PLAYING WITH NUMBERS

Concepts Covered

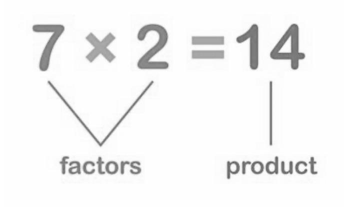
- Factors and multiples, Different types of numbers, Divisibility rules, Prime factorization, Highest Common Factor, Lowest Common Multiple.

Introduction

The study of mathematics involves numbers and various operations performed on them. While discussing operations such as multiplication and division, we often come across terms, such as, factors and multiples. These two terms are directly connected to one another. It is important to know how they are same as different from each other.

Factors and Multiples

When two or more natural numbers are multiplied, the product obtained is called the multiple of either of the numbers. Each of these numbers is called a factor of the product.



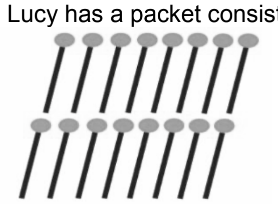
Example:

For instance, $2 \times 3 = 6$. Here, 6 is a multiple of 2 and 3, 2 and 3 are two factors of 6.

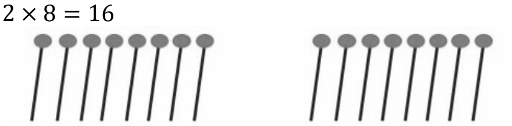
Multiplication Chart

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

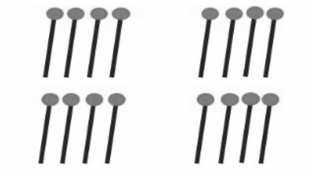
Let us further understand this concept with the help of an example .
Lucy has a packet consisting of 16 candies, which she wishes to distribute among her 2 friends.



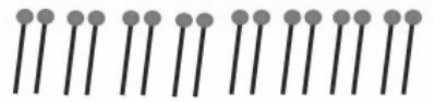
Now it would only be fair if each one gets the same number of candies.
$16 \div 2 = 8$
$2 \times 8 = 16$



Therefore, each friend will get 8 candies.
Now, 2 more friends joined the group.
Lucy has to distribute these candies equally among her 4 friends.
$16 \div (2+2) \Rightarrow 16 \div 4 = 4$
$4 * 4 = 16$
What happens if 4 more friends come?



$16 \div (4+4) \Rightarrow 16 \div 8 = 2$



$8 \times 2 = 16$
So, we see that 16 can be written as a product of two numbers in different ways.
$16 = 2 \times 8\,; 16 = 4 \times 4\,; 16 = 8 \times 2$
2 divides 16 completely $\Rightarrow 2$ is a factor of 16.
4 divides 16 completely $\Rightarrow 4$ is a factor of 16.
8 divides 16 completely $\Rightarrow 8$ is a factor of 16 and so on.
So, we see that $2,4,$and 8 are exact divisors of 16.
They are called the factors of 16.

Example:

Find the factors of 12 and 14.
Solution: 12 = 1 × 12 = 2 × 6 = 3 × 4
Therefore, the factors of 12 are 1, 2, 3, 4, 6, and 12.
14 = 1 × 14 = 2 × 7
Therefore, the factors of 14 are 1, 2, 7, and 14.

Example:

Find the first five multiples of 5.
Solution: 5 × 1 = 5; 5 × 2 = 10; 5 × 3 = 15; 5 × 4 = 20; 5 × 5 = 25.
5, 10, 15, 20 and 25 are the first five multiples of 5.

Example:

Find the factors of 36.
Solution: 36 = 1 × 36 ; 36 = 2 × 18 ; 36 = 3 × 12 ; 36 = 4 × 9 ; 36 = 6 × 6
Stop here, because both the factors (6) are same.
Thus, the factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Extended Learning

Sieve of Eratosthenes

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Steps :

1) Begin with 2 which is a prime number.
   Keep it but cross out all its multiples.
2) Next, the number 3 is prime.
   Thus, we keep it but cross out all its multiples.
   Some of these numbers have already been crossed out.
3) The next number not crossed out is 5.
   It is also prime.
   So, keep it and cross out all its multiples.
4) Continue this process keeping only the primes and striking off their multiples until we cannot strike off any more numbers.

Thus the prime numbers from 1 to 100 are :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Eratosthenes, probably, made holes in the paper instead of crossing out the numbers.

