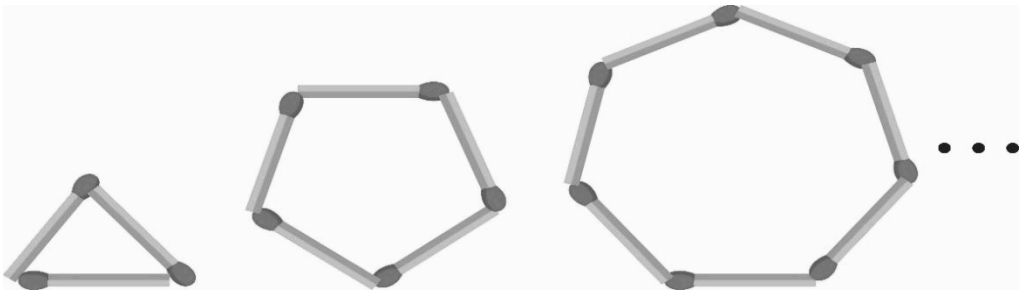

ALGEBRAIC EXPRESSIONS

Concepts Covered

- Introduction (Terms, Coefficient, Like Terms, Unlike Terms)
- Addition and Subtraction of Algebraic Expressions, Multiplication of Algebraic Expression
- Finding the Value of an Expression
- Polynomials and Types of Polynomials
- Factorization of polynomials
- Algebraic Identities
- Applications of Identities

Introduction

An algebraic expression is defined as “The combination of numbers, variables, constants and operators like plus, minus signs". For example,

Variable ← x + 2 → Constant

↓

Operator

Note: It is not necessary that all algebraic expressions must have constants, variables, and operators.

Now, let us discuss the basic terminologies related to algebraic expressions.

(i) Constant: A symbol having a fixed numerical value is called a constant.
Example: 1, 2, 3.14 etc., are examples of constants.

(ii) Variable: A symbol that may be assigned different values is called a variable. Alphabets like x, y, z, a, b, c, r etc., are used to denote variables.

(iii) Operators: Operators include all the operations like addition, subtraction, multiplication, division, equal to, etc.

(iv) Term: The various parts of an algebraic expression connected by + or – sign are called terms of the expression.
For Example: $2x + 4y^2 – 3z$ has three terms, namely, + 2x, + $4y^2$ and -3z.
$2xy – 4y^2$ has two terms namely + 2xy and $–4y^2$.

(v) Coefficients: The numerical part is called the **numerical coefficient** and the literal part, or the variable part is called the **literal coefficient.**
For Example: In 24xy, 24 is the numerical coefficient and xy is the literal coefficient.

(vi) Like terms and Unlike terms: The terms having the same literal factors are called like terms and those having different literal factors are called, unlike terms.
For Example: $–6x^2y$ and $4yx^2$ are like terms whereas 2xy, $– 3x^2$, $4xy^2$ are unlike terms.

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Example:

Write the following statements using variables, constants and arithmetic operations.
(i) One-fourth of the product of ' p ' and ' q '
(ii) 5 added to the three times of ' z'
(iii) x multiplied by 5 and added to ' 10 '

Solution: (i) $\frac{1}{4}$ pq **(ii)** 5+ 3z **(iii)** 5x+10

Example:

Write the following expressions in statements.
(i) $3m + 11$ (ii) $2y - 5$
Solution: (i) 11 added to the three times of m.
(ii) 5 subtracted from the 2 times of y.

Types of Algebraic Expressions

An algebraic expression is called a monomial, a binomial, or a trinomial on the basis of the number of terms that it contains.
1. Monomial: An algebraic expression that contains only one term is called a monomial.
For Example: $2x, 3, 4t, 9pq, 2c^2$ etc., each of these expressions contains only one term and hence is called a monomial.

2. Binomial: An algebraic expression that contains two unlike terms is called a binomial.
For Example:
(i) $6x + 2x^2$
The given expression is a binomial since it contains two unlike terms, i.e., $6x$ and $2x^2$ which are separated by a plus sign.
(ii) $6x - 2x^2$
The given expression is a binomial since it contains two unlike terms, $6x$ and $2x^2$ which are separated by a minus sign.
Note: If the terms, $6x$ and $2x^2$ are separated by a multiplication sign $(6x \times 2x^2)$ then it is not binomial because when we multiply two unlike terms then a monomial is formed.
$6x \times 2x^2 = (6 \times 2)(x \times x^2) = 12x^3$.

3. Trinomial: An algebraic expression that contains three unlike terms is called a trinomial.
For Example: (i) $x + x^2 - 3x^3$ is trinomial since it contains three unlike terms namely, x, x^2 and $-3x^3$.
(ii) $7y^3 + 8y^2 + 9y$ is also a trinomial since it contains three unlike terms namely, $9y, 8y^2$ and $7y^3$.

Note: A polynomial having four or more than four terms does not have a particular name. These are simply called **polynomials.**

Example:

Which of the following multinomials is monomial, binomial or trinomial?
(i) $1 + x + x^2$ (ii) $7mn$ (iii) $a^2 + b^2$
(iv) $3y^2 - x^2y^2 + 4x$ (v) $7x^2 - 2xy + 9y^2 - 11$
Solution: (i) Trinomial **(ii)** Monomial **(iii)** Binomial **(iv)** Trinomial **(v)** Polynomial

Addition and Subtraction of Algebraic Expressions

The sum or difference of several like terms is another like term whose coefficient is the sum or difference of those like terms.
In addition, or subtraction, we can add or subtract only like terms together means those terms having the same algebraic factors. If you have unlike terms, you cannot add or subtract them together. So, you can leave them as it is.

Example:

Add the following: 3xy, 10xy and 5xy.
Solution: The sum of the numerical coefficients of the given like terms is 3 + 10 + 5 = 18.
Hence, 3xy + 10xy + 5xy = 18xy.

