CIRCLES

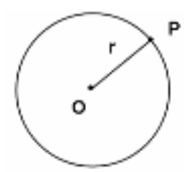
Concepts Covered

- Introduction to Circumference, Chord, Diameter, Secant and Tangent
- Concentric circles.
- Cyclic Quadrilaterals.
- Theorems based on the properties of Circles.

Definition

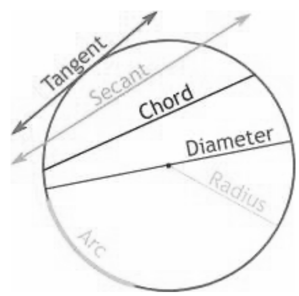
A circle is the locus of a point which moves in such a way that its distance from a fixed point is constant. The fixed point is called the centre of the circle and the constant distance, is the radius of the circle.

A circle with centre O and radius r is denoted by C (O, r).



A Circle with Radius r

Terms related to a Circle



Diameter

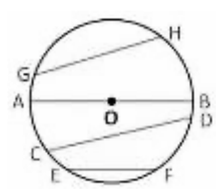
A diameter is a chord of a circle passing through the centre of the circle.
Thus, AOB is a diameter of a circle with centre O.
Diameter is the longest chord of a circle
Diameter = 2 × radius

Circumference

The perimeter of a circle is called its circumference.
Circumference = $\mathbf{2\pi r}$

Chord

A chord of a circle is a line segment joining any two points on the circle.
In the given figure, GH, CD, EF and AOB are the chords of the circle with centre O.



Secant

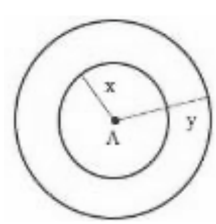
A line which intersects a circle in two distinct points is called a secant of the circle.

Tangent

A line that intersects the circle at exactly one point is called a tangent to the circle.

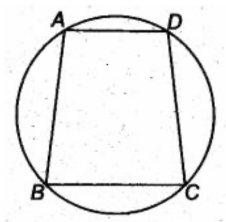
Concentric Circles

Circles which have the same centre and different radii are called concentric circles.
In the given figure C(A, x) and C(A, y) are concentric circles having the same centre A but different radii x and y respectively.



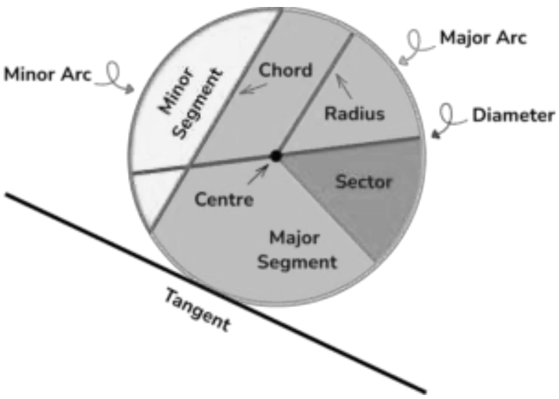
Cyclic Quadrilateral

A quadrilateral ABCD is called cyclic if all four vertices of it lie on a circle.



Cyclic Quadrilateral

Some Important parts of Circle

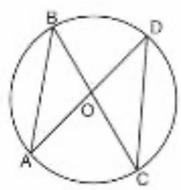


Theorems

Theorem 1

Statement: Equal chords of a circle subtend equal angles at the centre.

Given: AB and CD are chords of a circle with centre O, such that $AB = CD$.
To prove: $\angle AOB = \angle COD$



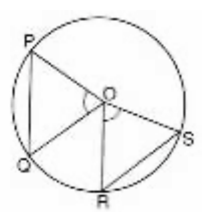
Proof: In $\triangle AOB$ and $\triangle COD$,

$AO = CO$	(Radii of the same circle)
$BO = DO$	(Radii of the same circle)
$AB = CD$	(Given)
$\therefore \triangle AOB \cong \triangle COD$	(SSS)
Hence, $\angle AOB = \angle COD$	[Corresponding part of the congruent triangle (CPCT)]

Theorem 2

Statement: If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Given: Two chords PQ and RS of a circle $C(O, r)$, such that $\angle POQ = \angle ROS$.
To prove: $PQ = RS$



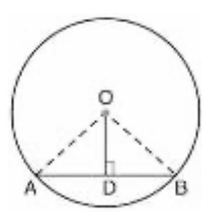
Proof: In $\triangle POQ$ and $\triangle ROS$,

$OP = OQ = OR = OS = r$	(Radii of the same circle)
and $\angle POQ = \angle ROS$	(Given)
$\therefore \triangle POQ \cong \triangle ROS$	(SAS)
$\therefore PQ = RS$	(CPCT)

Theorem 3

Statement: The perpendicular from the centre of a circle to a chord bisects the chord.

Given: AB is the chord of a circle with centre O and $OD \perp AB$.
To prove: $AD = DB$
Construction: Join OA and OB.



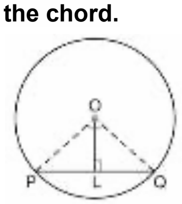
Proof: In $\triangle ODA$ and $\triangle ODB$,

$OA = OB$	(Radii of the same circle)
$OD = OD$	(Common)
$\angle ODA = \angle ODB$	(Each is a right angle)
$\triangle ODA \cong \triangle ODB$	(R.H.S.)
$AD = DB$	(CPCT)

Theorem 4

Statement: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Given: A chord PQ of a circle $C(O, r)$ and L is the mid-point of PQ.