Observations. Some observations about prime numbers are :

(i) 2 is the smallest prime number.

(ii) All prime numbers (except 2) are odd numbers.

(iii) The number of primes is unlimited.

(iv) Both the numbers 13 and 31 have the same digits and are prime.
Other such numbers between 1 and 100 are : 17, 71, 37, 73, 79, and 97.

(v) Every odd prime number can be expressed as a product of primes plus 1.

For example,

$3 = 2 + 1$; $5 = 2 \times 2 + 1$

$7 = 2 \times 3 + 1$; $43 = 2 \times 3 \times 7 + 1$

To find prime numbers between 100 and 400

We know that $20 \times 20 = 400$

So we adopt the following rule :

The given number will be prime if it is not divisible by any prime number less than 20.

Types of Numbers

(a) Even Number: A number which is exactly divisible by 2 is called an even number.
Example of even numbers are : 2,,4, 6, 8,………

(b) Odd Number: A number which is not exactly divisible by 2 is called an odd number.
Example of odd numbers are : 1, 13, 15, 25, 29, ...

(c) Prime Numbers: A natural number greater than 1, which has no factors except 1 and itself is called a prime number.
Examples of prime numbers are : 2, 3, 5, 11, 13, 17, ...

Note :
Every even number greater than 4 can be expressed as a sum of two odd prime numbers,
For example, 6 = 3 + 3; 18 = 5 + 13; 44 = 13 + 31.

(d) Composite Numbers : A number is composite if it has at least one factor other than 1 and itself.
Example of composite numbers are 4, 6, 8, 9, 10, 12, 14,........

Note :
1. 1 is neither prime nor composite.
2. Every natural number except 1 is, either a prime number or a composite number.
3. 2 is the only prime number which is even. All other prime numbers are odd.

(e) Twin Primes: Pairs of prime numbers that have a difference of 2 are called twin primes.
Example of twin primes are (3, 5), (5, 7), (11, 13), (17, 19),........

(f) Perfect Numbers : If the sum of all the factors of a number is twice the number, then the number is called a perfect number.
For example, 6 is a perfect number since the factors of 6 are 1, 2, 3, 6 and their sum
$1 + 2 + 3 + 6 = 2 \times 6$.

(g) Coprime Numbers: Two numbers are said to be coprime if they do not have a common factor other than 1.
Examples of coprime numbers are : (8, 15); (5, 9); (2, 11)

Note :
1. Two prime numbers are always coprime.
2. Two coprime numbers need not be both prime numbers.

(h) Prime Triplet : A set of successive prime numbers differing by 2 is called a prime triplet.
The only example of a prime triplet is (3, 5, 7).

Extended Learning

A Sphenic Number is a positive integer n which is product of exactly three distinct primes. The first few sphenic numbers are 30, 42, 66, 70, 78, 102, 105, 110, 114, …

Check Your Concept - 1

(i) Find the possible factors of 16, 19 and 36.

(ii) How many numbers are there which are both prime and even? Note down the list of such numbers.

(iii) What is the difference between two consecutive odd numbers? Also is it same as the difference between two consecutive even numbers?

(iv) Maximum numbers of factors a prime number can have?

(v) Why 1 is neither prime nor composite? Though 1 is only divisible by itself and also it has a factor which is 1 it-self.

Greatest Common Divisor or Highest Common Factor

The highest common factor (HCF) of two or more natural numbers is the largest factor in the set of common factors of those numbers. In other words, the HCF or the greatest common divisor (GCD) of two or more numbers is the largest number that divides each of them exactly.

Prime Factorization Method

When the numbers whose GCD has to be found are relatively small, this is the best method.
Here, we solve the given numbers into their prime factors and find out the product of common factors of given numbers.
This method can be easily applied to all numbers.

DID you KNOW

Knowledge of prime factors is applied in Cryptography (study of solving codes).

Finding HCF by Prime Factorisation
Step 1
Find the prime factors of the given numbers.
Step 2
Find the common factors and circle them.
STEP 3.
Multiply the common factors to get HCF.

Example:

Write all the prime factors of the following numbers:
(i) 18 **(ii) 124**
Solution:

(I)

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad 18 = 2 \times 3 \times 3$$

∴ The prime factors of 18 are 2 and 3 **.**

(II)

$$\begin{array}{r|l} 2 & 124 \\ \hline 2 & 62 \\ \hline & 31 \end{array} \qquad 124 = 2 \times 2 \times 31$$

∴ The prime factors of 124 are 2 and 31 .

