







CIRCLES

Concepts Covered

- Introduction to Circumference, Chord, Diameter, Secant and Tangent
- Concentric circles.
- Cyclic Quadrilaterals.
- Theorems based on the properties of Circles.

Definition

A circle is the locus of a point which moves in such a way that its distance from a fixed point is constant. The fixed point is called the centre of the circle and the constant distance, is the radius of the circle.

A circle with centre O and radius r is denoted by C (O, r).

Terms related to a Circle

Diameter

A diameter is a chord of a circle passing through the centre of the circle. Thus, AOB is a diameter of a circle with centre O. Diameter is the longest chord of a circle Diameter = 2 × radius

Circumference

The perimeter of a circle is called its circumference. Circumference = $2\pi r$

Chord

A chord of a circle is a line segment joining any two points on the circle. In the given figure, GH, CD, EF and AOB are the chords of the circle with centre O.

Secant

A line which intersects a circle in two distinct points is called a secant of the circle.

Tangent

A line that intersects the circle at exactly one point is called a tangent to the circle.

Concentric Circles

Circles which have the same centre and different radii are called concentric circles. In the given figure C(A, x) and C(A, y) are concentric circles having the same centre A but different radii x and y respectively.











Cyclic Quadrilateral

A quadrilateral ABCD is called cyclic if all four vertices of it lie on a circle.



Some Important parts of Circle



Cyclic Quadrilateral

Theorems

Theorem 1

Statement: Equal chords of a circle subtend equal angles at the centre.

Given: AB and CD are chords of a circle with centre 0, such that AB = CD. To prove: $\angle AOB = \angle COD$

Proof: In \triangle AOB and \triangle COD,

AO = CO BO = DO AB = CD $\therefore \triangle AOB \cong \triangle COD$ Hence, $\angle AOB = \angle COD$ (Radii of the same circle) (Radii of the same circle) (Given) (SSS) [Corresponding part of the congruent triangle (CPCT)]

Theorem 2

Statement: If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal. Given: Two chords PQ and RS of a circle C(0, r), such that $\angle POQ = \angle ROS$. To prove: PQ = RS

Proof: In \triangle POQ and \triangle ROS, OP = OQ = OR = OS = r and \angle POQ = \angle ROS

(Radii of the same circle) (Given) (SAS) (CPCT)

Theorem 3

 \therefore PQ = RS

 $\mathrel{\dot{\cdot}} \bigtriangleup \mathsf{POQ} \cong \bigtriangleup \mathsf{ROS}$

Statement: The perpendicular from the centre of a circle to a chord bisects the chord. Given: AB is the chord of a circle with centre 0 and $0D \perp AB$. To prove: AD = DB

Construction: Join OA and OB.

Proof: $In \triangle ODA$ and $\triangle ODB$,(Radii of the same circle)OA = OB(Radii of the same circle)OD = OD(Common) $\angle ODA = \angle ODB$ (Each is a right angle) $\triangle ODA \cong \triangle ODB$ (R.H.S.)AD = DB(CPCT)

Theorem 4

Statement: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord. Given: A chord PQ of a circle C(0, r) and L is the mid-point of PQ.

Proof: $OL \perp PQ$.

Construction: Joint OP and OQ. In \triangle OLP and \triangle OLQ, OP = OQ PL = QL OL = OL

(Radii of the same circle) (Given) (Common)











 $\begin{array}{ll} \therefore \bigtriangleup OLP \cong \bigtriangleup OLQ & (SSS) \\ \mbox{Also, } \angle OLP + \angle OLQ = 180^{\circ} & (Linear pair) \\ \therefore \angle OLP = \angle OLQ = 90^{\circ} \\ \mbox{Hence, } OL \perp PQ \end{array}$

Theorem 5

Statement: There is one and only one circle passing through three given non-collinear points.

Given: Three non-collinear points P, Q, and R.

To prove: There is one and only one circle passing through the points P, Q, and R. Construction: Draw line segments PQ and QR. Draw perpendicular bisectors MN and ST of PQ and RQ respectively. Since P, Q, and R are not collinear, MN is not parallel to ST and will intersect, at the point 0. Join OP, OQ and OR (Fig).

Proof: As O lies on MN, the perpendicular bisector of PQ,

In \triangle OXP and \triangle OXQ we have,

OX = OX(Common) $\angle OXP = \angle OXQ$ (Right angles) XP = XQ(MN is the perpendicular bisector) $\therefore \triangle OXP \cong \triangle OXQ$ (By SAS congruence Rule) 0P = 00(CPCT) Similarly, $\triangle 0Y0 \cong \triangle 0YR$ and 00 = 0R (CPCT) $\therefore OP = OO = OR = r$ (suppose) Taking 0 as the centre and r as the radius, draw a circle C(0, r) which will pass through P, O, and R. If possible, suppose there is another circle C(Q', r') passing through P, Q and R. Then 0' will lie on the perpendicular bisector MN of PQ and ST of QR. Since two lines cannot intersect at more than one point, O' must coincide with O. Since OP = r, O'P = r'. We have, r = r'Hence, C(Q', r') = C(Q, r)Hence, there is one and only one circle passing through the three non-collinear points PQ and R.





Theorem 6

Statement: Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).