Example:

Add $4x^2 - 3x + 1$ to the sum of $3x^2 - 2x$ and $x + 7$
Solution: Sum of $3x^2 - 2x$ and $x + 7$
$= (3x^2 - 2x) + (x + 7)$
$= 3x^2 - 2x + x + 7$
$= 3x^2 - x + 7$
Now, the required expression;
$= 4x^2 - 3x + 1 + 3x^2 - x + 7$
$= 4x^2 + 3x^2 - 3x - x + 1 + 7$
$= 7x^2 - 4x + 8$

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Example:

Add the following: $3x + 4y + 5z$ and $2x - 3y - 4z$.

Solution: Horizontal Method,

$= (3x + 4y + 5z) + (2x - 3y - 4z)$

$= (3x + 2x) + (4y - 3y) + (5z - 4z)$

$= (3 + 2)x + (4 - 3)y + (5 - 4)z = 5x + y + z$

Column Method

$$\begin{array}{r} 3x + 4y + 5z \\ +2x - 3y - 4z \\ \hline 5x + y + z \end{array}$$

Note: To subtract an algebraic expression from another, we change the sign (from '+' to '–' or from '–' to '+') of all the terms of the expression which is to be subtracted, and then the two expressions are added.

Example:

Subtract: $(2x^2 + 5x - 3)$ from $(-6x + 7x^2 - 4)$

Solution: $(-6x + 7x^2 - 4) - (2x^2 + 5x - 3) = -6x + 7x^2 - 4 - 2x^2 - 5x + 3$

First, we collect the like terms together;

$\Rightarrow 7x^2 - 2x^2 - 6x - 5x - 4 + 3$

$\Rightarrow (7 - 2)x^2 - (6 + 5)x - 4 + 3$

$\Rightarrow 5x^2 - 11x - 1$

Alternate method:

We subtract $(-6x + 7x^2 - 4)$ from $(2x^2 + 5x - 3)$ column wise.

To subtract this column-wise, we arrange the terms of the given expression in descending powers of x, and then subtract column-wise.

$$\begin{array}{r} 7x^2 - 6x - 4 \\ 2x^2 + 5x - 3 \\ \hline - \quad - \quad + \\ 5x^2 - 11x - 1 \end{array}$$

Multiplication of Algebraic Expression

The following are the rules of signs and the laws of exponents that are used in multiplication.

Rule 1: (i) Multiply numerical coefficients, (ii) Multiply literal coefficients, (iii) Multiply results.

For Example:

(i) $3x \times 5y = (3 \times 5) \times (x \times y) = 15 \times xy = 15xy$.

(ii) $2a \times 3b \times 5c = (2 \times 3 \times 5) \times (a \times b \times c) = 30abc$.

Rule 2: Signs in multiplication:

The product of two factors with like signs is positive, and the product of two factors with unlike signs is negative. Thus if a and b are two positive numbers then,

(i) $(+a) \times (+b) = +(ab)$; e.g. $2 \times 3 = 6$

(ii) $(+a) \times (-b) = -(ab)$; e.g. $\frac{2}{3} \times \frac{-5}{7} = \frac{-10}{21}$

(iii) $(-a) \times (+b) = -(ab)$; e.g. $\frac{-2}{3} \times \frac{5}{7} = \frac{-10}{21}$

(iv) $(-a) \times (-b) = +(ab)$; e.g. $\frac{-2}{3} \times \frac{-5}{7} = \frac{10}{21}$

Rule 3: Laws of exponents in multiplication:

To multiply the powers of the same base, keep the base and add the exponents.

Thus, $a.a^2 = a^{1+2} = a^3$, $\quad x^3.x^5.x = x^{(3+5+1)} = x^9$.

$a^3b^4.a^2b^7 = a^5b^{11}$, $\quad 3^2.3^3.x.x^4.x^5.y^2.y = 3^5\,x^{10}\,y^3$.

Example:

(i) $4a^2b \times -6a^3b^2c$, **(ii) $-\frac{6}{8}x^4yz \times 24x^2y^2z^3$.**

Solution: (i) $4a^2b \times -6a^3b^2c = (4 \times -6) \times (a^2b \times a^3b^2c) = -24a^5b^3c$.

(ii) $-\frac{6}{8}x^4yz \times 24x^2y^2z^3 = -\frac{6}{8} \times 24 \times (x^4yz \times x^2y^2z^3) = -18x^6y^3z^4$.

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Multiplication of a Monomial by a Monomial

Example:

Multiply:

(i) $\mathbf{3x^3y^2 \times 4x^2y}$ **(ii)** $\mathbf{\frac{4}{5}ab \times \frac{-3}{2}a^2bc}$ **(iii)** $\mathbf{\frac{-2}{3}x \times \frac{3}{4}y \times -\frac{4}{5}z}$

Solution: (i) $3x^3y^2 \times 4x^2y = (3 \times 4)x^{3+2}y^{2+1} = 12x^5y^3$

(ii) $\frac{4}{5}ab \times \frac{-3}{2}a^2bc = \left(\frac{4}{5} \times \frac{-3}{2}\right)a^{1+2}b^{1+1}c^1 = \frac{-6}{5}a^3b^2c$

(iii) $\frac{-2}{3}x \times \frac{3}{4}y \times -\frac{4}{5}z = \left(\frac{-2}{3} \times \frac{3}{4} \times \frac{-4}{5}\right)x \times y \times z = \frac{2}{5}xyz$

Multiplication of a Binomial by a Monomial

To multiply a binomial by a monomial use the following rule:
$a \times (b + c) = a \times b + a \times c$.

