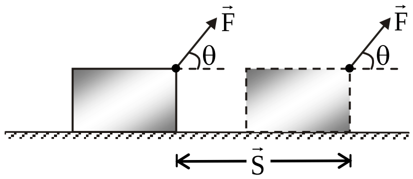
WORK, ENERGY & POWER

1. Work

(a) Work done by a constant force ($\vec{F}$) during displacement ($\vec{S}$) of point of application of force is



$W = \vec{F}.\vec{S}$;

$W = F_x(x_2 - x_1) + F_y(y_2 - y_1) + F_z(z_2 - z_1)$

Where $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

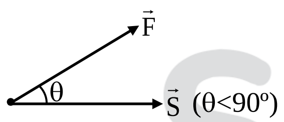
$\vec{S} = \vec{r}_2 - \vec{r}_1$

POSITIVE AND NEGATIVE WORK :

(b) The work is said to be positive if the angle between force and displacement is acute (θ < 90°) because

$W = \vec{F}.\vec{S} = F\,S\cos\theta$ and $\cos\theta = (+)$ve

when $\theta < 90^\circ$.

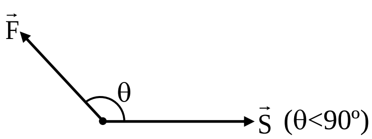


(Positive work)

The positive work means that the external force supplies energy to the system or body.

(c) The work is said to be negative if the angle between force and displacement is obtuse

(θ > 90°) because $\cos\theta = (-)$ve when θ > 90°.



(Negative work)

The negative work means that the force is extracting energy from the system.

(d) WORK IS RELATIVE :

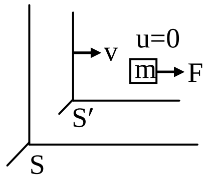
Work done by a force depends on the frame of reference. Suppose S & S' are two frames of reference and S' is moving with a constant velocity v w.r.t S. A block m is acted upon by a constant force F. The block is initially at rest in S - frame.

In - S- frame

Work done by F in time t

$W = F.S$

$$= F\left[0.t + \frac{1}{2}\left(\frac{F}{m}\right)t^2\right] = \frac{F^2t^2}{2m}$$



In S′ - frame

Work done by F in time t

$$W' = FS' = F\left[-vt + \frac{1}{2}\left(\frac{F}{m}\right)t^2\right]$$

Clearly $W \neq W'$

(e) LINE INTEGRAL OF FORCE

Work done by a variable force $\vec{F}$ to move from $\vec{r}_1$ to $\vec{r}_2$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}.d\vec{r}$$

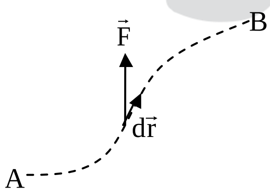
$d\vec{r} \rightarrow$ small displacement

When the force changes in three dimensions then we can consider it to be of the form

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

For calculating the work done from A to B while going through the path AB, we calculate it for an infinitesimal displacement $d\vec{r}$ i.e.

$$W = \int_A^B \vec{F}.d\vec{r}$$



Where $d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

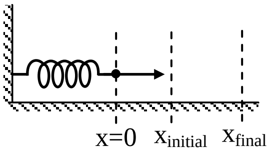
$$W = \int_A^B (F_x dx + F_y dy + F_z dz)$$

Now using path equation, eliminate y and z from F_x, x and z from F_y, x and y from F_z and integrate the above function

$$W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

(f) Area under force *vs* displacement graph gives the work done. (Area is considered with proper sign).

(g) WORK DONE BY SPRING FORCE



$W_{spring} = \frac{1}{2}k(x_{initial}^2 - x_{final}^2)$

(Elongation/compression) $x_{initial}$ & x_{final} are measured from natural length of spring.

2. Kinetic energy

(a) $K = \frac{1}{2}mv^2$

m → mass of the body

v → speed of the body

(b) WORK ENERGY THEOREM:

(i) The sum of the work done by all the forces [external as well as internal] acting on a particle (or a system of particles) is equal to the change in its kinetic energy.

Work done by all the forces

$\Sigma W = (K.E.)_{final} - (K.E.)_{initial}$

(ii) While applying work energy theorem, we must take into account not only the external but also the internal forces to calculate the total work done.

(iii) $W_{cosnservative\ force} + W_{non\ conservative\ force} = K.E_f - K.E_i$

$W_{by\ forces\ other\ than\ conservative\ force}$

$= (KE_f - KE_i) - W_{by\ conservative\ force}$

$= (KE_f - KE_i) + W_{against\ conservative\ force}$

(iv) $W_{by\ force\ other\ than\ conservative}$

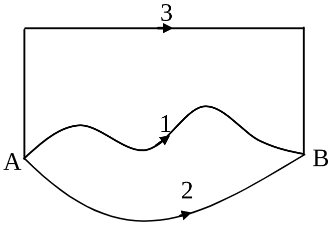
$= (KE_f - KE_i) + (PE_f - PE_i)$

3. Relation between conservative force and potential energy

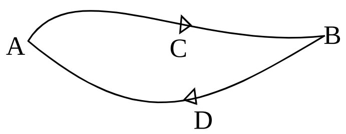
(a) Conservative Force

(i) If the work done by a force in moving a body from an initial location to a final location is independent of the path taken, then the force is conservative.

$W_1 = W_2 = W_3$



(ii) If the total work done by a force is zero, when a body is moved in a closed path, then the force is conservative.



$W_{ACBDA} = W_{ACB} + W_{BDA} = 0$

(iii) For conservative force

$\vec{\nabla} \times \vec{F} = \vec{0}$

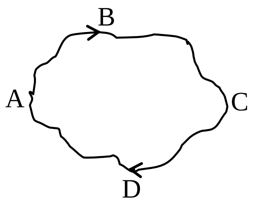
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \vec{0}$$

where $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

(b) Non-Conservative force

(i) The work done by a force in a closed path is non-zero.



$W_{ABCDA} \neq 0$

(ii) Work done by force is path dependent.

