

# TRIGONOMETRY

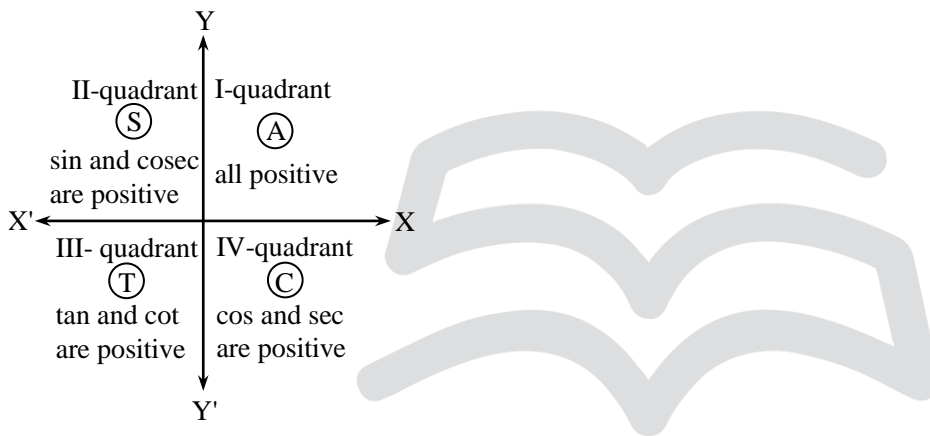
## 1. Fundamental Trigonometric Identities

- (i)  $\sin^2 \theta + \cos^2 \theta = 1$
- (ii)  $1 + \tan^2 \theta = \sec^2 \theta$
- (iii)  $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

## 2. Sign of Trigonometrical Ratios or Functions

A crude aid to memorise the signs of trigonometrical ratio in different quadrant is

“All Silver Tea Cups”



## 3. Trigonometric Ratios of Allied Angles

Allied angles	$(-\theta)$	$(90^\circ - \theta)$ or $\left(\frac{\pi}{2} - \theta\right)$	$(90^\circ + \theta)$ or $\left(\frac{\pi}{2} + \theta\right)$	$(180^\circ - \theta)$ or $(\pi - \theta)$	$(180^\circ + \theta)$ or $(\pi + \theta)$	$(270^\circ - \theta)$ or $\left(\frac{3\pi}{2} - \theta\right)$	$(270^\circ + \theta)$ or $\left(\frac{3\pi}{2} + \theta\right)$	$(360^\circ - \theta)$ or $(2\pi - \theta)$
$\sin \theta$	$-\sin \theta$	$\cos \theta$	$\cos \theta$	$\sin \theta$	$-\sin \theta$	$-\cos \theta$	$-\cos \theta$	$-\sin \theta$
$\cos \theta$	$\cos \theta$	$\sin \theta$	$-\sin \theta$	$-\cos \theta$	$-\cos \theta$	$-\sin \theta$	$\sin \theta$	$\cos \theta$
$\tan \theta$	$-\tan \theta$	$\cot \theta$	$-\cot \theta$	$-\tan \theta$	$\tan \theta$	$\cot \theta$	$-\cot \theta$	$-\tan \theta$

## 4. Sum and Difference Formulae

- (i)  $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$
- (ii)  $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$
- (iii)  $\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- (iv)  $\cot (A \pm B) = \frac{\cot A \cdot \cot B \mp 1}{\cot B \pm \cot A}$
- (v)  $\sin (A + B) \cdot \sin (A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
- (vi)  $\cos (A + B) \cdot \cos (A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

$$(vii) \tan (A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

$$(viii) \sin (A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$$

$$(ix) \cos (A + B + C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$$

## 5. Formulae for Product into Sum or Difference Conversion

$$(i) 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$(ii) 2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$(iii) 2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$(iv) 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$(v) \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$(vi) \sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

$$(vii) \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$(viii) \cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right)$$