Proof: $OL \perp PQ$.
Construction: Joint OP and OQ.
In $\triangle OLP$ and $\triangle OLQ$,

$OP = OQ$	(Radii of the same circle)
$PL = QL$	(Given)
$OL = OL$	(Common)

Check Your Concept - 1

(i) A circle of 30 cm diameter has a 24 cm chord What is the distance of the chord from the centre?

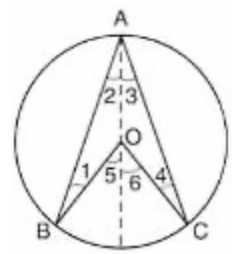
(ii) A chord of length 12 cm is drawn in a circle of radius 10 cm. Calculate its distance from the centre of the circle.

(iii) The radius of a circle is 13 cm and the length of one of its chords is 10 cm, find the distance of the chord from the centre.

Theorem 8

Statement: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given: O is the centre of the circle.
To prove: $\angle BOC = 2\angle BAC$
Construction: Join O to A.



Proof: In $\triangle AOB$,
$OA = OB$ (radii of the same circle)
$\Rightarrow \angle 1 = \angle 2$
Similarly in $\triangle AOC$, $\angle 3 = \angle 4$
Now, by exterior angle property,
$\angle 5 = \angle 1 + \angle 2$
$\angle 6 = \angle 3 + \angle 4$
$\Rightarrow \angle 5 + \angle 6 = \angle 1 + \angle 2 + \angle 3 + \angle 4$
$\Rightarrow \angle 5 + \angle 6 = 2\angle 2 + 2\angle 3 = 2(\angle 2 + \angle 3)$
$\Rightarrow \angle BOC = 2\angle BAC$

Theorem 9

Statement: Angles in the same segment of a circle are equal.

Given: Two angles $\angle ACB$ and $\angle ADB$ are in the same segment of a circle $C(O, r)$.
To prove: $\angle ACB = \angle ADB$
Construction: Join OA and OB.

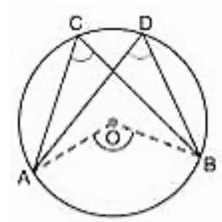


fig. (i)

Proof: In fig. (i), we know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc in the alternate segment.
Hence, $\angle AOB = 2\angle ACB$
$\angle AOB = 2\angle ADB$
So, $\angle ACB = \angle ADB$
In fig. (ii), we have,
Reflex $\angle AOB = 2\angle ACB$ and Reflex $\angle AOB = 2\angle ADB$
$2\angle ACB = 2\angle ADB$
$\therefore \angle ACB = \angle ADB$

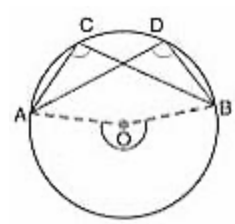
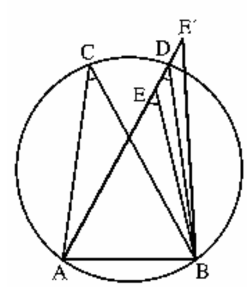


fig. (ii)

Theorem 10

Statement: If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e., they are concyclic).

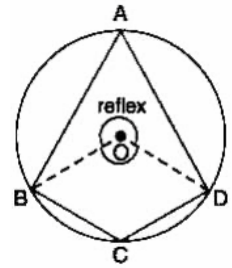
Given: In the figure., AB is a line segment, which subtends equal angles at two points C and D. That is $\angle ACB = \angle ADB$
To proof: To show that the points A, B, C and D lie on a circle let us draw a circle through the points A, C and B.



Proof: Suppose it does not pass-through point D. Then it will intersect AD (or extended AD) at a point, say E (or E').
If points A, C, E and B lie on a circle,
But it is given that $\angle ACB = \angle ADB$.
Therefore, $\angle AEB = \angle ADB$.
This is not possible unless E coincides with D.
Similarly, E' should also coincide with D.

Theorem 11

Statement: The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.



Given: Let ABCD be a cyclic quadrilateral
To prove: $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$
Construction: Join OB and OD.

Proof: $\angle BOD = 2\angle BAD$

$\angle BAD = \frac{1}{2}\angle BOD$

Similarly, $\angle BCD = \frac{1}{2}$ reflex $\angle BOD$

$\therefore \angle BAD + \angle BCD = \frac{1}{2}\angle BOD + \frac{1}{2}$ reflex $\angle BOD = \frac{1}{2}(\angle BOD +$ reflex $\angle BOD) = \frac{1}{2} \times 360°$

$\therefore \angle A + \angle C = 180°$

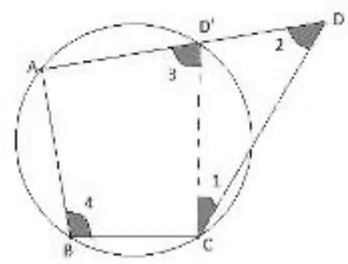
Similarly, $\angle B + \angle D = 180°$

Theorem 12

Statement: If the sum of a pair of opposite angles of a quadrilateral is $180°$, the quadrilateral is cyclic.

Given: The sum of pair of opposite angles of a quadrilateral is $180°$
To prove: The quadrilateral is cyclic.