(c) Potential Energy Curve

(i) Conservative force is defined as the negative gradient of the potential energy

(ii) $\vec{F}_C = -\vec{\nabla}\,U = -\text{grad}\,U$

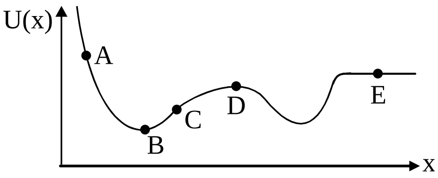
$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ = del operator

U = scalar potential energy function

(iii) In one dimension, the component of conservative force along a specified direction (say x -axis) is equal to the negative derivative of the potential energy w.r.t. distance along the x – axis.

i.e. $F_{c_x} = -\frac{\partial U}{\partial x}$

(iv) If we plot U(x) versus x curve for one dimensional motion then the negative of the slope is equal to the conservative force.



$F_x = -\frac{\partial U}{\partial x}$

At **A,** $\frac{dU}{dx} = (-)ve \quad F_x = (+)ve$

Force is directed along + (ve) x-axis

At **B**, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

Force = 0, position B = **stable**
equilibrium position

At **C**, $\frac{dU}{dx} = (+)ve, \quad F_x = (-)ve$

Force is directed along (–)ve x - axis

At **D**, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} < 0$

Force = 0 , position
D = **Unstable** equilibrium position

At **E,** $\frac{dU}{dx} = 0$ and

if we slightly displace the particle from position E, it experiences no force. Thus position E = **neutra**l equilibrium position.

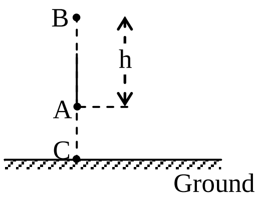
4. Gravitational potential energy

(a) The change in the potential energy of a system corresponding to a conservative internal force is defined as

$$U_f - U_i = -W_c = \int_i^f \vec{F}_c . d\vec{r}$$

where W_c = Work done by the internal conservative force on the system.

(b) Gravitational potential energy (Near the earth's surface)



$U_B - U_A = -W_{AB} = mgh$

5. Spring potential energy

$U = \frac{1}{2} kx^2$

$x \rightarrow$ elongation/compression in spring k $\rightarrow$ spring constant

6. Mechanical energy conservation

(a) Mechanical energy = kinetic energy + potential energy

$E = K + U$

(b) The total mechanical energy of a system remains constant if

(i) The internal forces are conservative .

(ii) and the external forces do no work.

This is called the principle of conservation of mechanical energy.

Note :– We can apply the principle of conservation of mechanical energy even in the presence of external forces. But in that case, external forces must not work on the system.

(c) MATHEMATICAL FORM

$U_f + K_f = U_i + K_i = E =$ constant

(**d)** Modified form of the work- energy theorem. According to the work-energy theorem,

$W_c + W_{nc} + W_{ext} = K_f - K_i$ 1

But $W_c = -(U_f - U_i)$

$W_{nc} + W_{ext} = (K_f + U_f) - (K_i + U_i)$

$W_{nc} + W_{ext} = E_f - E_i = \Delta E$ 2

(i) We generally come across with complicated integration technique while calculating the work done by the conservative force. To avoid this complexity, we use this modified form.

(ii) While applying this modified form , we must not include the work done by those conservative forces whose corresponding potential energy terms are to be included.

(e) If the internal forces are only conservative but the external forces can do work , then

$W_{ext} = E_f - E_i$ $[\because W_{nc} = 0]$

i.e. the work done by the external forces equals the change in the mechanical energy.

7. Power

(i) The time rate of doing work is called the power delivered by the force.

(ii) Instantaneous power

$$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$$

Where $\vec{F}$ = applied force

$\vec{v}$ = instantaneous velocity

(iii) Average power

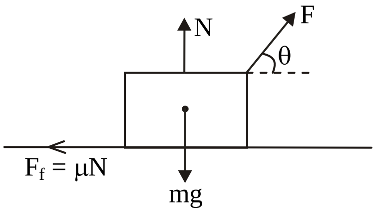
$$P_{avg} = \frac{\text{work done by force during given time interval}}{\text{given time interval}}$$

(iv) Area under Power versus time graph gives the work done

SOLVED EXAMPLES

Ex. 1 A person pulls a 6 kg block by a distance of s = 12 m along a horizontal surface at a constant speed. If the coefficient of kinetic friction is $\mu = 0.2$ and the cord pulling the block is at an angle of 45° with the horizontal, then calculate the work done by the person. (take $g = 10\ m/s^2$)

Sol.



Since there is no upward motion

$N + F \sin\theta = mg$ .....(1)

Since the motion along the displacement direction is without any acceleration

$F \cos\theta = \mu N$ .....(2)

from eq. (1) & (2)

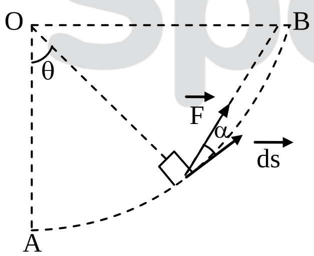
$$F = \frac{\mu mg}{\cos\theta + \mu \sin\theta}$$

$$= \frac{0.2 \times 6 \times 10}{\frac{1}{\sqrt{2}}(1+0.2)} = 10\sqrt{2} \text{ newton}$$

Work done $W = Fs \cos\theta$

$$= (10\sqrt{2}) \times 12 \times \frac{1}{\sqrt{2}} = 120 \text{ J}$$

Ex. 2 Figure shows a smooth circular path of radius R in the vertical plane which subtends an angle of $\pi/2$ at O. A block of mass m is taken from position A to B under the action of a constant force F which is always directed towards the point B, then determine the work done by this force.



Sol. From simple geometry

$$\alpha = \frac{\pi}{4} - \frac{\theta}{2}$$

$$W = \int dW = \int \vec{F}.\vec{ds}$$

$$W = \int F ds \cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$= \int_{\theta=0}^{\pi/2} \left(F \cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right) R\, d\theta$$

$[\because ds = R\, d\theta]$

$$= -RF\,2\left[\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]_0^{\pi/2}$$

$$= -2RF\left[\sin\left(\frac{-\pi}{9}\right)-\sin\left(\frac{\pi}{9}\right)\right]$$

$$= \frac{4RF}{\sqrt{2}} = 2\sqrt{2}\ RF$$

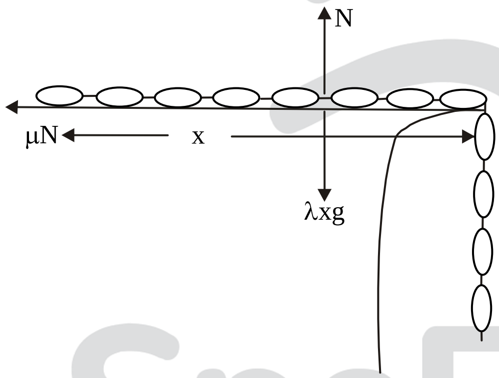
Ex.3 A chain of mass m and length ℓ rests on a rough–surfaced table so that one of its ends hangs over the edge. The chain starts sliding off the table all by itself provided the over hanging part equals η of the chain length. What will be the total work performed by the friction forces acting on the chain by the moment it slides completely off the table?