## 6. Trigonometrical Ratios of Multiple Angles

$$(i) \sin 2\theta = 2 \sin\theta \cos\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$(ii) \cos 2\theta = \cos^2\theta - \sin^2\theta = 2 \cos^2\theta - 1 = 1 - 2 \sin^2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$(iii) \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$(iv) \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$(v) \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$(vi) \tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

$$(vii) \sin (\theta/2) = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$(viii) \cos (\theta/2) = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$(ix) \tan (\theta/2) = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \left| \frac{1 - \cos \theta}{\sin \theta} \right| = \left| \frac{\sin \theta}{1 + \cos \theta} \right|$$

$$(x) \sqrt{1 + \sin 2A} = |\sin \theta + \cos \theta|$$

$$(xi) \sqrt{1 - \sin 2A} = |\sin \theta - \cos \theta|$$

## 7. Some Useful Identities

$$(i) \tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$$

$$(ii) \tan \theta = \cot \theta - 2 \cot 2\theta$$

$$(iii) \tan 3\theta = \tan \theta \cdot \tan(60^\circ - \theta) \cdot \tan(60^\circ + \theta)$$

$$(iv) \tan(A + B) - \tan A - \tan B = \tan A \cdot \tan B \cdot \tan(A + B)$$

$$(v) \sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$$

$$(vi) \cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$$

## 8. Sine, Cosine and Tangent of some Angles less than 90°

	15°	18°	$\left(22\frac{1}{2}\right)^\circ$	36°
sin	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
cos	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

## SOLVED EXAMPLES

**Ex.1** The value of the expression

$$1 - \frac{\sin^2 y}{1 + \cos y} + \frac{1 + \cos y}{\sin y} - \frac{\sin y}{1 - \cos y} \text{ is equal to}$$

- (A) 0                                      (B) 1  
(C)  $\sin y$                                 (D)  $\cos y$

**Sol.** Taking L.C.M.

$$\begin{aligned} & \left[ 1 - \frac{\sin^2 y}{1 + \cos y} \right] + \left[ \frac{1 + \cos y}{\sin y} - \frac{\sin y}{1 - \cos y} \right] \\ &= \frac{1 + \cos y - \sin^2 y}{1 + \cos y} + \frac{1 - \cos^2 y - \sin^2 y}{\sin y \cdot (1 - \cos y)} \\ &= \frac{\cos y + \cos^2 y}{1 + \cos y} + 0 = \cos y \end{aligned}$$

**Ans.[D]**

**Ex.2** If  $A + B = \frac{\pi}{3}$  and  $\cos A + \cos B = 1$  then

- (A)  $\cos(A - B) = \frac{1}{3}$   
(B)  $|\cos A - \cos B| = \sqrt{\frac{2}{3}}$   
(C)  $\cos(A - B) = -\frac{1}{3}$   
(D)  $|\cos A - \cos B| = \frac{1}{2\sqrt{3}}$

**Sol.**  $\cos A + \cos B = 1$

$$\text{so, } 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} = 1$$

$$\text{or } 2 \cdot \frac{\sqrt{3}}{2} \cdot \cos \frac{A-B}{2} = 1$$

$$\therefore \cos \frac{A-B}{2} = \frac{1}{\sqrt{3}}$$

$$\therefore \cos(A - B) = 2 \cos^2 \left( \frac{A-B}{2} \right) - 1$$

$$= \frac{2}{3} - 1 = -\frac{1}{3}$$

$$|\cos A - \cos B| = \left| 2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2} \right|$$

$$= 2 \cdot \frac{1}{2} \cdot \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$$

**Ans.[B, C]**

**Ex.3** The value of  $\tan 63^\circ - \cot 63^\circ$  is equal to

(A)  $\frac{2}{\sqrt{5}+1} \cdot \sqrt{10-2\sqrt{5}}$

(B)  $\frac{2}{\sqrt{5}+1} \cdot \sqrt{10+2\sqrt{5}}$

(C)  $\frac{\sqrt{5}+1}{4} \cdot \sqrt{5-\sqrt{5}}$

(D) None of these

**Sol.** Value =  $-\frac{\cos 126^\circ}{\frac{1}{2} \sin 126^\circ} = -2 \cot 126^\circ$

=  $-2 \cot (90 + 36)^\circ = 2 \tan 36^\circ$

=  $\frac{4 \cdot \sin 18^\circ \cos 18^\circ}{\cos 36^\circ}$

=  $\frac{4 \cdot \frac{\sqrt{5}-1}{4} \cdot \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}}{\frac{\sqrt{5}+1}{4}}$