Example:

Multiply : (i) $\mathbf{3x^2 + 4xy}$ **by** $\mathbf{2x}$ **(ii)** $\mathbf{-4a[a + 3b]}$ **(iii)** $\mathbf{-4a[a - 3b]}$

Solution: (i) $2x[3x^2 + 4xy] = 2x \times 3x^2 + 2x \times 4xy = 6x^3 + 8x^2y$
(ii) $-4a[a + 3b] = -4a \times a - 4a \times 3b = -4a^2 - 12ab$
(iii) $-4a[a - 3b] = -4a \times a - 4a \times (-3b) = -4a^2 + 12ab$.

Multiplication of a Binomial by a Binomial

Suppose we have to multiply two binomials (a + b) and (c + d). We can do so by using the distributive law of multiplication over addition, twice as shown below.

$(a + b) \times (c + d) = a \times (c + d) + b \times (c + d)$ (Applying distributive law)
$= (a \times c + a \times d) + (b \times c + b \times d)$ (Applying distributive law again)
$= ac + ad + bc + bd$

Thus, to multiply any two binomials, we multiply each term of one binomial by each term of the other and then add the products.

Remark: This method is known as the **horizontal method.**

Example:

Multiply 3a + 2b and 5a + 7b.

Solution: $(3a + 2b)(5a + 7b) = 3a(5a + 7b) + 2b(5a + 7b)$
$= 15a^2 + 21ab + 10ab + 14b^2$
$= 15a^2 + 31ab + 14b^2$

Second Method: (Column Method). Recall that you can multiply two whole numbers by writing them one below the other. Likewise, you can multiply two binomials by writing them one below the other as shown below:

$$\begin{array}{l} 3a + 2b \\ \underline{\times\ 5a + 7b} \\ 15a^2 + 10ab \\ \underline{\quad\quad + 21ab + 14b^2} \\ 15a^2 + 31ab + 14b^2 \end{array}$$

Example:

Simplify: (5x – 3y) (–x + 7y) – (2x + 5y) (3x – 4y).

Solution: We have,

$(5x - 3y)(-x + 7y) - (2x + 5y)(3x - 4y) = 5x(-x + 7y) - 3y(-x + 7y) - [2x(3x - 4y) + 5y(3x - 4y)]$
$= (-5x^2 + 35xy + 3xy - 21y^2) - (6x^2 - 8xy + 15xy - 20y^2)$
$= (-5x^2 + 38xy - 21y^2) - (6x^2 + 7xy - 20y^2)$
$= -5x^2 + 38xy - 21y^2 - 6x^2 + 7xy - 20y^2$
$= (-5x^2 - 6x^2) + (38xy - 7xy) - (21y^2 - 20y^2) = -11x^2 + 31xy - y^2$.

Multiplication of a Binomial and a Trinomial

Recall that a trinomial is an algebraic expression that contains three terms. The distributive law of multiplication can be extended to the product of an algebraic expression containing any number of terms. Thus, we have;

$(a + b)(x + y + z) = a(x + y + z) + b(x + y + z)$
$= ax + ay + az + bx + by + bz$

Rule: Multiply each term of the binomial with every term of the trinomial and add all the products so obtained.

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Example:

Find the product (3x –2) ($2x^2$ + 3x – 5).
Solution: Horizontal Method:

Product = (3x – 2) ($2x^2$ + 3x – 5)
= 3x ($2x^2$ + 3x – 5) – 2($2x^2$ + 3x – 5)
= $6x^3$ + $9x^2$ – 15x – $4x^2$ – 6x + 10
= $6x^3$ + ($9x^2$ – $4x^2$) – 15x – 6x + 10
= $6x^3$ + $5x^2$ – 21x + 10.

Finding the value of an Expression

We know that the algebraic expression contains both variables and constants. So, we can find the value of expression only if we know the value of the variables.
For Example:
(i) We have $3a + 5$
Here, a is the variable.
Now, if a $= 2$, then
$3 \times 2 + 5 = 6 + 5 = 11$
Hence, the value of this algebraic expression is 11.

Note: The value of an algebraic expression changes as the value of the variable change.
Now, if $a = 3$, then
$3 \times 3 + 5 = 9 + 5 = 14$
Hence, the value of this algebraic expression is 14.

(ii) What should be the value of x, if $9p^2 + p + 3x =$ 47 when $p = 2$
When $p = 2$;
$\Rightarrow\ 9p^2 + p + 3x = 47$
$\Rightarrow\ 9(2)^2 + 2 + 3x = 47$
$\Rightarrow\ 9(2 \times 2) + 2 + 3x = 47$
$\Rightarrow\ 9 \times 4 + 2 + 3x = 47$
$\Rightarrow\ 36 + 2 + 3x = 47$
$\Rightarrow\ 38 + 3x = 47$
$\Rightarrow\ 38 - 38 + 3x = 47 - 38$
$\Rightarrow\ 3x = 47 - 38 \Rightarrow\ 3x = 9$
$\Rightarrow\ x = 3$
Hence, the value of x is 3.

Example:

Write the expression $5x^2 - 4 - 3x^2 + 6x + 8 + 5x - 13$ in its simplified form. Find its value when $x = -2$.
Solution: Let us simplify the given algebraic expression by adding and subtracting the like and unlike terms,

$\Rightarrow 5x^2 - 4 - 3x^2 + 6x + 8 + 5x - 13$ = ($5x^2$-$3x^2$) + (6x+5x) + (-4+8-13)
$\Rightarrow$2x^2 + 11x -9
Now, x= -2;
= 2 $(-2)^2$ + 11 (-2) - 9
= –23

Example:

Simple interest is given by $I = \frac{PTR}{100}$. If $P = ₹900$, $T = 2$ years; and $R = 5\%$, find the simple interest.
Solution: By the given formula,

Simple Interest (I) = $\frac{P \times T \times R}{100}$
$\Rightarrow \frac{900 \times 2 \times 5}{100}$ = 90 Rs.