Sol. Initially for the chain

$$\lambda\eta\ell g = \mu\lambda(1-\eta)\,\ell g$$

$$\mu = \frac{\eta}{1-\eta} \qquad ...(1)$$

Let at an arbitrary moment of time, the length of the chain on the table is x.



Net friction force $F_f = \mu N = \mu\lambda xg$

Infinitesimal work done by friction

$$dW = -\vec{F_f}\cdot\vec{dr} = -F_f\,ds$$

$$dW = -(\mu\lambda xg)(-dx) = \frac{\lambda\eta gx\,dx}{1-\eta}$$

Hence the total work done

$$W = \int dW = \frac{\lambda\eta g}{1-\eta}\int_{(1-\eta)\ell}^{0} x\,dx$$

$$W = -(1-\eta)\,\eta\,\frac{mg\ell}{2} \quad [\because \lambda = \frac{m}{\ell}]$$

Ex.4 An object of mass 5kg falls from rest through a vertical distance of 20 m and reaches a velocity of 10 m/s. . How much work is done by the push of the air on the object ?

(A) 350 J (B) 750 J

(C) 200 J (D) 300 J

Sol. **(B)** The following two forces are acting on the body

(i) Weight mg is acting vertically downward

(ii) The push of the air is acting upward.

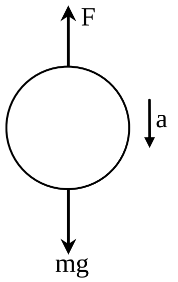
As the body is accelerating down ward, the resultant force is (mg – F).

Workdone by the resultant force to fall through a vertical distance of 20m is (mg – F) × 20 joule.

Gain in the kinetic energy = $\frac{1}{2}mv^2$

Now the work by the resultant force is equal to the change in kinetic energy i.e.

$(mg - F)\,20 = \frac{1}{2}mv^2$ (from work-energy theorem)



or $(50 - F)\,20 = \frac{1}{2} \times 5 \times (10)^2$

or $50 - F = 12.5$

or $F = 50 - 12.5$

$\therefore F = 37.5$ N

Work done by the force

$= -37.5 \times 20 = -750$ joule

(the negative sign. is used because the push of the air is upwards while the displacement is downwards.)

Ex.5 A plank of mass M and length L is placed at rest on a smooth horizontal surface. A small block of mass m is projected with a velocity v_o from the left end of it as shown in the figure. The coefficient of friction between the block and the plank is μ, and its value is such that the block becomes stationary w.r.t the plank before it reaches the other end.

(i) Find the work done by the friction force on the block during the period it slides on the plank. Is the work positive or negative?

(ii) Calculate the work done on the plank during the same period. Is the work positive or negative?

(iii) Also, determine the net work done by friction. Is it positive or negative ?

Sol. From Free Body Diagrams :-

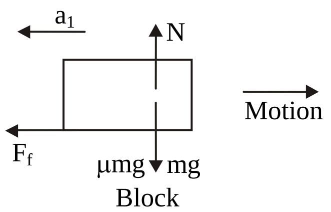
For Block $a_1 = \frac{F_f}{m} = \mu g$

velocity at time t

$v_1 = v_0 - \mu gt$

For Plank $a_2 = \frac{F_f}{M} = \frac{\mu mg}{M}$

velocity at time t, $v_2 = \frac{\mu mgt}{M}$

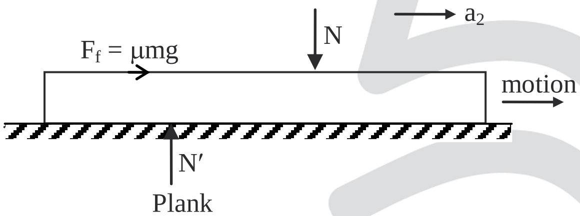


Finally both moves with common velocity i.e. $v_1 = v_2$

$v_0 - \mu gt = \frac{\mu mgt}{M}$

$t = \frac{M v_0}{(M+m)\mu g}$

$\therefore$ common velocity $v = \frac{mv_0}{M+m}$



(i) Work done by the friction on the block = Change in the kinetic energy of the block

$W_1 = K_f - K_i = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$

$\therefore W_1 = \frac{1}{2} m\left[\left(\frac{mv_0}{M+m}\right)^2 - v_0^2\right]$

$= -\frac{1}{2}\frac{mM}{(m+M)^2}(M + 2m)\, v_0^2$

= negative

(ii) Work done by the friction on the plank = Change in the kinetic energy of the plank

$W_2 = \frac{1}{2} Mv^2 - 0$

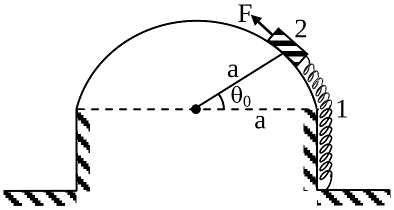
$W_2 = \frac{1}{2} M\left(\frac{mv_0}{M+m}\right)^2$

$= \frac{1}{2}\frac{m^2 M v_0^2}{(m+M)^2}$ = positive

(iii) Net work done by the friction is : -

$W = W_1 + W_2 = -\frac{1}{2}\frac{mM}{(m+M)} v_0^2$ = negative

Ex.6 In the position 1 (figure) the spring of constant K is undeformed. Find the work done by the force $\vec{F}$ (which is always directed along the tangent to the smooth hemi spherical surface) on the small block of mass m to shift it from the position 1 to position 2 slowly.