=  $\frac{\sqrt{5}-1}{\sqrt{5}+1} \sqrt{10+2\sqrt{5}}$

=  $\frac{\sqrt{(\sqrt{5}-1)^2(10+2\sqrt{5})}}{\sqrt{5}+1}$

=  $\frac{2}{\sqrt{5}+1} \cdot \sqrt{10-2\sqrt{5}}$

**Ans.[A]**

**Ex.4** If  $\sin(\theta + \alpha) = a$  and  $\sin(\theta + \beta) = b$ , then  $\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta)$  is equal to

(A)  $1 - a^2 - b^2$

(B)  $1 - 2a^2 - 2b^2$

(C)  $2 + a^2 + b^2$

(D)  $2 - a^2 - b^2$

**Sol.**  $\sin(\theta + \alpha) = a$ ,  $\sin(\theta + \beta) = b$

$\theta + \alpha = \sin^{-1} a$ ,  $\theta + \beta = \sin^{-1} b$

$\therefore \alpha - \beta = \sin^{-1} a - \sin^{-1} b$

=  $\frac{\pi}{2} - \cos^{-1} a - \frac{\pi}{2} + \cos^{-1} b$

=  $\cos^{-1} b - \cos^{-1} a$

=  $\cos^{-1} \left( ab + \sqrt{1-b^2} \sqrt{1-a^2} \right)$

$\cos(\alpha - \beta) = ab + \sqrt{(1-a^2-b^2+a^2b^2)}$

$\Rightarrow \cos^2(\alpha - \beta) = a^2b^2 + 1 - a^2 - b^2 + a^2b^2 + 2ab \sqrt{1-a^2-b^2+a^2b^2}$

$\therefore \cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta)$

=  $2 \cos^2(\alpha - \beta) - 1 - 4ab \cos(\alpha - \beta)$

=  $2a^2b^2 + 2 - 2a^2 - 2b^2 + 2a^2b^2 + 4ab \sqrt{1-a^2-b^2+a^2b^2} - 1 - 4a^2b^2 - 4ab \sqrt{1-a^2-b^2+a^2b^2}$

$$= 1 - 2a^2 - 2b^2$$

Ans.[B]

**Ex.5** Prove that

$$\left( \frac{1}{\sec^2 \alpha - \cos^2 \alpha} + \frac{1}{\cos^2 \alpha - \sin^2 \alpha} \right) \times \sin^2 \alpha \cos^2 \alpha = \frac{1 - \sin^2 \alpha \cos^2 \alpha}{2 + \sin^2 \alpha \cos^2 \alpha}$$

**Sol.** LHS =  $\left( \frac{\cos^2 \alpha}{1 - \cos^4 \alpha} + \frac{\sin^2 \alpha}{1 - \sin^4 \alpha} \right) \sin^2 \alpha \cos^2 \alpha$

$$= \frac{\cos^4 \alpha \sin^2 \alpha}{(1 - \cos^2 \alpha)(1 + \cos^2 \alpha)} + \frac{\sin^4 \alpha \cos^2 \alpha}{(1 - \sin^2 \alpha)(1 + \sin^2 \alpha)}$$

$$= \frac{\cos^4 \alpha \sin^2 \alpha}{\sin^2 \alpha (1 + \cos^2 \alpha)} + \frac{\sin^4 \alpha \cos^2 \alpha}{\cos^2 \alpha (1 + \sin^2 \alpha)}$$

$$= \frac{\cos^4 \alpha}{1 + \cos^2 \alpha} + \frac{\sin^4 \alpha}{1 + \sin^2 \alpha}$$