Example:

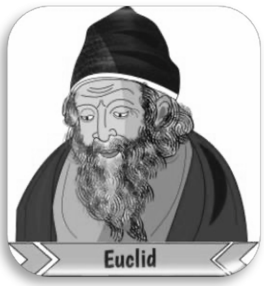
Find the HCF of 12 and 56 by the prime factorization method.
Solution:

$12 = 2 \times 6$ $\quad$ $56 = 2 \times 28$
$= 2 \times 2 \times 3$ $\quad$ $56 = 2 \times 2 \times 14$
$= 2^2 \times 3^1;$ $\quad$ $56 = 2^3 \times 7^1$
$\therefore \mathrm{HCF} = 2^2 = 4$ (Product of all common prime factors with their least exponents)

Division Method:

When the numbers whose GCD has to be found are very large,
it is time consuming to use the prime factorization method.
In this case, we use the method of ‘Long Division’.

DID you KNOW



Division method was proposed by Euclid (a Greek mathematician).

Step 1: Divide the larger number by the smaller number.
If the remainder is zero, then the divisor is the GCD, otherwise move to Step 2.
Step 2: Let the divisor in step 2 be the dividend now, and the remainder of step 1 becomes the divisor of step 2.
If the remainder is zero, then the divisor is HCF. Otherwise, step 2 has to be repeated.

Example:

Find the H.C.F. of 345 and 506.
Solution:

```
345) 506
    -345
161) 345 (2
    -322
23) 161 (7
   -161
      0
```

The last divisor is 23.
∴ H.C.F. of 345 and 506 is 23.
To find the H.C.F. of three numbers, first we find the H.C.F. of any two numbers.
Then treating this H.C.F. as one number and third number as another number,
we find their H.C.F. by the method stated above.
The H.C.F. so found will be the H.C.F. of the three numbers.

Example:

Find the H.C.F. of 219, 2628, 2190 and 8833.
Solution:
First we find the H.C.F. of 219 and 2628.

```
219) 2628 (12
    -219
     438
    -438
       0
```

Now we find the H.C.F. of 219 and 2190.

```
219) 2190 (10
    -219
       0
```

∴ H.C.F. of 219, 2628 and 2190 = 219.
Now we find the H.C.F. of 219 and 8833.

```
219) 8833 (40
    -876
73) 219(3
   -219
      0
```

Hence the H.C.F. of 219, 2628, 2190 and 8833 is 73.

Example:

Find the largest number that will divide 20, 57, and 85 leaving remainders 2, 3, and 4 respectively.
Solution:
Clearly, the required number is the HCF of the number 20 – 2 = 18, 57 – 3 = 54 and 85 – 4 = 81.
18 = 2 × 3 × 3
54 = 2 × 3 × 3 × 3
81 = 3 × 3 × 3 × 3
∴ Required HCF = 3 × 3 = 9
Hence, the required number = 9.

Example:

A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose?
Solution: Largest size of the tile = H.C.F. of 378 cm and 525 cm = 21 cm.
Common Multiple
Number which are exactly divisible by two or more numbers are called their common multiples.
Let us find the common multiple of 12 and 18.
Multiples of 12 are 12, 24, 36, 48, 60, 72 .....
Multiples of 18 are 18, 36, 54, 72 .....
The common multiple of 12 and 18 are 36, 72 ....

Example:

The length, breadth and height of a room are 8m 25cm, 6m 75cm and 4m 50cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.
Solution: We have, 8m 25cm = 825cm, 6m 75cm = 675cm 4m 50cm = 450cm
Length of the longest rod in cm is the HCF of 825, 675 and 450.
$825 = 3 \times 5 \times 5 \times 11$
$675 = 3 \times 3 \times 3 \times 5 \times 5$
$450 = 2 \times 3 \times 3 \times 5 \times 5$
$\therefore$ HCF 825, 675 and 450 = $3 \times 5 \times 5 = 75$
Hence, the required length of rod = 75.

Least Common Multiple (L.C.M.)

To find the LCM of two or more numbers, we adopt the following steps :
(i) Find the multiples of the given numbers.
(ii) Select their common multiples.
(iii) Take the smallest of the above common multiples.
Finding LCM by Prime Factorisation
Step 1.
Find the prime factors of the given numbers.
Step 2.
LCM is the product of all the prime factors with greatest powers.