Sol. Let us locate the block at an arbitrary angular position $\theta < \theta_o$

$dW = \vec{F} . \vec{dr} = Fds = F (Rd\theta)$ ...(1)

Now from the condition of equilibrium of the block , we have

$F = mg \cos \theta + KR \theta$ ...(2)

from eq. (1) & (2)

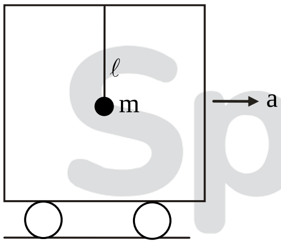
$dW = mg R \cos \theta\, d\theta + KR^2 \theta\, d\theta$

Hence the sought work

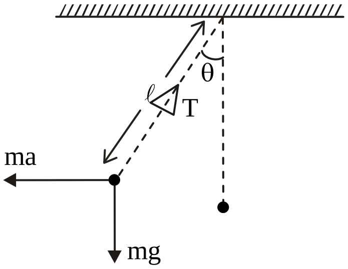
$$W = \int dW = \int_0^{\theta_0} \cos\theta\, d\theta + KR^2 \int_0^{\theta_0} \theta\, d\theta$$

$$W = mg R \sin \theta_0 + \frac{KR^2\theta_0{}^2}{2}$$

Ex.7 A pendulum of mass m and length is suspended from the ceiling of a trolley which has a constant acceleration a in the horizontal direction as shown in the figure . Find the maximum deflection θ of the pendulum from the vertical.



Sol. Free Body Diagram w.r.t trolley



Initial velocity of the mass m = 0

Velocity at maximum deflection = 0

$\therefore \Delta K = 0$

Applying work- energy theorem, we get

$W_g + W_{ps} + W_T = \Delta K$

W_g = work done by gravity

$= -\,mg\ell\,(1 - \cos\theta)$

W_{ps} = work done by pseudo force

$= ma\ell\,\sin\theta$

W_T = work done by tension = 0

(θ = maximum deflection angle) $(\because \vec{T} \perp \vec{ds})$

$\therefore -\,mg\ell\,(1-\cos\theta) + ma\ell\,\sin\theta + 0 = 0$

$$\frac{1-\cos\theta}{\sin\theta} = \frac{a}{g}$$

$$\tan\frac{\theta}{2} = \frac{a}{g}$$

Hence $\theta = 2\tan^{-1}\left(\frac{a}{g}\right)$

Ex.8 A stone with weight W is thrown vertically upward in the air with initial speed v_o. If a constant force F due to air drag acts on the stone through out its flight :

(a) Show that the maximum height reached by the stone is :

$$h = \frac{v_0^2}{2g\left[1+\frac{F}{W}\right]}$$

(b) Show that the speed of the stone upon impact with the ground is

$$v = v_0\left(\frac{W-F}{W+F}\right)^{1/2}$$

Sol. (a) From work – energy theorem,

For upward motion, work done by

(gravity+airdrag)=change in the kinetic energy

$$-\frac{W}{g}gh - Fh = 0 - \frac{1}{2}\frac{W}{g}v_0^2$$

$$h = \frac{v_0^2}{2g\left[1+\frac{F}{W}\right]} \qquad ...(1)$$

(b) For downward motion

$$+\frac{W}{g}gh - Fh = \frac{1}{2}\frac{W}{g}v^2 - 0$$

$$h = \frac{v_0^2}{2g\left[1-\frac{F}{W}\right]} \qquad ...(2)$$

Dividing eq. (2) by (1), we have

$$\frac{v^2}{v_0^2} = \frac{W-F}{W+F} \Rightarrow v = v_0\left(\frac{W-F}{W+F}\right)^{1/2}$$

Ex.9 The potential energy ϕ (in Joule) of a particle of mass 1 kg. moving in the XY plane obeys the law $\phi = 3x + 4y$, where (x, y) are the co- ordinates of the particle in metre. If the particle is at rest at (6,4) at time t = 0, then find out.

(i) Its acceleration

(ii) The work done by the external force in moving the particle from the position of rest of the particle, to the instant of the particle crossing the X - axis

(iii) The speed of the particle when it crosses the y – axis.

(iv) The co-ordinate of the particle at time t = 4 sec.

Sol. (i) $\phi = 3x + 4y$

$$\vec{F} = -\nabla\phi = -\hat{i}\frac{\partial\phi}{\partial x} - \hat{j}\frac{\partial\phi}{\partial y}$$

$$\vec{F} = -(3\hat{i} + 4\hat{j})\ N$$

Acceleration $\vec{a} = \frac{\vec{F}}{m} = -\ 3\hat{i} + 4\hat{j}\ m/s^2$ $\quad(\because m = 1\ kg)$

(ii) $\because$ acceleration is constant,

$\therefore$ position at time t, $\vec{r} = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\vec{r} = (6\hat{i} + 4\hat{j}) + 0 + \frac{1}{2}(-3\hat{i} + 4\hat{j})t^2 \quad ...(1)$$

$= x\hat{i} + y\hat{j}$

When the particle is crossing x -axis, y = 0

$4 - 2t_1^2 = 0$ hence $t_1 = \sqrt{2}$ sec

Work done $W = \vec{F}\cdot(\vec{r} - \vec{r}_0) = k_f - k_i$

$W = (3\hat{i} + 4\hat{j})\cdot(3\hat{i} + 4\hat{j}) \times 2$

= 9 + 16 = 25 J

(iii) When the particle is crossing y –axis,

x = 0 hence from eq. ....(1)

$6 - \frac{3}{2}t_2^2 = 0$ or $t_2 = 2$ sec.

$\vec{v} = \vec{u} + \vec{a}t_2 = 0 + \vec{a}t_2$

$v = a t_2 = \left(\sqrt{3^2 + 4^2}\right) \times 2 = 10$ m/s

(iv) Using eq.(1), at t = 4 sec.

$$\vec{r} = (6\hat{i} + 4\hat{j}) - \frac{1}{2} \times (3\hat{i} + 4\hat{j})\ 4 \times 4$$

$= -\ 18\hat{i} - 28\hat{j}$

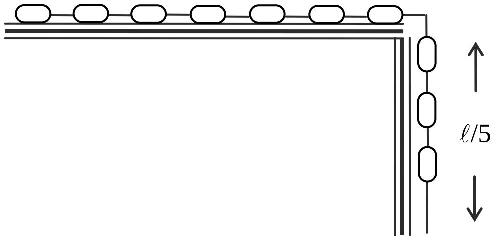
$\therefore$ Coordinates of the particle = (–18, – 28)

Ex.10 A uniform chain is held on a frictionless table with one-fifth of its length hanging over the edge. If the chain has a length ℓ m and a mass m, how much work is required to pull the hanging part back on the table?