$$= \frac{\cos^4 \alpha (1 + \sin^2 \alpha) + \sin^4 \alpha (1 + \cos^2 \alpha)}{(1 + \cos^2 \alpha)(1 + \sin^2 \alpha)}$$

$$= \frac{\cos^4 \alpha + \sin^4 \alpha + \sin^2 \alpha \cos^2 \alpha (\cos^2 \alpha + \sin^2 \alpha)}{1 + (\sin^2 \alpha + \cos^2 \alpha) + \sin^2 \alpha \cos^2 \alpha}$$

$$= \frac{(\cos^2 \alpha + \sin^2 \alpha)^2 - 2 \sin^2 \alpha \cos^2 \alpha + \sin^2 \alpha \cos^2 \alpha}{2 + \sin^2 \alpha \cos^2 \alpha}$$

$$= \frac{1 - \sin^2 \alpha \cos^2 \alpha}{2 + \sin^2 \alpha \cos^2 \alpha} = \text{RHS}$$

**Ex.6** If  $\sec \alpha = 13/5$  ( $0 < \alpha < \frac{\pi}{2}$ ), find the value of  $\frac{2 - 3\cot \alpha}{4 - 9\sqrt{\sec^2 \alpha - 1}}$ .

**Sol.**  $\sec \alpha = \frac{13}{5} \Rightarrow \sec^2 \alpha - 1 = \frac{169}{25} - 1$

$$\Rightarrow \tan^2 \alpha = \frac{144}{25} \Rightarrow \tan \alpha = \frac{12}{5}$$

Now, the given expression is equal to

$$\frac{2 - 3\cot \alpha}{4 - 9\tan \alpha} = \frac{2 - 3 \cdot \frac{5}{12}}{4 - 9 \cdot \frac{12}{5}} = \frac{(24 - 15)5}{12(20 - 108)}$$

$$= \frac{-45}{12 \cdot 88} = -\frac{15}{352}$$

**Ex.7** If  $\cos x = \tan y$ ,  $\cos y = \tan z$  and  $\cos z = \tan x$ , prove that  $\sin x = \sin y = \sin z = 2 \sin 18^\circ$ .

**Sol.**  $\cos^2 x = \tan^2 y = \sec^2 y - 1 = \cot^2 z - 1$

$$\Rightarrow 1 + \cos^2 x = \cot^2 z = \frac{\cos^2 z}{1 - \cos^2 z}$$

$$= \frac{\tan^2 x}{1 - \tan^2 x}$$

$$\Rightarrow 2 - \sin^2 x = \frac{\sin^2 x}{\cos^2 x - \sin^2 x} = \frac{\sin^2 x}{1 - 2\sin^2 x}$$

$$\Rightarrow (2 - \sin^2 x)(1 - 2\sin^2 x) = \sin^2 x$$

$$\Rightarrow 2\sin^4 x - 6\sin^2 x + 2 = 0$$

$$\Rightarrow \sin^4 x - 3\sin^2 x + 1 = 0$$

$$\Rightarrow \sin^2 x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{1}{2}(3 \pm \sqrt{5})$$

$$\text{then } \frac{1}{2}(3 + \sqrt{5}) > 1,$$

$$\text{so, } \sin^2 x = \frac{1}{2}(3 - \sqrt{5}) < 1,$$

$$\sin x = \frac{\sqrt{5}-1}{2} = 2 \sin 18^\circ$$

Similarly,  $\sin y = 2 \sin 18^\circ$  and  $\sin z = 2 \sin 18^\circ$ .