Example:

Find the LCM of 12 and 15 by the prime factorization method.
Solution:
$12 = 4 \times 3 = 2^2 \times 3^1$
$15 = 3 \times 5 = 3^1 \times 5^1$
$\mathrm{LCM}(12,15) = 2^2 \times 3^1 \times 5^1 = 60$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 3 & 15 \\ \hline & 5 \end{array}$$

Example:

Find the LCM of 24 and 40.
Solution: $\therefore$ The LCM of 24 and 40
$= 2 \times 2 \times 2 \times 3 \times 5$
$= 120$

$$\begin{array}{r|l} 2 & 24, 40 \\ \hline 2 & 12, 20 \\ \hline 2 & 6, 10 \\ \hline & 3, 5 \end{array}$$

Finding LCM by Common Division
Step 1.
Write the given numbers in a row separated by commas.
Step 2.
Divide these numbers by the least prime numbers which divides at least one of the given numbers.
Step 3.
Write the quotients and the numbers that are not divisible by the prime numbers in the second row. Then repeat Steps 2 and 3 with the rows and continue till the numbers in a row are all 1.
Step 4.
The LCM is found out by multiplying all the prime divisors and the quotients other than 1.

Example:

Find the L.C.M. of $28, 36, 45$ and 60.
Solution:

2	28, 36, 45, 60
2	14, 18, 45, 30
3	7, 9, 45, 15
3	7, 3, 15, 5
5	7, 1, 5, 5
	7, 1, 1, 1

Relationship between LCM and HCF
The LCM and HCF of two given numbers are related to the given numbers by the following relationship.

The product of two numbers = The product of their LCM and HCF

Where, LCM denotes the LCM of the given numbers and HCF denotes the HCF of the given numbers.

Example:

Let us take two numbers, say 16 and 24.
Solution:
The HCF of 16 and 24 is 8.
The LCM of 16 and 24 is 48.
Since 8 is factor of 48, so we can say that HCF of the numbers is a factor of their LCM.
Product of HCF and LCM = 8 × 48 = 384
Product of Numbers = 16 × 24 = 384
So we can say that the product of two numbers is equal to the product of their HCF and LCM.
Let a and b be the two numbers, then a × b = HCF × LCM

Example:

LCM of two numbers is 378 and their HCF is 9. If one of the numbers is 63, then find the other number.
Solution: We know that $\mathrm{LCM} \times \mathrm{HCF} =$ Product of two numbers
Let the products of two numbers be N_2
$378 \times 9 = 63 \times \mathrm{N}_2$
$378 = 7\mathrm{N}_2$
$\mathrm{N}_2 = \dfrac{378}{7} = 54$
∴ The other number is 54.

Example:

The HCF of two numbers is 29 and their LCM is 1160. If one of the numbers is 145, find the other.
Solution:
We know that
Product of the number = Product of their HCF and LCM
Required No. $\times 145 = 29 \times 1160$
$\text{Required No.} = \dfrac{29 \times 1160}{145} = 232$

Properties of HCF and LCM

1. The HCF of 6 and 10 is 2. So 2 is a factor of both 6 and 10. Also, 2 is the smallest amongst 2, 6 and 10.
2. The LCM of 6 and 10 is 30. 30 is a multiple of both 6 and 10. Also, 30 > 10 and 6, i.e., it is the greatest amongst 6, 10 and 30.
3. Consider two numbers, 35 and 39.
Now, 35 = 1 × 5 × 7
39 = 1 × 3 × 13
Common factor = 1
□35 and 39 are co-prime numbers.
HCF of 35 and 39 = 1
Thus, HCF of two or more co-prime numbers is 1.
4. Again consider 35 and 39 which are co-prime.
LCM of 35 and 39 = 3 × 5 × 7 × 13 = 35 × 39

Thus, the LCM of co-prime numbers = the product of the co-primes.
5. HCF of 6, 10 = 2
LCM of 6, 10 = 30
Also, 30 = 2 × 15 = 2 × 3 × 5
i.e., 2 is a factor of LCM.
Thus, HCF is a factor of LCM. In other words, LCM is a multiple of HCF.

6. 2 and 3 are prime numbers.
HCF of 2 and 3 is 1.
HCF of two or more prime numbers is 1.

