2 - 2\sqrt{3}x + 4$

(A) forms an empty set (B) is only one

(C) is only two (D) is greater than 2

Q.34 If $\tan^2[\pi(x+y)] + \cot^2[\pi(x+y)] = 1 + \sqrt{\dfrac{2x}{1+x^2}}$ where x, y ∈ R, then least positive value of y is

(A) $\frac{5}{4}$ (B) $\frac{1}{4}$

(C) $\frac{3}{4}$ (D) 2

Q.35 Solve for x, $\sum_{r=1}^{n} \sin(rx) \cdot \sin(r^2x) = 1$

(A) $\frac{(2m+1)\pi}{n(n-1)}$ (B) $\frac{(4m-1)\pi}{(n+1)}$

(C) $\frac{(2m+1)\pi}{n(n+1)}$ (D) None of these

Q.36 The set of values of x for which $\sin x \cdot \cos^3 x > \cos x \cdot \sin^3 x$, $0 \le x \le 2\pi$, is-

(A) $(0, \pi)$ (B) $\left(0, \frac{\pi}{4}\right)$

(C) $\left(\frac{\pi}{4}, \pi\right)$ (D) None of these

Q.37 If $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$, $\theta \in [0, 2\pi]$, then

(A) $\theta \in \left(0, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}\right\}$ (B) $\theta \in \left(\frac{\pi}{2}, \pi\right) - \left\{\frac{3\pi}{4}\right\}$

(C) $\theta \in \left(\pi, \frac{3\pi}{2}\right) - \left\{\frac{5\pi}{4}\right\}$ (D) $\theta \in (0, \pi) - \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$

Q.38 If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$, the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is -

(A) π (B) 2π

(C) $\frac{5\pi}{2}$ (D) None of these

Q.39 Find the general solution of x, $\cos^2 2x + \cos^2 3x = 1$

(A) $(2k+1)\frac{\pi}{10}$, k ∈ I (B) $(\pi k+1)\frac{\pi}{10}$; k ∈ I

(C) $(2k-1)\frac{\pi}{10}$, k ∈ I (D) None of these

Q.40 The equation $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$ is solvable for -

(A) $-\frac{1}{2} \le \alpha \le \frac{1}{2}$ (B) $-3 \le \alpha \le 1$

(C) $-\frac{3}{2} \le \alpha \le \frac{1}{2}$ (D) $-1 \le \alpha \le 1$

Q.41 x_1 and x_2 are two positive values of x for which 2 cos x, |cos x| and 3 $\sin^2 x - 2$ are in G.P.

The minimum value of $|x_1 - x_2|$ is equal to

(A) $\frac{4\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\cos^{-1}\left(\frac{2}{3}\right)$

Q.42 The set of values of x satisfying the equation $2^{\tan\left(x-\frac{\pi}{4}\right)} - 2.(0\cdot 25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}} + 1 = 0$ is-

(A) A set of 2 values; $\left(\frac{\pi}{2}, \frac{5\pi}{4}\right)$ (B) An infinite set

(C) An empty set (D) None of these

Q.43 Find the general solution of x, sin 5x + sin x + $2\sin^2 x = 1$

(A) $\frac{n\pi}{3} + (-1)^n \frac{\pi}{18}, \frac{\pi k}{2} + \frac{\pi}{4}$ when n, k ∈ I (B) $\frac{n\pi}{3} + (-1)^n \frac{\pi}{9}, \frac{\pi k}{2} + \frac{\pi}{4}$; n, k ∈ I

(C) $\frac{n\pi}{3} - (-1)^n \frac{\pi}{9}, \frac{\pi k}{2} - \frac{\pi}{4}$ when n, k ∈ I (D) None of these

Q.44 If $\max_{\theta \in R}$ {5sin θ + 3sin (θ – α)} = 7 then the set of possible values of α is

(A) $\left\{x \mid x = 2n\pi \pm \frac{\pi}{3}; n \in Z\right\}$ (B) $\left\{x \mid x = 2n\pi \pm \frac{2\pi}{3}; n \in Z\right\}$

(C) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ (D) None of these

Q.45 If $0 \le x \le 3\pi$, $0 \le y \le 3\pi$ and cos x sin y = 1 then the possible number of values of the ordered pair (x, y) is

(A) 6 (B) 12

(C) 8 (D) 15

Q.46 If $x \neq \frac{n\pi}{2}$ and $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$, then all solutions of x are given by

(A) $2n\pi + \frac{\pi}{2}$ (B) $(2n + 1)\pi - \frac{\pi}{2}$

(C) $n\pi + (-1)^n \frac{\pi}{2}$ (D) None of these

ANSWER KEY

EXERCISE

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	A	C	A	B	A	B	A	B	A	B	C	C	A,C
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	B	D	B	B	A	B	D	A	C	B	A	D	A	B
Q.No.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	A	A	B	B	C	B	D	B	A	C	C	C	A	A	A
Q.No.	**46**														
Ans.	D														