**Ex.8** Given  $\tan 15^\circ = 2 - \sqrt{3}$ , find  $\tan 7\frac{1}{2}^\circ$ .

**Sol.** We know that

$$\tan\left(\frac{A}{2}\right) = \frac{\pm\sqrt{1+\tan^2 A}-1}{\tan A} \quad \dots(1)$$

Putting  $A = 15^\circ$  we have, from equation (1),

$$\tan 7\frac{1}{2}^\circ = \frac{\pm\sqrt{1+(2-\sqrt{3})^2}-1}{2-\sqrt{3}}$$

$$= \frac{\pm\sqrt{8-4\sqrt{3}}-1}{2-\sqrt{3}}$$

Now  $\tan 7\frac{1}{2}^\circ$  is positive, so we must take the positive sign.

$$\text{Hence, } \tan 7\frac{1}{2}^\circ = \frac{+(\sqrt{6}-\sqrt{2})-1}{2-\sqrt{3}}$$

$$= (\sqrt{6}-\sqrt{2}-1)(2+\sqrt{3})$$

$$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2 = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$$

**Ex.9** Prove that  $\frac{\cos 3\theta + \cos 3\phi}{2\cos(\theta-\phi)-1} = (\cos \theta + \cos \phi) \cos(\theta + \phi) - \sin(\theta + \phi) (\sin \theta + \sin \phi)$

**Sol.** LHS = 
$$\frac{2\cos\left(\frac{3\theta+3\phi}{2}\right)\cos\left(\frac{3\theta-3\phi}{2}\right)}{2\left(2\cos^2\frac{\theta-\phi}{2}-1\right)-1}$$

$$\begin{aligned}
 &= \frac{2 \cos\left(\frac{3\theta + 3\phi}{2}\right) \left(4 \cos^3\left(\frac{\theta - \phi}{2}\right) - 3 \cos\left(\frac{\theta - \phi}{2}\right)\right)}{4 \cos^2\left(\frac{\theta - \phi}{2}\right) - 3} \\
 &= 2 \cos\left(\frac{3\theta + 3\phi}{2}\right) \cos\left(\frac{\theta - \phi}{2}\right) \\
 \text{RHS} &= 2 \cos\left(\frac{\theta + \phi}{2}\right) \cos\left(\frac{\theta - \phi}{2}\right) \cos(\theta + \phi) - 2 \sin\left(\frac{\theta + \phi}{2}\right) \cos\left(\frac{\theta - \phi}{2}\right) \sin(\theta + \phi) \\
 &= 2 \cos\left(\frac{\theta - \phi}{2}\right) \left[\cos\left(\frac{\theta + \phi}{2}\right) \cos(\theta + \phi) - \sin\left(\frac{\theta + \phi}{2}\right) \sin(\theta + \phi)\right] \\
 &= 2 \cos\left(\frac{\theta - \phi}{2}\right) \cos\left(\frac{3\theta + 3\phi}{2}\right) = \text{LHS}
 \end{aligned}$$

**Ex.10** Prove that  $4 \sin 27^\circ = (5 + \sqrt{5})^{1/2} - (3 - \sqrt{5})^{1/2}$

**Sol.**  $\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5} + 1}{4}$

$$\Rightarrow \cos 54^\circ = \sqrt{1 - \left(\frac{\sqrt{5} + 1}{4}\right)^2} = \frac{1}{4} \sqrt{10 - 2\sqrt{5}}$$

$$\Rightarrow 1 - 2 \sin^2 27^\circ = \frac{1}{4} \sqrt{10 - 2\sqrt{5}}$$

$$\Rightarrow 2 \sin^2 27^\circ = 1 - \frac{1}{4} \sqrt{10 - 2\sqrt{5}}$$

$$\Rightarrow 16 \sin^2 27^\circ = 8 - 2 \sqrt{10 - 2\sqrt{5}}$$

$$= [(5 + \sqrt{5})^{1/2} - (3 - \sqrt{5})^{1/2}]^2$$

$$\Rightarrow 4 \sin 27^\circ = (5 + \sqrt{5})^{1/2} - (3 - \sqrt{5})^{1/2} \quad [\because \sin 27^\circ \text{ is positive}]$$