Divisibility Test

Consider two natural numbers a and b. When a is divided by b, if a remainder of zero is obtained, we say that a is divisible by b.
For Example :
12 is divisible by 3 because when 12 is divided by 3, the remainder is zero. Also, we say that 12 is not divisible by 5, because 12 when divided by 5 leaves a remainder of 2.

Tests of Divisibility
Test of divisibility lets us know whether a number is divisible by another or not without doing the calculations. We now study the methods to test the divisibility of natural numbers with 2, 3, 4, 5, 6, 8, 9 and 11 without performing actual division.

Test of Divisibility by 2
A natural number is divisible by 2, if its units digit is divisible by 2, i.e., the units place is either 0, 2, 4, 6 or 8.
For Example:
The numbers 4096, 23,548 and 34,052 are divisible by 2 as they end with 6, 8 and 2, respectively.

Test of Divisibility by 3
A natural number is divisible by 3, if the sum of its digits is divisible by 3.
For Example:
Consider the number 21,43,251. The sum of the digits of 21,43,251 is (2 + 1 + 4 + 3+ 2 + 5 + 1), i.e., 18. As 18 is divisible by 3, the number 21,43,251 is divisible by 3.

Test of Divisibility by 4
A natural number is divisible by 4, if the number formed by its last two-digits in the same order (ten's digit and unit's digit) is divisible by 4.
For Example:
4096, 52, 216, 548 and 4000 are all divisible by 4 as the numbers formed by taking the last two-digits in each case is divisible by 4.

Test of Divisibility by 5
A natural number is divisible by 5, if its unit's digit is either 0 or 5.
For Example:
The numbers 4095 and 235060 are divisible by 5 as they have 5 and 0 in their respective units' places.

Test of Divisibility by 6
A number is divisible by 6, if it is divisible by both 2 and 3.
For Example :
Consider the number 7,53,618. Since its units digit is 8, it is divisible by 2. Also its sum of digits is 7 + 5 + 3 + 6 + 1 + 8 = 30. As 30 is divisible by 3,7,53,618 is divisible by 3. Hence, 7,53,618 is divisible by 6.

Test of Divisibility by 8
A number is divisible by 8, if the number formed by its last three-digits in the same order (hundreds, tens and units digits) is divisible by 8.

For Example:
15,840, 5432 and 7096 are divisible by 8 as the numbers formed by the last three-digits in each case are divisible by 8.
Test of Divisibility by 9
A natural number is divisible by 9, if the sum of its digits is divisible by 9.
For Example :
Consider the number 1,25,847. The sum of digits = 1 + 2 + 5 + 8 + 4 + 7 = 27.
As 27 is divisible by 9, the number 1,25,847 is divisible by 9.

Test of Divisibility by 11
A number is divisible by 11, if the difference between the sum of the digits in odd places and the sum of the digits in even places of the number is either 0 or a multiple of 11.
For Example :
Consider the number 95,82,540. Now, (the sum of digits in odd places) - (the sum of digits in even places) = (9 + 8 + 5 + 0) - (5 + 2 + 4) = 11, which is divisible by 11.
Hence, 95,82,540 is divisible by 11.

Extended Learning

Some More Divisibility Rules :

Let us observe a few more rules about the divisibility of numbers.

(i) One of the factor of 18 is 9. A factor of 9 is 3. Is 3 a factor of 18? Yes, it is. Take any other factor of 18, say 6. Now, 2 is a factor of 6 and it also divides 18. Check this for the other factors of 18.

Consider 24. It is divisible by 8 and the factors of 8 i.e., 1, 2, 4 and 8 also divide 24.

So, we may say that if a number is divisible by another number then it is divisible by each of the factors of that number.

(ii) The number 80 is divisible by 4 and 5. It is also divisible by 4 × 5 = 20, and 4 and 5 are co-primes.

Similarly, 60 is divisible by 3 and 5 which are co-primes. 60 is also divisible by 3 × 5 = 15,

If a number is divisible by two co-prime numbers, then it is divisible by their product also.

(iii) The numbers 16 and 20 are both divisible by 4. The number 16 + 20 = 36 is also divisible by 4.

If two given numbers are divisible by a number, then their sum is also divisible by that number.

(iv) The number 35 and 20 are both divisible by 5. Their difference 35 – 20 = 15 also divisible by 5.

If two given number are divisible by a number, then their difference is also divisible by that number.