(A) mg/10 (B) mg/5

(C) mg/50 (D) mg/2

Sol.(C) Mass of the hanging part of the chain = (m/5); the weight mg/5 acts at the center of the gravity of the hanging chain, i.e., at a distance = ℓ /10 below the surface of a table .

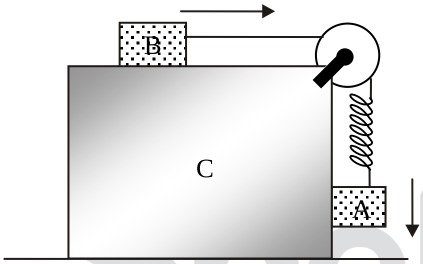


The gain in the potential energy in pulling the hanging part on the table.

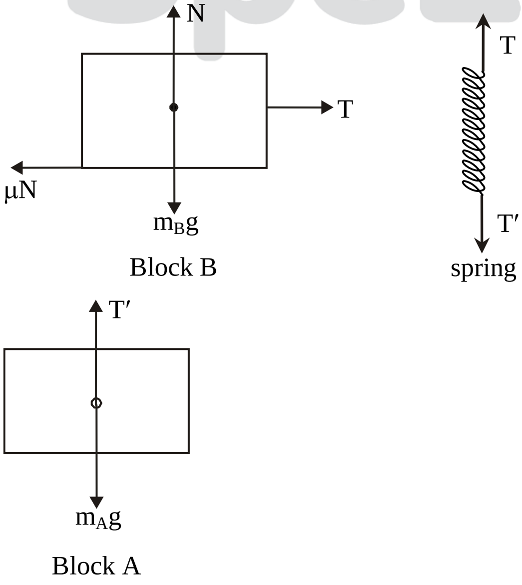
$$U = \frac{mg}{5} \times \frac{\ell}{10} = \frac{mg\ell}{50}$$

$\therefore$ Work done = U = mg /50

Ex.11 Two blocks A and B are connected to each other by a string and a spring ; the string passes over a frictionless pulley as shown in figure . Block B slides over the horizontal surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of the blocks is 0.2 .The force constant of the spring is 1960 N/M. If the mass of block A is 2 kg, calculate the mass of block B and the energy stored in the spring .



Sol. Free Body Diagram



Since Block A and B has no acceleration and spring is massless.

$N = m_B g$ ....(1)

$T = \mu N = \mu\, m_B\, g$ ....(2)

$T = T'$ ....(3)

$T' = m_A g$ ....(4)

From eq.(2), (3) & (4)

$\mu\, m_B g = m_A g$

$m_B = \frac{m_A}{\mu} = \frac{2}{0.2} = 10$ kg .

Extension in the spring = x (say)

$T = kx$

$x = \frac{T}{k} = \frac{19.6}{1960} = 0.01$ m

$\therefore$ Energy stored in the spring

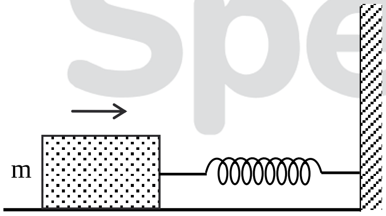
$U = \frac{1}{2} kx^2$

$= \frac{1}{2} \times 1960 \times (0.01)^2 = 0.098$ J

Ex.12 A 2 kg block collides with a horizontal weightless spring of force constant 2 N/m.The block compresses the spring 4 metre from the rest position. The speed of the block at the instant of collision (i) If the surface on which the block slides is frictionless, (ii) if the coefficient of kinetic friction between the block and the horizontal surface is 0.25 .

(A) 4 m/s, 4m/s (B) 4m/s , 5.96m/s
(C) 5m /s,7.96 m/s (D) 5 m/s, 5m/s

Sol. **(B)** The situation is shown in fig.



(i) When the spring is compressed, the energy E stored is given by

$E = \frac{1}{2} kx^2 = \frac{1}{2} \times 2 \times 4^2 = 16$ Joule

This amount of energy is imparted to the block. Let v be the velocity of the block at the instant of collision. When spring is compressed, the velocity of block becomes zero. Hence, the energy lost by the block, is–

$= \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$

$= \frac{1}{2} mv^2$ as $v_0 = 0$.

$\therefore \frac{1}{2} mv^2 = 16$ Joule or $\frac{1}{2} \times 2\, v^2 = 16$ or v = 4 m/sec.

(ii) Work done by the block against friction,

W = Force × Distance

= μ R. distance (∴ F = μ R)

= μ (mg) d (∴ R = m g)

= 0.25 × 2 × 9.8 × 4

= 19.6 Joule

Now total energy lost by the block

= 16 Joule + 19.6 Joule

= 35.6 Joule

If v_1 be the velocity of the spring at the instant of collision, we have

$$\frac{1}{2} m v_1^2 = 35.6 \text{ Joule}$$

$$\frac{1}{2} \times 2 \times v_1^2 = 35.6$$

$$v_1^2 = 35.6$$

or v_1 = 5.96 m/sec.

Ex.13 An ideal massless spring can be compressed 2 metre by a force of 200N. This spring is placed at the bottom of a frictionless inclined plane which makes an angle θ = 30° with the horizontal. A 20 kg mass is released from rest at the top of the inclined plane and is brought to rest momentarily after compressing the spring 4 metre. Through what distance does the mass slide before coming to rest ?

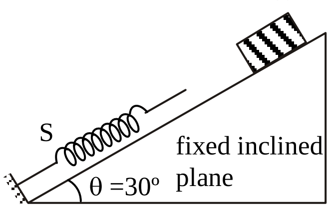
(A) 4.17 m (B) 1.00 m

(C) 8.17 m (D) 2.17 m

Sol.(C) As the spring is compressed by 2 metre with the application of a force of 200 N. Hence its force constant k is given by

$$k = \frac{F}{2} = \frac{200}{2} = 100 \text{ N/ m}$$

Let ℓ be the distance along the inclined plane in which the mass travels before it comes to rest. Now applying the conservation of energy.