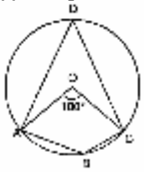
Proof: Let us assume that the quadrilateral $ABCD$ is not cyclic i.e. Let point D not lie on the circle which makes the quadrilateral non-cyclic. Now, let us do a construction such that join CD' where D' is the point of intersection of side AD with the circle. Now, ABCD' is cyclic $\Rightarrow \angle 3 + \angle 4 = 180°$



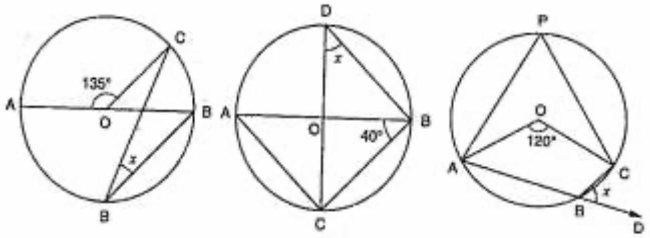
Now, it is given that the sum of pair opposite angles of a quadrilateral $ABCD$ is $180°$. Therefore, $\angle 2 + \angle 4 = 180°$
From the above two equations, we get $\angle 3 + \angle 4 = \angle 2 + \angle 4 \Rightarrow \angle 3 = \angle 2$
Now, in triangle CDD', by external angle property
$\angle 3 = \angle 1 + \angle 2$
$\Rightarrow \angle 1 = 0$, hence the side CD' and CD coincides
$\Rightarrow$ Point D lies on the circle

Check Your Concept – 2

(i) In fig., O is the centre of the circle, and the measure of arc ABC is 100°. Determine ∠ADC and ∠ABC.



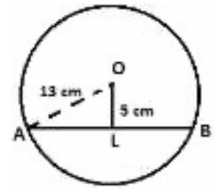
(ii) If O is the centre of the circle, find the value of x in each of the following figures:



Solved Examples

(1) Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 13 cm.

Solution: Let AB be a chord of a circle with centre O and radius $13\ \mathrm{cm}$. Draw $\mathrm{OL} \perp \mathrm{AB}$. Join OA.



Here, $\mathrm{OL} = 5\ \mathrm{cm}, \mathrm{OA} = 13\ \mathrm{cm}$

In the right triangle OLA,

$\mathrm{OA}^2 = \mathrm{OL}^2 + \mathrm{AL}^2$

$13^2 = 5^2 + \mathrm{AL}^2$

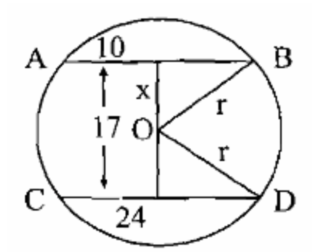
$\mathrm{AL}^2 = 144$

$\mathrm{AL} = 12$

Since the perpendicular from the centre to a chord bisects the chord. Therefore,

$\mathrm{AB} = 2\mathrm{AL} = 2 \times 12 = \mathbf{24\ cm}$

(2) AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm, If the chords are on the opposite sides of the centre and the distance between them is 17 cm, the radius of the circle is



Solution: $\mathrm{r}^2 = \mathrm{x}^2 + (5)^2$

Also, $\mathrm{r}^2 = (17 - \mathrm{x})^2 + (12)^2$

So, $\mathrm{x}^2 + (5)^2 = (17 - \mathrm{x})^2 + (12)^2$

$\mathrm{x}^2 + 25 = (17 - \mathrm{x})^2 + 144$

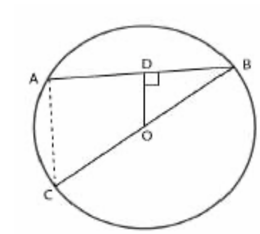
$\Rightarrow \mathrm{x} = 12$

$\therefore \mathrm{r}^2 = 144 + 25 = 169$

$\Rightarrow \mathrm{r} = \mathbf{13\ cm}$

(3) In the adjoining figure OD is perpendicular to the chord AB of a circle with centre O. If BC is a diameter, show that AC || DO and $\mathbf{AC = 2\ \times OD}$.

Solution: Perpendicular from the centre of a circle to a chord bisects the chord



We know that OB ⊥ AB

From the figure, we know that D is the midpoint of AB

We get, AD = BD

We also know that O is the midpoint of BC

We get, OC = OB

Consider △ABC

Using the midpoint theorem

We get OB ∥ AC and

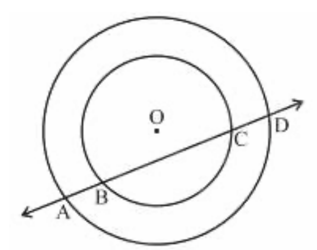
OD = 12 × AC

By cross multiplication

AC = 2 × OD

Therefore, it is proved that AC ∥ DO and AC = 2 × OD

(4) If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure).



Solution: We know that, $\mathrm{OA} = \mathrm{OD}$ and $\mathrm{OB} = \mathrm{OC}$.

They are the radius of respective circles.

In $\triangle\ \mathrm{OBC}$, we know that $\mathrm{OB} = \mathrm{OC}$, so $\angle\mathrm{OBC} = \angle\mathrm{OCB}$

$\therefore \angle\mathrm{OCD} = \angle\mathrm{OBA}$

In $\triangle\ \mathrm{OAD}$, we know that $\mathrm{OA} = \mathrm{OD}$, so $\angle\mathrm{OAD} = \angle\mathrm{ODA}$

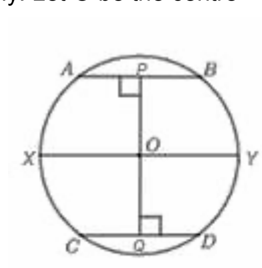
Since, $\angle\mathrm{OCD} = \angle\mathrm{OBA}$ and $\angle\mathrm{OAD} = \angle\mathrm{ODA}$, we get $\angle\mathrm{AOB}$ in $\triangle\ \mathrm{OAB}$ is equal to $\angle\mathrm{COD}$ in $\triangle\ \mathrm{OCD}$.