Example:

Find the least number which when decreased by 7 is exactly divisible by $12, 16, 18, 21$ and 28 .
Solution:

2	12,	16,	18,	21,	28,
7	6,	8,	9,	21,	14,
3	6,	8,	9,	3,	2,
2	2,	8,	3,	1,	2,
	1,	4,	3,	1,	1,

LCM $= 2 \times 7 \times 3 \times 2 \times 4 \times 3 = 1008$
Required number $= 1008 + 7 = 1015$

Example:

When 21 is added to a number, it is divided exactly by $3, 8, 9, 12, 16$ and 18. How many such numbers exist? Find the least of them.
Solution:

We know that the least number divisible by $3, 8, 9, 12, 16$ and 18 is their LCM.
Therefore, the required number must be 21 less then their LCM

2	3,	8,	9,	16,	18,
2	3,	4,	9,	8,	9,
2	3,	2,	9,	4,	9,
3	3,	1,	9,	2,	9,
3	1,	1,	3,	2,	3,
	1,	1,	1,	2,	1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 2 = 144$
Hence, the required number $= (144 - 21) = 123$
There exists many such numbers (i.e., all the multiples of 123) and least of them is 123.

Example:

During a morning walk, four boys start from the same point together. Their steps measure $70\ \text{cm}, 65\ \text{cm}, 75\ \text{cm}$ and $80\ \text{cm}$, respectively. At what distance from the starting point will they meet again?

Solution: The distance covered by each one of them is required to be same and minimum both. The required minimum distance each should walk must be the LCM of their steps in cm.

2	70, 65, 75, 80,
5	35, 65, 75, 40,
	7, 13, 15, 8

$\therefore$ $\text{LCM} = 2 \times 5 \times 7 \times 13 \times 15 \times 8 = 109200$

$\therefore$ They will step off together again after a distance of $109200\ \text{cm} = 1092\ \text{m}$.

Check Your Concept - 2

(i) Comment on the LCM and HCF of two prime numbers.

(ii) Find the LCM and HCF of following numbers.

(a) 15,20 (b) 18, 24 (c) 7,49

(iii) Find the prime numbers which can divide the following numbers.

(a) 90 (b) 70 (c) 126

Solved Examples

(1) Without actual division, show that 11 is a factor of 1,111.

Solution : The sum of the digits at the odd places = 1 + 1 = 2
The sum of the digits at the even places = 1 + 1 = 2
The difference of the two sums = 2 – 2 = 0
Therefore, 1,111 is divisible by 11 because the difference of the sums is zero.

(2) Is there any natural number having no factor at all?

Solution : No, because we get 1 when we divide a number by the number itself.
Hence, every natural number is a factor of itself.

(3) Without actual division show that 11 is a factor of 11, 00,011

Solution : The sum of the digits at the odd places = 1 + 0 + 0 + 1 = 2
The sum of the digits at the even places = 1 + 0 + 1 = 2
The difference of the two sums = 2 – 2 = 0
Therefore, 11, 00,011 is divisible by 11 because the difference of the sums is zero.

(4) If number is divisible by 24 then, by what other numbers will that number be divisible?

Solution : Since the number is divisible by 24, it will be divisible by all the factors of 24.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
Hence, the number is also divisible by 1, 2, 3, 4, 6, 8 and 12.

(5) Test the divisibility of 7020 by 6.

Solution : Rule: A number is divisible by 6 if it is divisible by 2 as well as 3.
Here, the units digit = 0. Thus, the given number is divisible by 2.
Also, the sum of the digits = 7 + 0 + 2 + 0 = 9, which is divisible by 3.
So, the given number is divisible by 3. Hence, 7,020 is divisible by 6.

(6) 5*2 is a three-digit number with * as a missing digit. If the number is divisible by 6, the missing digit is.

Solution : Since we know that if a number is divisible by 6 then definitely, it will definitely be divisible by 2 and 3.
Using the divisibility rule of 2, we know
that the sum of the given digits = 5 + 2 = 7
Multiple of 3 greater than 7 is 9. So, we get 9 − 7 = 2
Hence, the required digit is 2.

(7) The HCF of two numbers is 28 and their LCM is 336. If one number is 112, then the other number is

Solution : We have, one number =112.

Let other number be x. HCF = 28, LCM = 336

Now, product of two numbers = HCF × LCM

⇒ 112 × x = 28 × 336 ⇒ x = (28×336)/112 ⇒ x = 84.