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Check Your Concept - 1

(i) Add 4x + 3y – 5z, –7z + 5x – 8y and – y –3x + 2z.
(A) 6x – 6y + 10z (B) 6x – 6y –10z
(C) 6x + 6y –10z (D) 6x – 9y –10z

(ii) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$.
(A) $8p^2 + 8q^2 - 5pq$ (B) $8p^2 + 8q^2 + 5pq$
(C) $8p^2 - 8q^2 - 5pq$ (D) $8p^2 - 8q^2 + 5pq$

(iii) Simplify: 3(a + b) – 2(2a – b) + 4a – 7.
(A) 3a - 5b – 7 (B) 3a + 5b + 7
(C) 3a + 5b – 7 (D) 3a - 5b + 7

(iv) The value of $(-5x^2y) \times \left(-\frac{2}{3}xy^2z\right) \times \left(\frac{8}{15}xyz^2\right) \times \left(-\frac{1}{4}z\right)$ is.
(A) $-\frac{4}{9}x^4y^4z^4$ (B) $\frac{4}{9}x^4y^4z^4$
(C) $-\frac{4}{9}x^3y^3z^3$ (D) $\frac{4}{9}x^3y^3z^3$

Polynomials

Definition: An algebraic expression f(x) of the form $f(x) = a_0 + a_1 + a_2x^2 + \ldots\ldots + a_nx^n$. Where, $a_0, a_1, a_2 \ldots\ldots, a_n$ are real numbers and all the index of x are non–negative integers is called a polynomial in x and the highest Index n is called the degree of the polynomial, if $a_n \neq 0$. Here $a_0, a_1, x, a_2x^2 \ldots\ldots, a_nx^n$ are called the terms of the polynomial and $a_0, a_1\ a_2, \ldots\ldots, a_n$ are called various **coefficients** of the polynomial f(x). A polynomial in x is said to be in **standard form** when the terms are written either in increasing order or decreasing order of the indices of x in various terms.
For example: 6x, $x^3 + 3x^2 + 9x + 7$, $3x^2 - 4xy + 7y^2$ etc. are all polynomials.

Different types of Polynomials

Generally, we divide the polynomials into two categories;

(A) Based on degrees

There are four types of polynomials based on degrees, which are discussed below:

(i) Linear Polynomials: A polynomial of degree one is called a linear polynomial. The general form of a linear polynomial is **ax + b**, where a and b are any real constants and a ≠ 0.

(ii) Quadratic Polynomials: A polynomial of degree two is called a quadratic polynomial. The general form of a quadratic polynomial is $\mathbf{ax^2 + bx + c}$, where a ≠ 0.

(iii) Cubic Polynomials: A polynomial of degree three is called a cubic polynomial. The general form of a cubic polynomial is $\mathbf{ax^3 + bx^2 + cx + d}$, where a ≠ 0.

(iv) Biquadratic Polynomials: A polynomial of degree four is called a biquadratic polynomial. The general form of a biquadratic polynomial is $\mathbf{ax^4 + bx^3 + cx^2 + dx + e}$, where a ≠ 0.

(v) Constant Polynomial: A polynomial having only one term which is a constant is called a constant polynomial. The degree of a constant polynomial is 0.
For example, 7, −2, and 5 are constant polynomials.

Note: A polynomial of degree five or more than five does not have any particular name. Such a polynomial is usually called a polynomial of degree five or six and so on.

(B) Based on the number of Variables

(i) Polynomial in one variable: An algebraic expression of the form $a + bx + cx^2 + dx^3$, where a, b, c and d are constants and x is a variable is called a polynomial. **The powers of the variable involved are non–negative integers.**
For Example: 6 + 8a is a polynomial of degree 1 and $4x^3 - 2x + 3$ is a polynomial of degree 3.

(ii) Polynomial in two or more variables: It is an algebraic expression involving two or more variables with non–negative integral powers.
In such a polynomial, the degree of any term is the sum of the powers of the variables in that term. The greatest sum is the degree of the polynomial.
For Example: **(i)** $4x + 4y^2 + 4xy$ is a polynomial in x and y and its degree is 2
(ii) $11x^2y^2 + 4xy - 2xy^2$ is a polynomial in x and y of degree 4.

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Example:

Find the degree of each algebraic expression.

(i) $3x - 15$ **(ii)** $xy + yz$ **(iii)** $2y^2z + 9yz - 7z - 11x^2y^2$

(iv) $2y^2z + 10yz$ **(v)** $pq + p^2q - p^2q^2$

Solution: (i) 1 (ii) 2 (iii) 4 (iv)3 (v) 4

Standard Form of an Expression

Consider the expression $3x + 5x^2 - 9$. The degrees of first, second and third terms are 1,2, and 0 respectively. Thus, the degrees of terms are not in the descending order. By re-arranging the terms in such a way that their degrees are in descending order; we get the expression $5x^2 + 3x - 9$. Now the expression is said to be in standard form.
Now consider the expression, $3c + 6a - 2b$. Degrees of all the terms in the expression are same. Thus, the expression is said to be already in standard form.

Example:

Write the following expressions in standard form.

(i) $3x + 18 + 4x^2$ **(ii)** $8 - 3x^2 + 4x$ **(iii)** $-2m + 6 - 3m^2$ **(iv)** $y^3 + 1 + y + 3y^2$

Solution: (i) $4x^2 + 3x + 18$ **(ii)** $-3x^2 + 4x + 8$ **(iii)** $-3m^2 - 2m + 6$ **(iv)** $y^3 + 3y^2 + y + 1$

Algebraic Identities

What is an Identity?