## EXERCISE

**Q.1** If  $\sin x + \sin^2 x = 1$ , then  $\cos^8 x + 2 \cos^6 x + \cos^4 x = \dots$

- (A) 0                                      (B) -1  
 (C) 2                                        (D) 1

**Q.2** If  $\sin \theta = \frac{1}{\sqrt{2}}$  and  $\frac{\pi}{2} < \theta < \pi$ . Then the value of  $\frac{\sin \theta + \cos \theta}{\tan \theta}$  is

- (A) 0                                        (B) 1  
 (C)  $\frac{1}{\sqrt{2}}$                                     (D)  $\sqrt{2}$

**Q.3** The expression  $3[\sin^4 \left\{ \frac{3}{2} \pi - \alpha \right\} + \sin^4 (3\pi + \alpha)] - 2[\sin^6 \left( \frac{1}{2} \pi + \alpha \right) + \sin^6 (5\pi - \alpha)]$  is equal to

- (A) 0            (B) 1            (C) 3            (D)  $\sin 4\alpha + \cos 6\alpha$

**Q.4** If  $\frac{\pi}{2} < \alpha < \pi$ ,  $\pi < \beta < \frac{3\pi}{2}$ ;  $\sin \alpha = \frac{15}{17}$  and  $\tan \beta = \frac{12}{5}$ , the value of  $\sin (\beta - \alpha)$  is

- (A)  $\frac{-21}{221}$                                         (B)  $\frac{21}{221}$   
 (C)  $\frac{-171}{221}$                                       (D)  $\frac{171}{221}$

**Q.5**  $\frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta} =$

- (A)  $\tan 3\theta / \tan \theta$                         (B)  $\cot 3\theta / \cot \theta$   
 (C)  $\tan 3\theta \tan \theta$                         (D)  $\cot 3\theta \cot \theta$

**Q.6** If  $\tan A = 1/3$  and  $\tan B = 1/7$  then the value of  $2A + B$  is

- (A)  $30^\circ$                                       (B)  $60^\circ$   
 (C)  $45^\circ$                                       (D)  $145^\circ$

**Q.7**  $\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ =$

- (A) 0                                        (B)  $3/4$   
 (C)  $-3/4$                                       (D)  $-4/3$

**Q.8**  $2 \sin \left( \frac{5\pi}{12} \right) \sin \left( \frac{\pi}{12} \right) =$

- (A)  $-\frac{1}{2}$                                       (B)  $\frac{1}{2}$   
 (C)  $\frac{1}{4}$                                         (D)  $\frac{1}{6}$

**Q.9**  $\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ} =$

- (A) 1 (B) -1  
(C) 0 (D) None

**Q.10** If  $m \sin \theta = n \sin (\theta + 2\alpha)$ , then  $\tan (\theta + \alpha) \cot \alpha =$

- (A)  $\frac{1-n}{1+n}$  (B)  $\frac{m+n}{m-n}$   
(C)  $\frac{m-n}{m+n}$  (D) None

**Q.11**  $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$  when  $n$  is odd, is

- (A)  $2 \cot^n \left(\frac{A-B}{2}\right)$  (B) zero  
(C)  $2 \tan^n \left(\frac{A-B}{2}\right)$  (D) None

**Q.12**  $2 \sin^2 \beta + 4 \cos (\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta)$  is equal to

- (A)  $\sin 2\alpha$  (B)  $\cos 2\beta$   
(C)  $\cos 2\alpha$  (D)  $\sin 2\beta$

**Q.13** If  $\tan \beta = \cos \theta \cdot \tan \alpha$ , then  $\tan^2 \left(\frac{\theta}{2}\right) =$

- (A)  $\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)}$  (B)  $\frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)}$  (C)  $\frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)}$  (D)  $\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)}$

**Q.14**  $\frac{1 - \cos 4\alpha}{\sec^2 2\alpha - 1} + \frac{1 + \cos 4\alpha}{\cos^2 2\alpha - 1}$  is equal to

- (A) 0 (B) 2  
(C) 1 (D) 4

**Q.15**  $\cos^2 \left(\frac{\pi}{16}\right) + \cos^2 \left(\frac{3\pi}{16}\right) + \cos^2 \left(\frac{5\pi}{16}\right) + \cos^2 \left(\frac{7\pi}{16}\right)$  is equal to