$\therefore$ From SAS congruency, we can say that $\triangle\ \mathrm{OAB}$ and $\triangle\ \mathrm{OCD}$ are congruent.

So, $\mathbf{AB = CD}$.

(5) Prove that the line joining the mid-points of two parallel chords of a circle passes through the centre.

Solution: Let AB and CD be two parallel chords having P and Q as their mid-points respectively. Let O be the centre of the circle. Join OP and OQ and though O, draw XY ∥ AB or CD.



Now, P is the mid-point of AB

OP ⊥ AB, ∠BPO = 90°

But, XY ∥ AB.

Therefore, ∠XOQ=∠BPO [Corresponding angles]

∠XOQ = 90°

Similarly, Q is the mid-point of CD

OQ ⊥ CD, ∠DQO = 90°

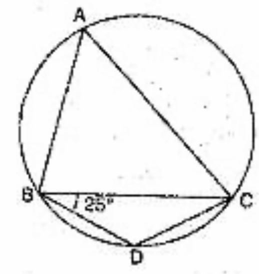
But, XY ∥ CD. Therefore, ∠POX = ∠DQO = 90° [Corresponding angles]

∠POX + ∠XOQ = 90° + 90° = 180°

POQ is a straight line.

Hence, PQ is a straight line passing through the centre of the circle.

(6) In figure, BD = DC and ∠DBC = 25°, find the measurement of ∠BAC.



Solution: It is given that,

$BD = DC \Rightarrow \angle CBD = \angle BCD$ (angles opposite to equal side)

Now, In$\triangle BCD$ we have

$\Rightarrow \angle CBD + \angle BCD + \angle BDC = 180^\circ$ (sum of angles of a triangle)

$\Rightarrow \angle BDC = 180^\circ - 50^\circ = 130^\circ$ (as $\angle CBD = \angle BCD = 25^\circ$)

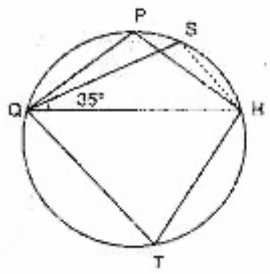
From the property of the circle, we know that the angle subtended by the same chord on the minor segment is twice the angle on the major segment.

So,

$\Rightarrow 2\angle BAC = \angle BDC \Rightarrow \angle BAC = \frac{130^\circ}{2} = \mathbf{65^\circ}$

(7) In fig., Δ PQR is an isosceles triangle with PQ = PR and m∠ PQR = 35°. Find m∠QSR and m∠QTR.

Solution: Given: $\triangle PQR$ is an isosceles triangle with $PQ = PR$ and $m\angle PQR = 35^\circ$



In $\triangle PQR$:

$\angle PQR = \angle PRQ = 35^\circ$ (Angle opposite to equal sides)

Again, by angle sum property

$\angle P + \angle Q + \angle R = 180^\circ$

$\angle P + 35^\circ + 35^\circ = 180^\circ$

$\angle P + 70^\circ = 180^\circ$

$\angle P = 180^\circ - 70^\circ$

$\angle P = 110^\circ$

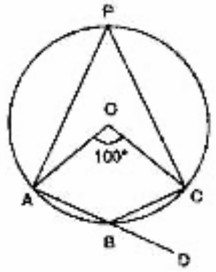
Now, in quadrilateral SQTR,

$\angle QSR + \angle QTR = 180^\circ$ (Opposite angles of a quadrilateral)

$110^\circ + \angle QTR = 180^\circ$

$\angle QTR = \mathbf{70}^\circ$

(8) In the given figure, O is the centre of the circle. Find ∠CBD.



Solution: ABCP is a cyclic quadrilateral

∠APC = ∠CBD

∠APC = $\frac{1}{2}$ ∠AOC

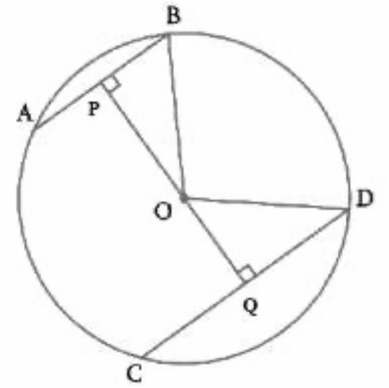
⇒ ∠APC = 50°

∠APC = angle CBD

⇒ angle CBD = **50°**

(9) Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution: Also, $AP = PB = 2.5$ cm and $CQ = QD = 5.5$ cm



(Perpendicular from the centre to a chord of the circle bisects the chord.)

In the right triangles QAP and OCQ, we have

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$\therefore r^2 = x^2 + (2.5)^2 \ldots\ldots\ldots\ldots (1)$

and $r^2 = (6 - x)^2 + (5.5)^2 \ldots\ldots\ldots (2)$

$\Rightarrow x^2 + (2.5)^2 = (6 - x)^2 + (5.5)^2$

$\Rightarrow x^2 + 6.25 = 36 - 12x + x^2 + 30.25$

$12x = 60$

$\therefore x = 5$

Join OA and OC.

Let the radius of the circle be r cm and O be the centre

Draw $OP \perp AB$ and $OQ \perp CD$.

We know, $OQ \perp CD, OP \perp AB$ and $AB \parallel CD$.

Therefore, points P, O and Q are collinear. So, $PQ = 6$ cm.

Let $OP = x$.