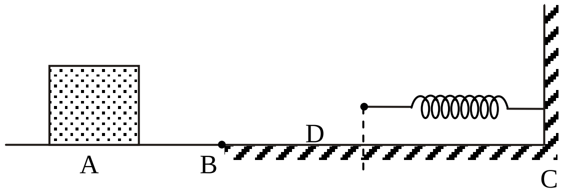


$$\frac{1}{2} kx^2 = mg\,h = mg\ell \sin\theta$$

or $\frac{1}{2} \times 100 \times 4^2 = 20 \times 9.8 \times \ell \times \frac{1}{2}$

or ℓ = 800/98 = 8.17 m.

Ex.14 A 0.5 kg , block slides from the point A on a horizontal track with an initial speed 3m/s towards a weight less horizontal spring of length 1m and force constant 2 N/m . The part AB of the track is frictionless and the part BC has the coefficient of static and kinetic friction as 0.22 and 0.20 respectively. (see figure) If the distance AB and BD are 2m and 2.14m respectively, find the total distance through which the block moves before it comes to rest completely. ($g = 10\ m/s^2$)



Sol. Kinetic energy of the block

$= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 3^2 = 2.25$ J

Path AB is frictionless .

In the path BD , work done against friction

$= \mu_K mgs$

$= 0.2 \times 0.5 \times 10 \times 2.14 = 2.14$ J

So at D, kinetic energy

$= 2.25 – 2.14 = 0.11$ J

Now if the spring is compressed by x, from energy conservation.

$0.11 = \frac{1}{2}kx^2 + \mu_K mgx$

$0.11 = \frac{1}{2} \times 2 \times x^2 + 0.2 \times 0.5 \times 10x$

$x^2 + x – 0.11 = 0$

$\Rightarrow$ x = 0.1 m [$\because$ x = –1.1 is in admissible]

Compressed spring exerts a force

$F = kx = 2 \times 0.1 = 0.2$ N

Limiting (maximum) static frictional force between block and track

$F_{fmax} = \mu_s mg = 0.22 \times 0.5 \times 10 = 1.1$ N

$\therefore\ F < F_{f\,max}$

The block will no move back

So, the total distance moved by the block

$= 2 + 2.14 + 0.1 = 4.24$ m

Ex.15 A particle moving in a straight line is acted by a force, which works at a constant power and changes its velocity from u to v in passing over a distance x. The time taken will be -

(A) $x\left(\frac{v-u}{v^2+u^2}\right)$ (B) $x\left(\frac{v+u}{v^2+u^2}\right)$

(C) $\frac{3}{2}\ x\left(\frac{v^2-u^2}{v^3-u^3}\right)$ (D) $x\left(\frac{v}{u}\right)$

Sol. **(C)**

The force acting on the particle $= \frac{mdv}{dt}$

Power of the force = $\left(\frac{mdv}{dt}\right)$ v = k (constant)

$m\frac{v^2}{2} = kt + c$ ...(1)

at t = 0, v = u $\therefore c = \frac{mu^2}{2}$

Now from (1),

$m\frac{v^2}{2} = kt + \frac{mu^2}{2}$

$\frac{1}{2}m(v^2 - u^2) = kt$ ...(2)

Again $\frac{mdv}{dt}v = k$

$m.v\frac{dv}{dx}v = k$

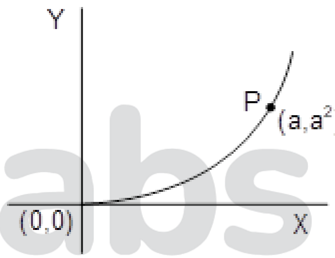
$\int_u^v mv^2dv = \int_0^x k\,dx$

Intergrating, $\frac{1}{3}m(v^3 - u^3) = kx$ ...(3)

From eqn (2) and (3),

$t = \frac{3}{2}\left(\frac{v^2 - u^2}{v^3 - u^3}\right)x$

Ex.16 Under the force $\vec{F} = xy^2\hat{i} + yx^2\hat{j}$, a particle is moving along a parabolic path given by $y = x^2$. Find the work done by this force in moving the particle from (0, 0) to (a, a^2)



Sol. Given $\vec{F} = xy^2\hat{i} + yx^2\hat{j}$

But $\vec{dr} = dx\,\hat{i} + dy\,\hat{j}$

$\therefore W = \int_{path}\vec{F}.\vec{dr}$

$= \int_{path} xy^2dx + yx^2dy$

$= \int_{x=0}^{a} xy^2dx + \int_{y=0}^{a^2} xy^2dy$

Eliminating y^2 and x consecutively from (i) and (ii) function, we have

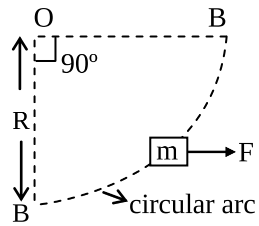
$W = \int_0^a x^5dx + \int_0^{a^2} y^2dy$

$= \frac{a^6}{6} + \frac{a^6}{3} = \frac{a^6}{2}$

EXERCISE # 1

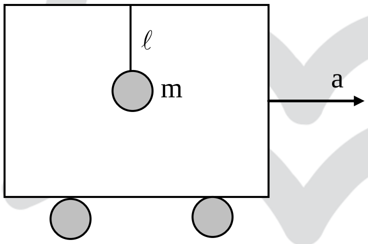
Work

Q.1 A block of mass m is taken from A to B under the action of a constant force F. Work done by this force is–



(A) FR (B) $\frac{\pi}{2}$FR (C) $\frac{FR}{\sqrt{2}}$ (D) $\frac{FR}{4}$

Q.2 A pendulum of mass m and length ℓ is suspended from the ceiling of a trolley which has a constant acceleration **a** in the horizontal direction as shown in figure. Work done by the tension is (In the frame of trolley) –



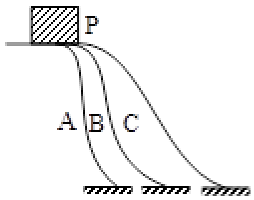
(A) $-mg\ell(1 - \cos\theta)$ (B) $ma\ell \sin\theta$
(C) $ma\ell \cos\theta$ (D) zero