(8) What is the smallest number which when divided by 24, 36 and 54 gives a remainder of 5 each time?

Solution : We have to find prime factorization of 24, 36, and 54.

Prime factorization of 24 = 2 × 2 × 2 × 3

Prime factorization of 36 = 2 × 2 × 3 × 3

Prime factorization of 54 = 2 × 3 × 3 × 3

Therefore, Required LCM = 2 × 2 × 2 × 3 × 3 × 3 = 216

Thus, 216 is the smallest number exactly divisible by 24, 36, and 54.

To get the remainder as 5 : Smallest number = 216 + 5 = 221

(9) There are 153 apples and 119 oranges. These fruits are to be arranged in heaps containing the same number of fruits. What is the greatest number of fruits possible in each heap is?

Solution : The greatest number of fruits in each heap is the HCF of 153 and 119.

```
119 ) 153 ( 1
     −119
      34 ) 119 ( 3
          −102
            17 ) 34 ( 2
                −34
                 0
```

∴ HCF (153, 119) = 17

Hence, the greatest number of fruits possible in each heap is 17.

(10) What least number be assigned to * so that number 653*47 is divisible by 11?

Solution : We know that the sum of the digits at odd places = 6 + 3 + 4 = 13

In the same way the sum of the digits at even places = 5 + * + 7 = 12 + *

Difference between them = 13 − [12 + *] = 1 − *

So, 6, 53,*47 is divisible by 11, then 1 − * must be zero or multiple of 11

It can be written as

1 − * = 0 or 11 and * = 1 or −10

We know that * is a digit, so it must be 1.

(11) Without making any actual division find that 7007 is divisible by 7.

Solution : 7007

= 7007 + 7

= 7$\times$ $(1000 + 1)$

= 7×1001

Clearly, 7007 is divisible by 7.

(12) If a number is divisible by both 7 and 16 then by which other number will that number be always divisible?

Solution : Since the number is divisible by 7 and 16, they are the factors of that number.

So, the number will be divisible by the common factor of 7 and 16.

The factors of 7 are 1 and 7.

The factors of 16 are 1, 2, 4, 8, and 16.

(13) Two tankers contain 850 litres and 680 litres of kerosene oil respectively. What is the maximum capacity of a container which can measure the kerosene out of both the tankers when used an exact number of times?

Solution : Capacities of two tankers are 850 L and 680 L.

Maximum capacity of a container is the HCF of 850 and 680.

680) 850 (7
−680

170) 680 (4
−680

0

∴ HCF=170; So, maximum capacity of container = 170 L.

(14) Three boys steps off together from the same spot. Their steps measure 63 cm, 70 cm, and 77 cm, respectively.
What is the minimum distance each should cover so that all can cover same distance in complete steps?

Solution : The minimum distance all boys should cover is the LCM of 63 cm, 70 cm and 77 cm.

2	63, 70, 77
3	63, 35, 77
3	21, 35, 77
5	7, 35, 77
7	7, 7, 77
11	1, 1, 11
	1, 1, 1

∴ LCM = 2×3×3×5×7×11 = 6930

So, 6930 cm is the minimum distance each should cover so that all can cover distance in complete steps.

(15) Find two numbers nearest to 10000 which are exactly divisible by each of 2, 3, 4, 5, 6 and 7.

Solution : The numbers which are exactly divisible by 2, 3, 4, 5, 6 and 7 are the multiples of the LCM of the given numbers.

∴ LCM = 2 × 2 × 3 × 5 × 7 = 420

Now, dividing 10000 by 420, we get remainder = 340

∴ Number just less than 10000 and exactly divisible by the given numbers

= 10000 – 340 = 9660

Number just greater than 10000 and exactly divisible by the given numbers

= 10000 + (420 – 340) = 10080

(16) Find the value of a + b + c, if 373a is divisible by 9, 473b is divisible by 11 and 371 c is divisible by 6.

Solution : 373 a is divisible by 9. ⇒3+7+3+a=13+a should be divisible by 9.

⇒ a=5, 473b is divisible by 11.

⇒ 4+3−7−b=0 or 11⇒b=0 or 11

∴ b=0, 371c is divisible by 6.

⇒ 371c is divisible by both 2 and 3.

⇒ c is an even number and 3 + 7 + 1 + c is divisible by 3.