Then, $OQ = (6 - x)$cm

And $OA = OC = r$

Putting $x = 5$ in (1), we get

$r^2 = 52 + (2.5)^2 = 25 + 6.25 = 31.25 \Rightarrow r^2 = 31.25 \Rightarrow r = 5.6$

Hence, the radius of the circle is 5.6 cm

Putting $x = 5$ in (1), we get

$r^2 = 52 + (2.5)^2 = 25 + 6.25 = 31.25 \Rightarrow r^2 = 31.25 \Rightarrow r = 5.6$

Hence, the radius of the circle is $\mathbf{5.6\ cm}$

(10) The length of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?

Solution: $AB = 6\text{ cm}, CD = 8\text{ cm}$

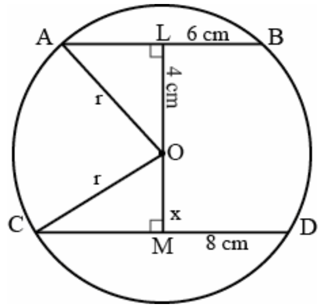
Let $OL \perp AB$ and $OM \perp CD$

$\therefore OL = 4\text{ cm}$

Let $OM = x\text{ cm}$

Let r be the radius of the circle

Now in right $\triangle OLA$



$OA^2 = OL^2 + AL^2 = 4^2 + \left(\frac{6}{2}\right)^2$

$r^2 = 16 + 9 = 25 \ldots\ldots\ldots\ldots(i)$

and in right $\triangle OMC$

$OC^2 = OM^2 + CM^2$

$r^2 = x^2 + \left(\frac{8}{2}\right)^2$

$= x^2 + (4)^2 = x^2 + 16 \ldots\ldots\ldots..(ii)$

From (i) and (ii),

$x^2 + 16 = 25$

$\Rightarrow x^2 = 25 - 16 = 9$

$\Rightarrow x^2 = (3)^2$

$\therefore x = 3\text{ cm}$

$\therefore$ Distance $= \mathbf{3\ cm}$

(11) Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita threw a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Nisha is 24 cm each, what is the difference between Ishita and Nisha.

Solution: Let Ishita at A, Isha at b and Nisha at C.

$\therefore AB = BC = 24\text{ cm}$.......(given)

and $ROA = OC = 20\text{ m}$ (radii)

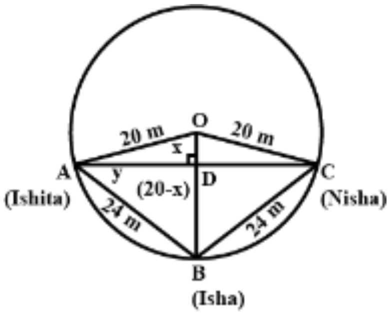
Let $OD = x\text{ m}$

$\therefore BD = OB - OD = (20 - x)\text{m}$

Let $AC = (2y)\text{m}$

Hence, $AD = (y)\text{m}$

In $\triangle$ODA,



$OD^2 + DA^2 = OA^2$

$x^2 + y^2 = (20)^2 \ldots\ldots$ (Pythagoras Theorem)

$\Rightarrow x^2 + y^2 = 400$

In $\triangle ADB, AD^2 + BD^2 = AB^2$

$\Rightarrow y^2 + (20 - x)^2 = (24)^2$

$\therefore y^2 + 400 - 40x + x^2 = 576$

$\therefore (x^2 + y^2) + 400 - 576 = 40x$

$\therefore 400 + 400 - 576 = 40x$

$\Rightarrow x = \frac{224}{40} = 5.6\text{ m}$

Substituting this value in

$x^2 + y^2 = 400$ we get

$(5.6)^2 + y^2 = 400$

$\therefore 31.36 + y^2 = 400$

$\Rightarrow y^2 = 400 - 31.36$

$\Rightarrow y^2 = 368.64$

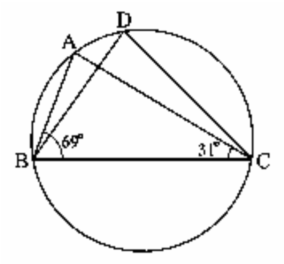
$\Rightarrow y = 19.2\text{ m}$

Now, $AC = 2y = 2 \times 19.2\text{ m} = 38.4\text{ m}$

Hence, the distance between Ishita and Nisha $= \mathbf{38.4\ m}$

(12) In the given figure, ∠ABC = 69° and ∠ACB=31°. Find ∠BDC.

Solution: The correct option is $D\ 80°$



Given that $\angle ABC = 69°$ and $\angle ACB = 31°$.

Using the angle sum property in $\triangle ABC$, we have

$\angle BAC + \angle ABC + \angle ACB = 180°$

i.e., $\angle BAC + 69° + 31° = 180°$

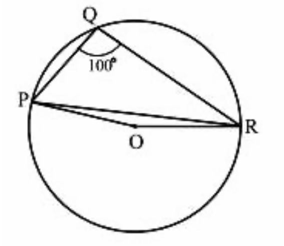
$\Rightarrow \angle BAC = 180° - 100° = 80°$

Now, since the angles in the same segment are equal,

$\angle BAC = \angle BDC$.

$\therefore \angle BDC = \mathbf{80°}$

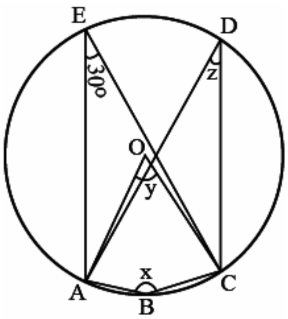
(13) In the figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.