Consider the equality $(a + 1)(a + 2) = a^2 + 3a + 2$
We shall evaluate both sides of this equality for some value of a, say a = 10.
LHS = (a + 1)(a + 2) = (10 + 1) (10 + 2) = 11 × 12 = 132
RHS = $a^2 + 3a + 2 = 10^2 + 3 \times 10 + 2 = 100 + 30 + 2 = 132$
∴ LHS = RHS.
Thus, the values of the two sides of the equality are equal for a = 10.
Let us now take a = –5
LHS = (a + 1) (a + 2) = (–5 + 1) (–5 + 2) = (–4) × (–3) = 12
RHS = $a^2 + 3a + 2 = (-5)^2 + 3(-5) + 2 = 25 - 15 + 2 = 10 + 2 = 12$
Thus, for a = –5, also LHS = RHS.

We shall find that for any value of a, LHS = RHS such **equality, true for every value of the variable in it, is called an identity.** Thus**,**
$(a + 1)(a + 2) = a^2 + 3a + 2$ is an identity.

Standard Identities

Identity 1. $\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$

Let us first consider the product (a + b) (a + b) or $(a + b)^2$
$(a + b)^2 = (a + b)(a + b)$
$= a(a + b) + b(a + b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$ (since ab = ba)
Thus,
$\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$

Clearly, this is an identity, since the expression on the RHS is obtained from the LHS by actual multiplication. One may verify that for any value of a and any value of b, the values of the two sides are equal.

Identity 2. $\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$

$(a - b)^2 = (a - b)(a - b) = a(a - b) - b(a - b)$
We have , $a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$
Thus,
$\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$

Identity 3. $\mathbf{(a + b)(a - b) = a^2 - b^2}$

$(a + b)(a - b) = a(a - b) + b(a - b)$
$\Rightarrow a^2 - ab + ba - b^2 = a^2 - b^2$ (since ab = ba)
or
$\mathbf{(a + b)(a - b) = a^2 - b^2}$

The Identities (1), (2) and (3) are known as **standard identities.**

Identity 4. $\mathbf{(x + a)(x + b) = x^2 + (a + b)x + ab}$

$(x + a)(x + b) = x(x + b) + a(x + b)$

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$= x^2 + xb + ax + ab$

$= x^2 + bx + ax + ab$ (Since xb = bx)

$= x^2 + ax + bx + ab$ (Since bx + ax = ax + bx)

$= x^2 + (a + b)\,x + ab$ (Taking x as a common factor from ax and bx)

$$\mathbf{(x + a)\,(x + b) = x^2 + (a + b)x + ab}$$

Example:

Find the following products using Identity I:

(i) (y + 2) (y + 6) **(ii) (x + 5) (x – 3)**

Solution: (i) Let us compare, (y + 2) (y + 6) with (x + a) (x + b). We observe that;

x = y, a = 2, b = 6

Therefore, using Identity 4, we have

$(y + 2)\,(y + 6) = y^2 + (2 + 6)y + 2 \times 6$

$= y^2 + 8y + 12$

(ii) Let us compare, (x + 5) (x – 3) with (x + a) (x + b). We observe that;

a = 5, b = –3

Therefore, using Identity 4, we have;

$(x + 5)(x - 3) = x^2 + \{5 + (-3)\}x + 5 \times (-3)$

$= x^2 + 2x - 15$

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Solved Examples

Level – 1

(1) Find the product of the following binomials $(6x^2 – 7y^2)$ $(6x^2 – 7y^2)$.

Solution: We have,

$(6x^2 – 7y^2)(6x^2 – 7y^2)$

$= (6x^2 – 7y^2)^2$

$= (6x^2)^2 – 2 \times 6x^2 \times 7y^2 + (7y^2)^2$

$= \mathbf{36x^4 – 84x^2y^2 + 49y^4}$

(2) Add: $5x^2 - \frac{1}{3}x + \frac{5}{2} - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}$ and $-2x^2 + \frac{1}{5}x - \frac{1}{6}$.

Solution: Required sum,

$= 5x^2 - \frac{1}{3}x + \frac{5}{2} - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3} - 2x^2 + \frac{1}{5}x - \frac{1}{6}$

$= 5x^2 - \frac{1}{2}x^2 - 2x^2 - \frac{1}{3}x + \frac{1}{2}x + \frac{1}{5}x + \frac{5}{2} - \frac{1}{3} - \frac{1}{6}$ [Collecting like terms]

$= \frac{10x^2 - x^2 - 2x^2}{2} + \frac{-10x + 15x + 6x}{30} + \frac{45 - 6 - 3}{18}$ [Adding like terms]

$= \mathbf{\frac{5x^2}{2} + \frac{11}{30}x + 2}$

(3) Evaluate: $(2x^2 + 3y)$ $(3x^2 – 2y)$.

Solution: $2x^2 + 3y)(3x^2 – 2y)$

$= 2x^2(3x^2 – 2y) + 3y(3x^2 – 2y)$

$= 2x^2 \times 3x^2 + 2x^2 \times (– 2y) + 3y \times 3x^2 + 3y \times (–2y)$

$= 6x^4 – 4x^2y + 9x^2y – 6y^2$

$\mathbf{= 6x^4 + 5x^2y – 6y^2}$

(4) Evaluate the following expression if a = 2, b = – 1, c = 1: $a^2 + b^2 + c^2 – ab – bc – ca$.

Solution: $a^2 + b^2 + c^2 – ab – bc – ca$

$= (2)^2 + (–1)^2 + (1)^2 – 2 \times (–1) – (–1) \times 1 – 1 \times 2$

$= 4 + 1 + 1 + 2 + 1 – 2$

$= 9 – 2 = 7$

So, $a^2 + b^2 + c^2 – ab – bc – ca$

At a = 2, b = – 1, c = 1 has value **7**.

(5) Classify into monomials, binomials and trinomials: $4y – 7z$, y^2, $x + y – xy$, 100, $ab – a – b$, $5 – 3t$, $4p^2q – 4pq^2$, 7mn, $z^2 – 3z + 8$, $a^3 + b^3$, $z^2 + z$, $1 + x + x^2$.