- (A) 0 (B) 1 (C) 2 (D) 3

**Q.16** If  $\sin A = \frac{336}{625}$ , where  $450^\circ < A < 540^\circ$ , then  $\sin \frac{A}{4}$  is

- (A)  $3/5$  (B)  $-3/5$   
(C)  $4/5$  (D)  $-4/5$

**Q.17**  $\cot 5^\circ - \tan 5^\circ - 2 \tan 10^\circ - 4 \tan 20^\circ - 8 \cot 40^\circ$  is equal to

- (A) 0 (B)  $8 \tan 40^\circ$   
(C)  $8 \tan 80^\circ$  (D) None of these

**Q.18** If  $0^\circ < \theta < 180^\circ$  then  $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2(1 + \cos \theta)}}}}}$  there being n number of 2's, is equal to

- (A)  $2 \cos \frac{\theta}{2^n}$                       (B)  $2 \cos \frac{\theta}{2^{n-1}}$   
 (C)  $2 \cos \frac{\theta}{2^{n+1}}$                       (D) None of these

**Q.19** The sign of the product  $\sin 2 \sin 3 \sin 5$  is

- (A) Negative                      (B) Positive  
 (C) 0                      (D) None of these

**Q.20**  $\frac{\cos 20^\circ + 8 \sin 70^\circ \sin 50^\circ \sin 10^\circ}{\sin^2 80^\circ}$  is equal to

- (A) 1                      (B) 2  
 (C)  $\frac{3}{4}$                       (D) None of these

**Q.21** It is known that  $\sin \beta = \frac{4}{5}$  and  $0 < \beta < \pi$  then the value of  $\frac{\sqrt{3} \sin(\alpha + \beta) - \frac{2}{\cos \pi/6} \cos(\alpha + \beta)}{\sin \alpha}$  is

- (A) Independent of  $\alpha$  for all  $\beta$  in  $(0, \pi)$   
 (B)  $\frac{5}{13}$  for  $\tan \beta < 0$   
 (C)  $\frac{3(7 + 24 \cot \alpha)}{15}$  for  $\tan \beta > 0$   
 (D) None of these

**Q.22** The value of the expression  $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{10\pi}{7} - \sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14}$  is

- (A) 0                      (B)  $-\frac{1}{4}$   
 (C)  $\frac{1}{4}$                       (D)  $-\frac{1}{8}$

**Q.23**  $\frac{\sin 7x + 6 \sin 5x + 17 \sin 3x + 12 \sin x}{\sin 6x + 5 \sin 4x + 12 \sin 2x} =$

- (A)  $\cos x$                       (B)  $2 \cos x$   
 (C)  $\sin x$                       (D)  $2 \sin x$

**Q.24** If product of  $\sin 1^\circ \sin 3^\circ \sin 5^\circ \dots \sin 89^\circ = \frac{1}{2^n}$  then n equals

- (A) 44                      (B)  $\frac{89}{2}$   
 (C) 45                      (D) None

**Q.25** If  $x \in \left(\pi, \frac{3\pi}{2}\right)$  then  $4 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4 \sin^4 x + \sin^2 2x}$  equals

(A) 2

(B) -2

(C) 3

(D) -3

**Q.26**  $1 + \operatorname{cosec} \frac{\pi}{4} + \operatorname{cosec} \frac{\pi}{8} + \operatorname{cosec} \frac{\pi}{16}$  equals

(A)  $\cot \frac{\pi}{8}$

(B)  $\cot \frac{\pi}{16}$

(C)  $\cot \frac{\pi}{32}$

(D) None

## ANSWER KEY

### EXERCISE

<b>Q.No.</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>
<b>Ans.</b>	D	A	B	D	C	C	C	B	C	B	B	C	C	B	C
<b>Q.No.</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>				
<b>Ans.</b>	C	A	A	A	B	D	B	B	B	A	C				

SpeEdLabs