Solution: Given:

$\angle PQR = 100°$
$\angle POR = 2 \times \angle PQR = 2 \times 100° = 200°$
$\therefore \angle POR = 360° - 200° = 160°$
In $\triangle OPR$
$\Rightarrow OP = OR$ ... Radii of the circle
$\Rightarrow \angle OPR = \angle ORP$
Now,
$\angle OPR + \angle ORP + \angle POR = 180°$ ...Sum of the angles in a triangle
$\Rightarrow \angle OPR + \angle OPR + 160° = 180°$
$\Rightarrow 2\angle OPR = 180° - 160°$
$\Rightarrow \angle OPR = 10°$

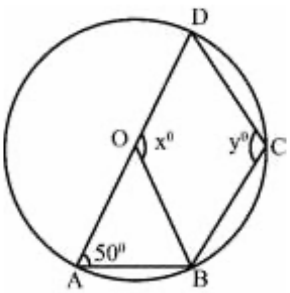
(14) In fig., O is the centre of the circle. If ∠CEA = 30°, find the values of x, y and z.



Solution: $\angle AEC$ and $\angle ADC$ are in the same segment.
$\therefore \angle AEC = \angle ADC = 30°$
$\therefore z = 30°$
ABCD is a cyclic quadrilateral.
$\therefore \angle B + \angle D = 180°$
$\Rightarrow x + z = 180°$
$\Rightarrow x + 30° = 180° \Rightarrow x = 180° - 30° = 150°$
Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part of the circle.
$\therefore \angle AOC = 2\angle D = 2 \times 30° = 60°$
$\therefore y = 60°$
Hence, $x = \mathbf{150}°, y = \mathbf{60}°$ and $z = \mathbf{30}°$

(15) In fig., O is the centre of the circle and ∠DAB = 50°. Calculate the values of x and y.

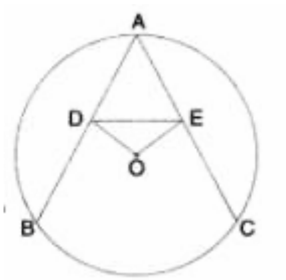
Solution: O is the centre of the circle and $\angle DAB = 50°$.



$OA = OB$ (Radii of a circle)
$\Rightarrow \angle OBA = \angle OAB = 50°$
In $\triangle OAB$, we have:
$\angle OAB + \angle OBA + \angle AOB = 180°$
$\Rightarrow 50° + 50° + \angle AOB = 180°$
$\Rightarrow \angle AOB = (180° - 100°) = 80°$
Since AOD is a straight line, we have:
$\therefore x = 180° - \angle AOB$
$= (180° - 80°) = 100°$
i.e., $x = 100°$
The opposite angles of a cyclic quadrilateral are supplementary.
ABCD is a cyclic quadrilateral.
Thus, $\angle DAB + \angle BCD = 180°$
$\angle BCD = (180° - 50°) = 130°$
$\therefore y = 130°$
Hence, $x = 100°$ and $y = \mathbf{130}°$

(16) In fig., OD and OE are respectively perpendiculars to chords AB and AC of a circle whose centre is O. If OD = OE, prove that ADE is an isosceles triangle.

Solution: Given: In the figure, AB and AC are two equal chords of a circle whose centre is O. $OD \perp AB$ and OE 1AC.



To Prove: ADE is an isosceles triangle.
Proof: $\because AB = AC$
$\therefore OD = OE$
$\because$ Equal chords are equidistant from the centre
$\therefore$ In $\triangle ODE$
$\angle ODE = \angle OED$
Angle opposite to equal sides
$\Rightarrow 90° - \angle ODE = 90° - \angle OED$
$\Rightarrow \angle ODA - \angle ODE = \angle OEA - \angle OED$
$\Rightarrow \angle ADE = \angle AED$
$\therefore AD = AE$
Sides opposite to equal angles
$\therefore \triangle ADE$ is an isosceles triangle.

Exercise

FILL IN THE BLANKS

(1) All points lying inside/outside a circle are called__________points/_________points.
(2) Circles having the same centre and different radii are called____________circles.
(3) A point whose distance from the centre of a circle is greater than its radius lies in___________of the circle.
(4) A continuous piece of a circle is___________of the circle.
(5) The longest chord of a circle is____________of the circle.
(6) An arc is a_____________when its ends are the ends of a diameter.
(7) Segment of a circle is the region between an arc and____________of the circle.
(8) A circle divides the plane, on which it lies, in_______________parts.

TRUE OR FALSE

(1) A circle is a plane figure.
(2) Line segment joining the centre to any point on the circle is a radius of the circle.
(3) If a circle is divided into three equal arcs each is a major arc.
(4) A circle has only a finite number of equal chords.
(5) A chord of a circle, which is twice as long as its radius is the diameter of the circle.
(6) Sector is the region between the chord and its corresponding arc.
(7) The degree measure of an arc is the complement of the central angle containing the arc.
(8) The degree measure of a semi-circle is 180^{o}.