Solution: Monomials → y^2, 100, 7mn

Binomials → $4y – 7z$, $5 –3t$, $4p^2q – 4pq^2$, $a^3 + b^3$, $z^2 + z$

Trinomials → $x + y – xy$, $ab – a – b$, $z^2 –3z + 8$, $1 + x + x^2$

(6) How much is $t^2 – 5t + 6$ greater than $t^2 + 5t – 6$?

Solution: Minuend → $t^2 – 5t + 6$

Subtrahend → $t^2 + 5t – 6$

(Change the sign of each term and add)

$t^2 - 5t + 6$

$-t^2 - 5t + 6$

$0.t^2 - 10 \cdot t + 12$

Difference → $0.t^2 – 10t + 12$

Hence, the difference = **– 10t + 12**

(7) From the sum of $2p^2 + 3pq$, $– p^2 – pq – q^2$ and $pq + 2q^2$, subtract the sum of $3p^2 – q^2$ and $– p^2 + pq + q^2$.

Solution: We have,

$[(2p^2 + 3pq) + (–p^2 – pq – q^2) + (pq + 2q^2)] – [(3p^2 – q^2) + (–p^2 + pq + q^2)]$

$= [2p^2 + 3pq – p^2 – pq – q^2 + pq + 2q^2) – [3p^2 – q^2 – p^2 + pq + q^2]$

$= (2p^2 – p^2 + 3pq – pq + pq – q^2 + 2q^2) – (3p^2 – p^2 – q^2 + q^2 + pq)$

$= (p^2 + 3pq + q^2) – (2p^2 + pq)$

$= p^2 + 3pq + q^2 – 2p^2 – pq$

$\mathbf{= – p^2 + 2pq + q}$

(8) Divide $18x^4 - 27x^3 + 6x$ by $3x$.

Solution: $\frac{18x^4 - 27x^3 + 6x}{3x} = \frac{18x^4}{3x} - \frac{27x^3}{3x} + \frac{6x}{3x}$

$\therefore = 6x^3 - 9x^2 + 2$

The required result $= \mathbf{6x^3 - 9x^2 + 2}$.

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(9) Find the sum of the expressions: $x + xy^2 + y + ax^2$; $2ax^2 - 2y + 3x$; $-5xy^2 - ax^2 - 2y$; $-5x + 3y - 2ax^2$.

Solution: Sum of given expressions,

$= (x + xy^2 + y + ax^2) + (2ax^2 - 2y + 3x) + (-5xy^2 - ax^2 - 2y) + (-5x + 3y - 2ax^2)$

$= (x + xy^2 + y + ax^2 + 2ax^2 - 2y + 3x) + (-5xy^2 - ax^2 - 2y - 5x + 3y - 2ax^2)$

$= (4x + 3ax^2 - y + xy^2) + (-5xy^2 - 3ax^2 - 5x + y)$

$= 4x + 3ax^2 - y + xy^2 - 5xy^2 - 3ax^2 - 5x + y$

$= (4 - 5)x + (3a - 3a)x^2 + (-1 + 1)y + (1 - 5)xy^2$

$= \mathbf{-x - 4xy^2}$

(10) Subtract the sum of $7x^2 - 4y^2$ and $4x^2 + 3xy - 4y^2$ from the sum of $5x^2 - 3xy - y^2$ and $x^2 + 2xy - 2y^2$.

Solution: Sum of $7x^2 - 4y^2$ and $4x^2 + 3xy - 4y^2$ is

$7x^2 - 4y^2 + 4x^2 + 3xy - 4y^2$

$= (7 + 4)x^2 + (-4 - 4)y^2 + 3xy$

$= 11x^2 - 8y^2 + 3xy$

Sum of $5x^2 - 3xy - y^2$ and $x^2 + 2xy - 2y^2$

$= 5x^2 - 3xy - y^2 + x^2 + 2xy - 2y^2$

$= (5 + 1)x^2 + (-3 + 2)xy + (-1 - 2)y^2$

$= 6x^2 - xy - 3y^2$

$\therefore$ Required difference $= (6x^2 - xy - 3y^2) - (11x^2 - 8y^2 + 3xy)$

$= 6x^2 - xy - 3y^2 - 11x^2 + 8y^2 - 3xy$

$= (6 - 11)x^2 + (-1 - 3)xy + (-3 + 8)y^2$

$= \mathbf{-5x^2 - 4xy + 5y^2}$

(11) (a) Simplify: $4ab(a - b) - 6a^2(b - b^2) - 3b^2(2a^2 - a) + 2ab(b - a)$

(b) Simplify: $x^2(x^2 + 1) - x^3(x + 1) - x(x^3 - x)$

Solution: (a) We have

$4a^2b - 4ab^2 - 6a^2b + 6a^2b^2 - 6a^2b^2 + 3ab^2 + 2ab^2 - 2a^2b$

By grouping similar expressions we get,

$\Rightarrow 4a^2b - 6a^2b - 2a^2b - 4ab^2 + 3ab^2 + 2ab^2 + 6a^2b^2 - 6a^2b^2$

$= \mathbf{-4a^2b + ab^2}$

(b) We have

$x^4 + x^2 - x^4 - x^3 - x^4 + x^2$

By grouping similar expressions we get,

$= x^4 - x^4 - x^4 - x^3 + x^2 + x^2$

$= \mathbf{-x^4 - x^3 + 2x^2}$

(12) If $2y + \frac{1}{2y} = 3$, then $16y^4 + \frac{1}{16y^4}$ is?