Q.3 The displacement x of a body of mass 1kg on horizontal smooth surface as a function of time t is given by $x = t^4/4$. The work done in the first one second is–

(A) $\frac{1}{4}$ J (B) $\frac{1}{2}$ J (C) $\frac{3}{4}$ J (D) $\frac{5}{4}$ J

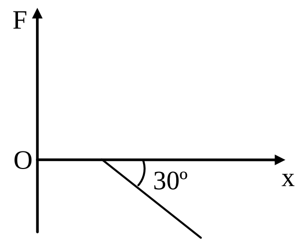
Q.4 A body moves a distance of 10m along a straight line under the action of a force of 5N. If the work done is 25 J , the angle which the force makes with the direction of motion of the body is-
(A) 0° (B) 30° (C) 60° (D) 90°

Q.5 A greased block P may slide along any of the three frictionless slopes A, B, or C, to reach the ground. The work done on the block by the block's weight Mg, are W_A , W_B , and W_C for the three slopes respectively. Then-



(A) $W_A < W_B < W_C$
(B) $W_A > W_B > W_C$
(C) $W_A = W_B = W_C$
(D) None of the above

Q.6 In a spring , it is found that the spring force F and the extension in the spring x are related as shown in figure. Then the value of the force constant of the spring is



(A) $\sqrt{3}$ (B) $\sqrt{3}/2$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}$

Energy

Q.7 A 5.0 kg block is thrown up a 30° incline with an initial speed v of 6.0 m/s. It is found to travel a distance d = 2.0 m up the plane as its speed gradually decreases to zero. then the loss in mechanical energy of the block due to friction in this process is

(A) 8 J (B) 41 J

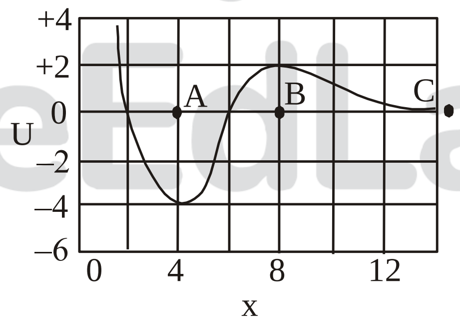
(C) 49 J (D) 90 J

Q.8 A long spring , when stretched by a distance x, has the potential energy U. On increasing the stretching to nx , the potential energy of the spring will be –

(A) U/n (B) nU

(C) n^2U (D) U/n^2

Q.9 For the potential energy(U) *vs* position(x) function shown in fig. there will be an unstable equilibrium at position



(A) A (B) B

(C) C (D) none of the above

Q.10 Energy required to accelerate a car from 10 to 20 ms^{-1} compared with that required to accelerate from 0 to 10 ms^{-1} in the same interval of time covering the same distance, is.

(A) twice (B) four times

(C) three times (D) same

Q.11 If a simple pendulum of length ℓ has the maximum angular displacement (θ) then the maximum KE of its bob of mass m is –

(A) (1/2) m (ℓ /g)

(B) (1/2) m (g / ℓ)

(C) mgℓ (1 – cos θ)

(D) (1/2) mgℓ sin θ

Q.12 Two springs A and B ($k_A = 2k_B$) are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, then energy that in B is-

(A) E/ 2 (B) 2E

(C) E (D) E/4

Power

Q.13 A block of mass M is allowed to slide down a fixed smooth inclined plane of angle θ and length ℓ .What is the power developed by the force of gravity when the block reaches the bottom ?

(A) $\sqrt{2m^2\ell(g\sin\theta)^3}$

(B) (2/3) m^3 ℓ g^2 sin θ

(C) $\sqrt{(2/3)m^2t^2g\cos\theta}$

(D) (1/3) m^3 ℓ g^2 sin θ

Q.14 A body of mass m is projected at an angle θ to the horizontal with initial velocity u. The mean power developed by the gravity over the time of flight is–

(A) mgu sinθ (B) mgu cosθ

(C) mg(gt – u) (D) zero

Q.15 An object of mass (m) is located on the horizontal plane at the origin O. The body acquires horizontal velocity V. The mean power developed by the frictional force during the whole time of motion is (μ = frictional coefficient)–

(A) μmgV

(B) $\frac{1}{2}$ μmgV

(C) μ mg $\frac{V}{4}$

(D) $\frac{3}{2}$ μmgV

Q.16 A 50 kg girl is swinging on a swing from rest. Then the power delivered when she was moving with a velocity of 2 m/s upwards in a direction making an angle 60º with the vertical is –

(A) 980 W (B) 490 W

(C) 490W (D) 245 W

Q.17 A bus of mass 1000 kg has an engine which produces a constant power of 50 kW. If the resistance to motion, assumed constant, is 1000 N. The maximum speed at which the bus can travel on level road and the acceleration when it is travelling at 25 m/s , will respectively be-

(A) 50 m/s , 1.0 m/s^2 (B) 1.0 m/s , 50 m/s^2
(C) 5.0 m/s, 10 m/s^2 (D) 10 m/s , 5 m/s^2

Q.18 It is found that the force required to row a boat in a river is proportional to the speed of the boat. When the speed of the boat is kept v km / hr, the power expended by the boat engine is 24 horse power. What shall be the power required, if one wishes to row the boat at a speed 2v km/hr –

(A) 48 hp (B) 96 hp
(C) 144 hp (D) 192 hp

Q.19 Power applied to a particle varies with time as $P = [3t^2 – 2t + 1]$ watts. Where t is time in seconds. Then the change in kinetic energy of particle between time t = 2s to t = 4s is –

(A) 46 J (B) 52 J
(C) 92 J (D) 104 J

EXERCISE # 2

Part- A: Only single correct answer type questions

Q.1 A 60 cm diameter hand wheel is rotated by exerting a force of 30 newton at the outer rim. If the wheel is turned through 1/2 revolution, then the work done is -

(A) zero (B) 18.0 joule

(C) 28.3 joule (D) 56.5 joule

Q.2 A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in time t will be –

(A) zero

(B) mgvt $\cos^2\theta$

(C) mgvt $\sin^2\theta$

(D) mgvt sin 2θ

Q.3 An object of mass m = 0.5 kg when attached to a vertical spring and lowered very slowly to its equilibrium position stretches the spring by 10cm. If the same object is attached to the same vertical spring but permitted to fall instead, then the maximum speed of the mass will be (take g = 10 m/s^2)