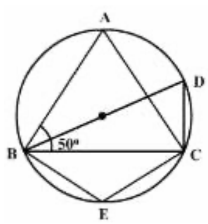
OBJECTIVE TYPE QUESTIONS

(1) The region between an arc and the two radii joining the centre of the end points of the arc is called a
(A) Segment (B) Semi circle
(C) Minor arc (D) Sector

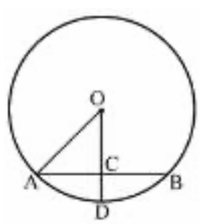
(2) In a circle with centre O and a chord BC, the point D lies on the same side BC as O. If $\angle BOC = 50°$, then $\angle BDC =$
(A) $25°$ (B) $100°$
(C) $75°$ (D) $150°$

(3) The region between the chord and either of the arc is called
(A) A Sector (B) A Semicircle
(C) A Segment (D) A Quarter circle

(4) AB is a chord of a circle with radius 'r'. If P is any point on the circle such that ∠APB is a right angle, then AB is equal to
(A) 3r (B) r
(C) 2r (D) r2



(5) In the figure, triangle ABC is an isosceles triangle with AB = AC and measure of angle ABC = 50°. Then the measure of angle BDC and angle BEC will be
(A) 60°,100° (B) 80°,100°
(C) 50°,100° (D) 40°,120°



(6) In the given figure if $OA = 5$ cm, $AB = 8$ cm and OD is perpendicular to AB, then CD is equal to
(A) 4 cm (B) 3 cm
(C) 5 cm (D) 2 cm

(7) Segment of a circle is the region between an arc and _________of the circle.
(A) Perpendicular (B) Radius
(C) Chord (D) Secant

(8) The degree measure of a semicircle is
(A) $0°$ (B) $90°$
(C) $360°$ (D) $180°$

(9) If chords AB and CD of congruent circles subtend equal angles at their centres, then
(A) $AB = CD$ (B) $AB > CD$
(C) $AB < AD$ (D) None of the above

(10) The angle subtended by the diameter of a semi-circle is
(A) 90 (B) 45
(C) 180 (D) 60

(11) The center of the circle lies in ___________ of the circle.
(A) Interior (B) Exterior
(C) Circumference (D) None of the above

(12) Equal _________ of the congruent circles subtend equal angles at the centers.
(A) Segments (B) Radii
(C) Arcs (D) Chords

(13) In a circle with centre O and a chord BC, points D and E lie on the same side of BC. Then, if $\angle\mathrm{BDC} = 80°$, then $\angle\mathrm{BEC} =$
(A) $80°$ (B) $20°$
(C) $160°$ (D) $40°$

(14) A regular octagon is inscribed in a circle. The angle that each side of the octagon subtends at the centre is
(A) $45°$ (B) $75°$
(C) $90°$ (D) $60°$

(15) AD is the diameter of a circle and AB is a chord. If $\mathrm{AD} = 50\ \mathrm{cm}, \mathrm{AB} = 48\ \mathrm{cm}$, then the distance of AB from the centre of the circle is
(A) $6\ \mathrm{cm}$ (B) $8\ \mathrm{cm}$
(C) $5\ \mathrm{cm}$ (D) $7\ \mathrm{cm}$

(16) A chord of a circle which is twice as long as its radius is a __________ of the circle.
(A) Diameter (B) Perpendicular
(C) Arc (D) Secant

(17) If there are two separate circles drawn apart from each other, then the maximum number of common points they have
(A) 0 (B) 1
(C) 2 (D) 3

(18) If a line intersects two concentric circles with centre O at $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D, then
(A $\mathrm{AB} = \mathrm{CD}$ (B) $\mathrm{AB} > \mathrm{CD}$
(C) $\mathrm{AB} < \mathrm{CD}$ (D) None of the above

(19) The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is
(A) 7 cm (B) 72 cm
(C) 10 cm (D) 5 cm

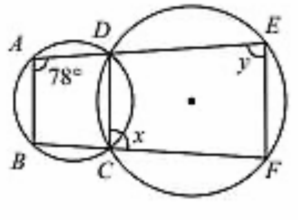
(20) A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 12 cm. Length of PQ is
(A) 12 cm (B) 13 cm
(C) 8.5 cm (D) 119 cm

(21) If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 800 then ∠POA is equal to
(A) 50° (B) 60°
(C) 70° (D) 80°

(22) Two circles touch each other externally at C and AB is a common tangent to the circle. Then ∠ACB=
(A) 60° (B) 45°
(C) 30° (D) 90°

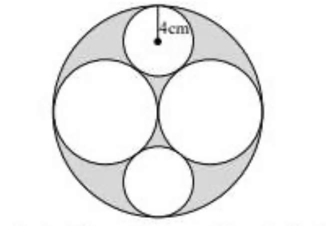
(23) ABC is a right-angled triangle, right-angled at B such that BC = 6 am and AB = 8 cm. A circle with centre O is inscribed in △ ABC. The radius of the circle is
(A) 1 cm (B) 2 cm
(C) 3 cm (D) 4 cm

(24) In the given figure, $\angle\mathrm{BAD} = 78°, \angle\mathrm{DCF} = \mathrm{x}$ and $\angle\mathrm{DEF} = \mathrm{y}$. The value of $\frac{2\mathrm{x}-\mathrm{y}}{9}$ is **(IMO – 2021-22)**
(A) $7°$ (B) $8°$
(C) $6°$ (D) $4°$



(25) Read the following statements carefully and choose the correct option. **(IMO – 2021-22)**
Statement-I: In a cyclic quadrilateral ABCD, if $\angle A - \angle C = 60°$, then the smaller of two is $60°$.
Statement-II: The angle subtended by an arc of a circle at the centre is half the angle subtended by it at any point on the remaining part of the circle.
(A) Both Statement-I and Statement-II are true.
(B) Both Statement-I and Statement-II are false.
(C) Statement-I is true but Statement-II is false.
(D) Statement-I is false but Statement-II is true.