Solution: Given, $2y + \frac{1}{2y} = 3$

Taking squares on both sides, we get

$\left(2y + \frac{1}{2y}\right)^2 = 3^2$

$\Rightarrow 4y^2 + \frac{1}{4y^2} + 2 = 9$

$\Rightarrow 4y^2 + \frac{1}{4y^2} = 7$

Again, taking squares on both sides, we get

$\left(4y^2 + \frac{1}{4y^2}\right)^2 = 7^2$

$\Rightarrow 16y^4 + \frac{1}{16y^4} + 2 = 49$

$\Rightarrow 16y^4 + \frac{1}{16y^4} = \mathbf{47}$

(13) If $C = (7x + 9)$ and $D = (4x^2 + 8x + 5)$, then the degree of the product CD is?

Solution: $C = 7x + 9$

$D = (4x^2 + 8x + 5)$

$C \times D = (7x + 9)(4x^2 + 8x + 5)$

$= 28x^3 + x^2(56 + 36) + x(72 + 35) + 45$

$= 28x^3 + 92x^2 + 107x + 45$

Degree $= \mathbf{3}$.

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(14) Simplify the following expressions:

(a) $(\mathbf{x}+\mathbf{y}+\mathbf{z})(\mathbf{x}+\mathbf{y}-\mathbf{z})$

(b) $\mathbf{x^2(x-3y^2)-xy(y^2-2xy)-x(y^3-5x^2)}$

(c) $\mathbf{2x^2(x+2)-3x(x^2-3)-5x(x+5)}$

Solution: (a) $(\mathbf{x}+\mathbf{y}+\mathbf{z})(\mathbf{x}+\mathbf{y}-\mathbf{z})$ $(\mathbf{a})(\mathbf{x}+\mathbf{y}+\mathbf{z})(\mathbf{x}+\mathbf{y}-\mathbf{z})$

$= x^2 + xy - xz + yx + y^2 - yz + zx + zy - z^2$

Now, adding the like terms and rearranging, we get

$\mathbf{x^2 + y^2 - z^2 + 2xy}$

(b) $\mathbf{x^2(x-3y^2)-xy(y^2-2xy)-x(y^3-5x^2)}$

$= x^3 - 3x^2y^2 - xy^3 + 2x^2y^2 - xy^3 + 5x^3$

Now, adding the like terms and rearranging, we get

$x^3 + 5x^3 - 3x^2y^2 + 2x^2y^2 - xy^3 - xy^3$

$= \mathbf{6x^3 - x^2y^2 - 2xy^3}$

(c) $\mathbf{2x^2(x+2)-3x(x^2-3)-5x(x+5)}$

$= 2x^3 + 4x^2 - 3x^3 + 9x - 5x^2 - 25x$

$= 2x^3 - 3x^3 - 5x^2 + 4x^2 + 9x - 25x$

$= \mathbf{-x^3 - x^2 - 16x}$

(15) (a) Calculate the volume of a cuboidal box whose dimensions are $\mathbf{5x \times 3x^2 \times 7x^4}$.

(b) Calculate the area of a rectangle whose length and breadths are given as $\mathbf{3x^2y\ m}$ and $\mathbf{5xy^2\ m}$ respectively.

Solution: (a) Given,

Length $= 5x$

Breadth $= 3x^2$

Height $= 7x^4$

Volume of cuboid = Length × Breadth × Height

$= 5x \times 3x^2 \times 7x^4$

$= 105(x \times x^2 \times x^4)$

By exponents rule: $x^a \times x^b = x^{(a+b)}$

$\Rightarrow 105(x \times x^2 \times x^4) = 105x^{1+2+4} = \mathbf{105x^7}$

(b) Given,

Length $= 3x^2y\ \ m$

Breadth $= 5xy^2\ m$

Area of rectangle = Length × Breadth

$= (3x^2y \times 5xy^2) = (3 \times 5) \times x^2y \times xy^2 = \mathbf{15x^3y^3\ \ m^2}$

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Exercise

FILL IN THE BLANKS

(1) Expression having 3 terms is known as _____.
(2) Dividend = _____ × quotient + remainder
(3) Degree of a constant term is _____.
(4) Algebraic expressions are formed from _____ and _____.
(5) Factors containing variables are said to be _____.
(6) The _____ is the numerical factor in the term.
(7) Any expression with one or more terms is called a _____.
(8) Terms which have the same algebraic factors are _____. Terms which have different algebraic factors are _____.
(9) The process of writing an algebraic expression as the product of two or more factors is called _____.
(10) 2xy and $-5x^2$ are _____ of the expression $2xy - 5x^2$.

TRUE OR FALSE

(1) Polynomial having degree four is known as a cubic polynomial.
(2) 5x + 2x is a monomial.
(3) Highest power of a variable is known as the Degree of a polynomial.
(4) 4b, - 4b are like terms.
(5) A monomial multiplied by another monomial always gives a monomial.
(6) 4x(x + 2)(x – 1) is an irreducible factor form.
(7) The value of $(x - y)^2$ is the same as $x^2 - y^2$.
(8) The polynomial $-3 + 8x - 9x^2 + 17x^3$ is in the standard form.
(9) The degree of the polynomial $3x^2 y + 5x^3y^2 + 9xy - 11$ is 3.
(10) $(2x^2 - 5)^2 = 2x^2 + 25 - 20x$.