(A) 1m/s

(B) $\sqrt{2}$ m/s

(C) $\frac{1}{\sqrt{2}}$ m/s

(D) $\frac{1}{2}$ m/s

Q.4 A ball loses 15% of its kinetic energy when it bounces back from a concrete floor. When it is thrown vertically downward with a speed v_0 from a certain height h, it bounces back to the same height .Then, if we neglect air resistance, the speed v_0 should be –

(A) zero (B) $\sqrt{3gh/17}$

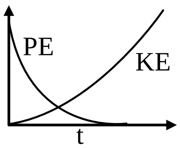
(C) $\sqrt{6gh/17}$ (D) None of these

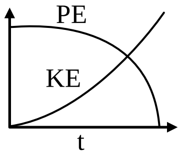
Q.5 A particle moves from a point $\vec{r}_1 = (2m)\hat{i} + (3m)\hat{j}$ to another point $\vec{r}_2 = (3m)\hat{i} + (2m)\hat{j}$ during which a certain force $\vec{F} = (5N)\hat{i} + (5N)\hat{j}$ acts on it. The work done by the force on the particle during the displacement is-

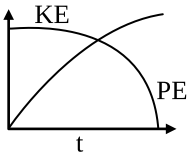
(A) 10 J (B) zero

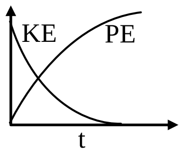
(C) 5 J (D) none

Q.6 A particle falls from rest under gravity. Its potential energy with respect to ground (PE) and its kinetic energy (KE) are plotted against time (t). Choose correct graph-

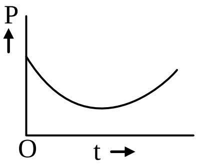
(A) 

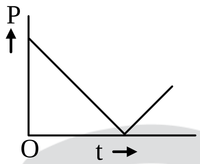
(B) 

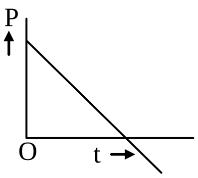
(C) 

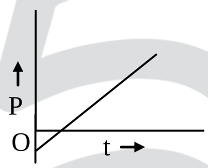
(D) 

Q.7 A stone is projected at time t = 0, with a speed V_0 and at angle θ with the horizontal in uniform gravitational field. The rate of work done (P) by the gravitational force plotted against time (t) will be as -

(A) 

(B) 

(C) 

(D) 

Q.8 A constant power P is applied to a car starting from rest. If a is the acceleration of the car at time t, then-

(A) $a \propto t$ (B) $a \propto \sqrt{t}$

(C) $a \propto \frac{1}{t}$ (D) $a \propto \frac{1}{\sqrt{t}}$

Q.9 A constant power P is applied to a car starting from rest .then if in time t the car travels a distance x, its kinetic energy will be proportional to -

(A) $x^{1/3}$ (B) $x^{2/3}$ (C) x (D) $x^{3/2}$

Q.10 A mass of 1 kg is acted upon by a single force $\vec{F} = (4\hat{i} + 4\hat{j})$ N. Due to force, mass is displaced from (0, 0) to (1m, 1m). If initially the speed of the particle was 2 m/s, its final speed should approximately be–

(A) 6 m/s (B) 4.5 m/s

(C) 8 m/s (D) 7.2 m/s

Q.11 A block of mass M is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F. If K.E. of the block increases by 20 J in 1 s, then–

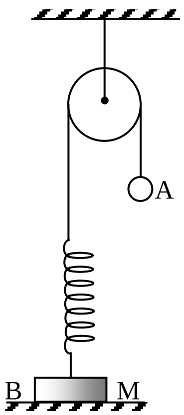
(A) tension in string is Mg

(B) tension in the string is F

(C) work done by the tension on the block is 20 J in 1 s

(D) work done by the force of gravity is 20 J in the above 1 s

Q.12 In the figure, the ball A is released from rest when the spring is at its natural (unstretched) length. For the block B, of mass M to leave contact with the ground at some stage, the minimum mass of A must be –



(A) 2M
(B) M
(C) M/2
(D) a function of M and the force constant of the spring

Q.13 A block hangs freely from the end of a spring. A boy then slowly pushes the block upwards so that the spring becomes strain free. The gain in gravitational potential energy of the block during this process is equal to–
(A) work done by the boy against the gravitational force acting on the block
(B) loss of energy stored in the spring minus the work done by the tension in the spring
(C) work done on the block by the boy plus the loss of energy stored in the spring
(D) work done on the block by the boy minus the work done by the tension in the spring

Q.14 If a man increases his speed by 2 m/sec, his K.E. is doubled. The original speed of the man is -
(A) $(2+\sqrt{2})$ m/s (B) $(2 + 2\sqrt{2})$ m/s
(C) 4 m/s (D) $(1 + \sqrt{2})$ m/s

Q.15 A bus and a car, moving with the same speed are brought to rest by applying the same retarding force then
(A) bus will come to rest in a shorter distance
(B) car will come to rest in a shorter distance
(C) both will come to rest in the same distance
(D) none of the above

Q.16 A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is-
(A) MgL (B) MgL / 3
(C) MgL/9 (D) MgL/18

Q.17 An elastic string of unstretched length L and force constant K is stretched by a small length x . it is further stretched by another small length y . The work done in the second stretching is –

(A) $\frac{1}{2}$ ky^2

(B) $\frac{1}{2}$ $k(x^2 + y^2)$

(C) $\frac{1}{2}$ $k(x + y)^2$

(D) $\frac{1}{2}$ $ky(2x + y)$

Q.18 A force $\vec{F} = -k(y\hat{i} + x\hat{j})$ (where k is a positive constant) acts on a particle moving in the xy-plane .Starting from the origin, the particle is taken along the positive x-axis to the point (a,0), and then parallel to the

y - axis to the point (a,a) .The total work done by the force $\vec{F}$ on the particle is–

(A) $-2ka^2$ (B) $2ka^2$

(C) $-ka^2$ (D) ka^2

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	A	D	B	C	C	C	B	C	B	C	C	B	A	D	B	B	A	B	A

EXERCISE # 2

PART-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	C	C	A	C	B	B	D	D	B	B	B	C	C	B	B	D	D	C