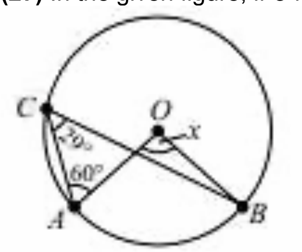
(26) The figure given below is made up of one big circle, two identical medium circles and two identical small circles. The ratio of the radius of the small circle to the radius of the medium circle is $2 : 3$. **(IMO – 2021-22)**



(a) What is the total area of the shaded part in the figure?
(b) What fraction of the big circle is unshaded?

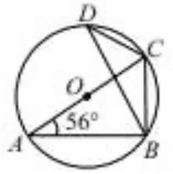
	(a)	(b)
(A)	$44\pi cm^2$	$5/18$
(B)	$40\pi cm^2$	$5/18$
(C)	$40\pi cm^2$	$13/18$
(D)	$44\pi cm^2$	$13/18$

(27) In the given figure, if O is the centre of the circle, then $x =$? **(IMO – 2020-21)**



(A) $29°$ (B) $40°$
(C) $58°$ (D) $38°$

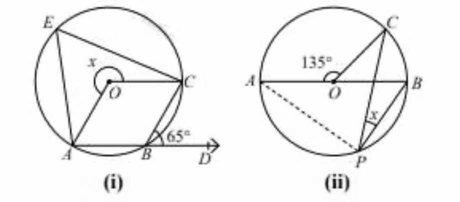
(28) In the given figure, O is the centre of the circle and $\angle BAC = 56°$. The measure of $\angle BDC$ is **(IMO – 2020-21)**



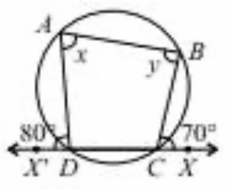
(A) $46°$ (B) $40°$
(C) $56°$ (D) $50°$

(29) Find the value of x in the given figures. **(IMO – 2020-21)**

	(i)	(ii)
(A)	$180°$	$40°$
(B)	$210°$	$35°$
(C)	$230°$	$22.5°$
(D)	$200°$	30



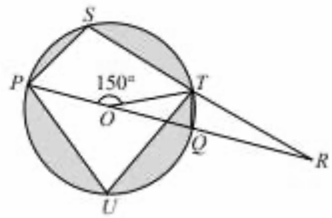
(30) $ABCD$ is a cyclic quadrilateral. If $\angle BCX = 70°$ and $\angle ADX' = 80°$, then find the values of x and y respectively.
(IMO – 2019-20)



(A) $70°, 80°$ (B) $70°, 70°$
(C) $80°, 70°$ (D) None of these

(31) Read the statements carefully and select the correct option. **(IMO – 2018-19)**
Statement-I: If two circles with centres A and B intersect each other at points M and N, then the line joining the centres AB bisects the common chord MN at right angle.
Statement-II: Two circles of radii 10 cm and 8 cm intersect each other and the length of the common chord is 12 cm. Then the distance between their centres is 8 cm.
(A) Both Statement-I and Statement-II are true.
(B) Both Statement-I and Statement-II are false.
(C) Statement-I is false but Statement-II is true.
(D) Statement-I is true but Statement-II is false.

(32) In the given figure, $\angle POT = 150°$ and O is the centre of circle. Find the measure of **(IMO – 2017-18)**



(i) $\angle RQT$
(ii) $\angle PUT$
(A) $75°, 75°$ (B) $105°, 75°$
(C) $105°, 105°$ (D) $75°, 105°$

(33) Read the given statements carefully. State 'T' for 'True' and 'F' for 'False'. **(IMO – 2017-18)**
(i) In a rhombus, the diagonals bisect each other at right angle.
(ii) A line drawn through the midpoint of a side of a triangle, parallel to another side bisects the third side.
(iii) Two chords are equal if the angle subtended by two chords at the centre of a circle are equal.
(iv) The angle in a semi-circle is an acute angle.

	(i)	(ii)	(iii)	(iv)
(A)	T	F	F	T
(B)	F	T	T	F
(C)	T	F	T	F
(D)	T	T	T	F

Answer Key

CHECK YOUR CONCEPT

(1) **(i)** 9 cm **(ii)** 8 cm **(iii)** 12 cm

(2) **(i)** $\angle ADC = 50°$ and $\angle ABC = 130°$ **(ii)** **(a)** $22\frac{1^0}{2}$, **(b)** 60°, **(c)** 60° **(iii)** 135°, 60°

FILL IN THE BLANKS

(1) Interior/Exterior
(2) Concentric
(3) The Exterior
(4) Arc
(5) Diameter
(6) Semi-Circle
(7) Chord
(8) Three

TRUE OR FALSE

(1) True
(2) True
(3) False
(4) False
(5) True
(6) False
(7) False
(8) True

OBJECTIVE TYPE QUESTIONS

(1)	(D)	**(6)**	(D)	**(11)**	(A)	**(16)**	(A)	**(21)**	(A)	**(26)**	(C)	**(31)**	(D)
(2)	(A)	**(7)**	(C)	**(12)**	(D)	**(17)**	(A)	**(22)**	(D)	**(27)**	(C)	**(32)**	(B)
(3)	(C)	**(8)**	(D)	**(13)**	(A)	**(18)**	(A)	**(23)**	(B)	**(28)**	(C)	**(33)**	(D)
(4)	(C)	**(9)**	(A)	**(14)**	(A)	**(19)**	(B)	**(24)**	(C)	**(29)**	(D)		
(5)	(B)	**(10)**	(C)	**(15)**	(D)	**(20)**	(D)	**(25)**	(C)	**(30)**	(A)		