OBJECTIVE TYPE QUESTIONS

(1) The value of $(x + y)^2$ is
(A) $x^2 + y^2$ (B) $x^2 - y^2$
(C) $x^2 + y^2 - 2xy$ (D) $x^2 + y^2 + 2xy$

(2) The value of $63x^3 y^3 z^5 \div (-7xy^2z^3)$ is
(A) $9x^2 yz^2$ (B) $-9x^2 yz^2$
(C) $-9x^4 y^5 z^8$ (D) $9x^4 y^5 z^8$

(3) Simplify $\left(2x + \frac{1}{3y}\right)^2 - \left(2x - \frac{1}{3y}\right)^2$.
(A) $\frac{4x}{3y}$ (B) $2\left(4x^2 + \frac{1}{9y^2}\right)$
(C) $\frac{8x}{3y}$ (D) $\frac{4y}{3x}$

(4) An expression is taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $-x^2 - y^2 + 6xy + 20$, then the expression is
(A) $4x^2 - 3y^2 - xy$ (B) $2x^2 - 5y^2 + xy + 40$
(C) $3y^2 - xy - 4x^2$ (D) $4x^2 + 3y^2 + xy$

(5) The value of $(-5x^2y) \times \left(-\frac{2}{3}xy^2z\right) \times \left(\frac{8}{15}xyz^2\right) \times \left(-\frac{1}{4}z\right)$ is
(A) $-\frac{4}{9}x^4y^4z^4$ (B) $\frac{4}{9}x^4y^4z^4$
(C) $-\frac{4}{9}x^3y^3z^3$ (D) $\frac{4}{9}x^3y^3z^3$

(6) The algebraic expression of the statement 'product of numbers a and b subtracted from 7 is
(A) ab – 7 (B) 7 – ab
(C) ab (D) 7ab

(7) Subtract $2x^3 + x^2 - 4x - 1$ from $5x^3 + 5x^2 + 9$.
(A) $-3x^3 + 4x^2 + 4x + 10$ (B) $3x^3 + 4x^2 + 4x + 10$
(C) $3x^3 + 4x^2 + 4x - 10$ (D) $-(3x^3 + 4x^2 + 4x + 10)$

(8) Subtract: $\frac{3}{2}x^2y + \frac{4}{5}y - \frac{1}{3}x^2yz$ from $\frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y$.
(A) $\frac{41}{15}x^2yz - \frac{5}{6}x^2y - \frac{3}{5}xyz - \frac{4}{5}y$ (B) $\frac{5}{6}x^2yz - \frac{3}{5}x^2y + \frac{41}{15}x^2yz - \frac{4}{5}y$
(C) $\frac{4}{5}x^2yz - \frac{5}{6}x^2y - \frac{3}{4}xyz + \frac{4}{7}y$ (D) $\frac{41}{15}x^2yz + \frac{5}{6}x^2y + \frac{4}{5}xyz + \frac{3}{5}y$

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(9) If 496 × 492 = $x^2 - 4$ (x > 0), then x = _____.
(A) 495 (B) 494
(C) 493 (D) 496

(10) For what value of K is $16x^2 + 24xy + K$ a perfect square?
(A) $9y^2$ (B) $18y^2$
(C) $3y^2$ (D) $16y^2$

(11) The value of 998^2 is _____.
(A) 996064 (B) 996004
(C) 998004 (D) 998064

(12) Simplify: $(y^3 - 2y^2 + 3y - 4)(y - 1) - (24 - 3)(y^2 - y + 1)$ **(2015)**
(A) $y^4 - 3y^3 - 16y^2 + 14y - 17$ (B) $y^4 - 3y^3 - 16y^2 - 14y - 17$
(C) $y^4 - 3y^3 + 16y^2 + 14y - 17$ (D) $y^4 + 3y^3 - 16y^2 + 14y - 17$

(13) The number of terms in the product of $(3x - 2)$ and $(2x + 3)$ is **(2011)**
(A) One (B) Two
(C) Three (D) Four

(14) Find the value of $6x^2(y - y^2) - 3y^2 + 2xy(x - y)$ for $x = 4$ and $y = 5$. **(2013)**
(A) -215 (B) 2035
(C) -2035 (D) 215

(15) What must be subtracted from $5a^2 - 10ab - 5\,b^2 - 1$ to get $4a^2 - 7ab - 4b^2 + 1$? **(2014)**
(A) $-a^2 + ab + b^2 - 2$ (B) $a^2 + 3ab + 4\,b^2 + 2$
(C) $a^2 - 3ab - b^2 - 2$ (D) $a^2 - 3ab - b^2 + 2$

(16) Simplify: $(p + q - r)^2 + (p - q - r)^2$ **(2013)**
(A) $2[(p - r)^2 + q^2]$ (B) $2[(p - q)^2 + r^2]$
(C) $2[(q - p)^2 + r^2]$ (D) $2[(p - q)^2 + q^2]$

(17) If $x + \frac{1}{x} = 7$ then the value of $x^2 + \frac{1}{x^2}$ is **(2015)**
(A) 47 (B) 34
(C) 42 (D) 24

(18) If $ab = 8$ and $a + b = 10$ then the value of $a^2 + b^2$ is **(2011)**
(A) 88 (B) 68
(C) 98 (D) 84

(19) If $a^2 + 16a + k$ is a perfect square, then find the value of k. **(2014)**
(A) 4 (B) 16
(C) 36 (D) 64

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Answer Key

CHECK YOUR CONCEPT

(1) **(i)** (B) **(ii)** (A) **(iii)** (C) **(iv)** (A)

FILL IN THE BLANKS

(1)	Trinomial	**(6)**	Coefficient
(2)	Divisor	**(7)**	Polynomial
(3)	Zero	**(8)**	Like terms and unlike terms
(4)	Variable and constant	**(9)**	Factorisation
(5)	Algebraic factor	**(10)**	Terms

TRUE OR FALSE

(1)	False	**(6)**	True
(2)	True	**(7)**	False
(3)	True	**(8)**	False
(4)	True	**(9)**	False
(5)	True	**(10)**	False

OBJECTIVE TYPE QUESTIONS

(1)	(D)	**(6)**	(B)	**(11)**	(B)	**(16)**	(A)
(2)	(B)	**(7)**	(B)	**(12)**	(A)	**(17)**	(A)
(3)	(C)	**(8)**	(A)	**(13)**	(C)	**(18)**	(D)
(4)	(A)	**(9)**	(B)	**(14)**	(C)	**(19)**	(D)
(5)	(A)	**(10)**	(A)	**(15)**	(C)		

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