⇒ c = 0, 2, 4, 6, 8 and 11 + c is divisible by 3.

⇒ c = 4

∴ a + b + c = 5 + 0 + 4 = 9

(17) The length, breadth and height of a room are 403 cm, 434 cm and 465 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Solution : Dimensions of room are 403 cm, 434 cm and 465 cm.

The longest tape which can measure all dimensions of the room exactly is the HCF of 403, 434 and 465.

$$
\begin{array}{r|l|l}
31 & 465 & 15 \\
 & -31 & \\
\hline
 & 155 & \\
 & -155 & \\
\hline
 & 0 &
\end{array}
\qquad
\begin{array}{r|l|l}
403 & 434 & 1 \\
 & -403 & \\
\hline
31 & 403 & 13 \\
 & -31 & \\
\hline
 & 93 & \\
 & -93 & \\
\hline
 & 0 &
\end{array}
$$

HCF = 31; So, length of longest tape = 31 cm.

Exercise

FILL IN THE BLANKS

(1) The number of factors of 24 is ______.
(2) The greatest 2-digit multiple of 8 is _____.
(3) The greatest common factor of 9 and 15 is _____.
(4) The number of prime factors of 48 is ______.
(5) The greatest pair of twin primes of 2-digit numbers is _____.
(6) The smallest even prime number is ______
(7) The LCM of 12 and 8 is________
(8) The greatest two-digit composite number is____
(9) The number of prime numbers between 1 to 20 is _____
(10) The LCM of two prime number is always ______ of the given prime numbers.

TRUE OR FALSE

(1) Two prime numbers are always co-prime.
(2) 1 is both prime and a composite number.
(3) All prime numbers are odd except 2.
(4) Every natural number is either a prime number or a composite number.
(5) HCF of two or more co-prime numbers is always 1.

OBJECTIVE TYPE QUESTION

(1) Which of the following is a prime number?
(A) 87 (B) 69
(C) 57 (D) 97

(2) Which of the following is not a sphenic number?
(A) 30 (B) 42
(C) 60 (D) 70

(3) Which of the following is a composite number?
(A) 37 (B) 47
(C) 57 (D) 67

(4) The HCF of which of the following pairs of composite numbers is different from that of the others?
(A) 12, 18 (B) 24, 30
(C) 18, 30 (D) 18, 45

(5) Which of the following is the greatest measure that can be used to measure the quantities 68 m, 102 m and 119 m?
(A) 34 (B) 51
(C) 13 (D) 17

(6) The LCM of a and b is 220. Which of the following can be the HCF of a and b?
(A) 33 (B) 15
(C) 20 (D) 12

(7) Which of the following pairs of numbers have their LCM 144?
(A) 18, 36 (B) 16, 96
(C) 24, 48 (D) 9, 16

(8) The HCF of the two numbers a and b is 12. Which of the following can be the LCM of a and b?
(A) 44 (B) 65
(C) 76 (D) 96

(9) Which of the following is divisible by 8, 15 and 6?
(A) 60 (B) 96
(C) 180 (D) 240

(10) Which of the following is not divisible by 9?
(A) 345672 (B) 278901
(C) 46938 (D) 96435

(11) Which of the following is divisible by 8?
(A) 7248882 (B) 836486
(C) 92304 (D) 726436

Answer Key

CHECK YOUR CONCEPT

(1) **(i)** ((1,2,4,8,16), (1,19), (1,2,3,4,6,9,12,18,36)

(ii) Only one which is 2

(iii) 2, Yes

(iv) Two

(v) Prime number has only two distinct factors while 1 has only one factor. While composite number has at least one factors other than 1 and itself.

(2) **(i)** LCM of two prime number is always multiple of the two given numbers and HCF is always 1.

(ii) (a) (60,5) (b) (72,6) (c) (49,7)

(iii) (a) (2,3,5) (b) (2,5,7) (c) (2,3,7)

FILL IN THE BLANKS

(1) 8
(2) 96
(3) 3
(4) 2
(5) (71, 73)
(6) 2
(7) 24
(8) 99
(9) 8
(10) Product

TRUE OR FALSE

(1) True
(2) False
(3) True
(4) False
(5) True

OBJECTIVE TYPE QUESTION

(1) (D)
(2) (C)
(3) (C)
(4) (D)
(5) (D)
(6) (C)
(7) (D)
(8) (D)
(9) (D)
(10) (C)
(11) (C)