Given: AB and CD are two equal chords of a circle. OM and ON are perpendicular from the centre to the chords AB and CD. To prove: OM = ON.

Construction: Join OA and OC.

Proof: In \triangle AOM and \triangle CON,

OA = OC(Radii of the same circle)MA = CN(Since OM and ON are perpendicular to the chords and it bisects the chord and AM =MB, CN = ND) $\angle OMA = \angle ONC = 90^{\circ}$ $\therefore \triangle AOM \cong \triangle CON$ (R.H.S.)



Theorem 7

 $\therefore OM = ON$

Statement: Chords equidistant from the centre of a circle are equal in length.

(CPCT)

Given: OM and ON are perpendiculars from the centre to the chords AB and CD and OM = ON. To prove: Chord AB = Chord CD. Construction: Join OA and OC.

Proof: $OM \perp AB \Rightarrow \frac{1}{2}AB = AM$ $ON \perp CD \Rightarrow \frac{1}{2}CD = CN$ Consider $\triangle AOM$ and $\triangle CON$, OA = OC (Radii of the same circle) OM = ON (Given) $\angle OMA = \angle ONC = 90^{\circ}$ (Given) $\triangle AOM \cong \triangle CON$ (RHS congruency) $AM = CN \Rightarrow \frac{1}{2}AB = \frac{1}{2}CD \Rightarrow AB = CD$ The two chords are equal if they are equidistant from the centre.

Equal chords of a circle are equidistant from the centre.







Check Your Concept - 1

(i) A circle of 30 cm diameter has a 24 cm chord What is the distance of the chord from the centre?

(ii) A chord of length 12 cm is drawn in a circle of radius 10 cm. Calculate its distance from the centre of the circle.

(iii) The radius of a circle is 13 cm and the length of one of its chords is 10 cm, find the distance of the chord from the centre.

Theorem 8

Statement: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given: 0 is the centre of the circle. To prove: $\angle BOC = 2 \angle BAC$ Construction: Join 0 to A.

Proof: In \triangle AOB,

 $\begin{array}{l} \mathsf{OA} = \mathsf{OB} \text{ (radii of the same circle)} \\ \Rightarrow \angle 1 = \angle 2 \\ \text{Similarly in } \triangle \text{ AOC}, \ \angle 3 = \angle 4 \\ \text{Now, by exterior angle property,} \\ \angle 5 = \angle 1 + \angle 2 \\ \angle 6 = \angle 3 + \angle 4 \\ \Rightarrow \angle 5 + \angle 6 = \angle 1 + \angle 2 + \angle 3 + \angle 4 \\ \Rightarrow \angle 5 + \angle 6 = 2\angle 2 + 2\angle 3 = 2(\angle 2 + \angle 3) \\ \Rightarrow \angle \text{BOC} = 2\angle \text{BAC} \end{array}$

Theorem 9

Statement: Angles in the same segment of a circle are equal.

Given: Two angles $\angle ACB$ and $\angle ADB$ are in the same segment of a circle C(0, r). To prove: $\angle ACB = \angle ADB$ Construction: Join OA and OB.

Proof: In fig. (i), we know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc in the alternate segment. Hence, $\angle AOB = 2 \angle ACB$

 $\angle AOB = 2 \angle ADB$ So, $\angle ACB = \angle ADB$ In fig. (ii), we have, Reflex $\angle AOB = 2 \angle ACB$ and Reflex $\angle AOB = 2 \angle ADB$ $\therefore \angle ACB = 2 \angle ADB$

Theorem 10

Statement: If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e., they are concyclic).

Given: In the figure., AB is a line segment, which subtends equal angles at two points C and D. That is $\angle ACB = \angle ADB$

To proof: To show that the points A, B, C and D lie on a circle let us draw a circle through the points A, C and B.

Proof: Suppose it does not pass-through point D. Then it will intersect AD (or extended AD) at a point, say E (or E'). If points A, C, E and B lie on a circle, But it is given that $\angle ACB = \angle ADB$. Therefore, $\angle AEB = \angle ADB$.

This is not possible unless E coincides with D.

Similarly, E' should also coincide with D.











Theorem 11

Statement: The sum of either pair of opposite angles of a cyclic quadrilateral is 180[°]. Given: Let ABCD be a cyclic quadrilateral

To prove: $\angle A + \angle C = 180^{\circ}$ and $\angle B + \angle D = 180^{\circ}$ Construction: Join OB and OD.

Proof: ∠BOD = 2∠BAD ∠BAD = $\frac{1}{2}$ ∠BOD Similarly, ∠BCD = $\frac{1}{2}$ reflex ∠BOD \therefore ∠BAD + ∠BCD = $\frac{1}{2}$ ∠BOD + $\frac{1}{2}$ reflex ∠BOD = $\frac{1}{2}$ (∠BOD + reflex ∠BOD) = $\frac{1}{2}$ × 360° \therefore ∠A + ∠C = 180° Similarly, ∠B + ∠D = 180°

Theorem 12

Statement: If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic. Given: The sum of pair of opposite angles of a quadrilateral is 180°

To prove: The quadrilateral is cyclic.

Proof: Let us assume that the quadrilateral ABCD is not cyclic i.e. Let point D not lie on the circle which makes the quadrilateral non-cyclic. Now, let us do a construction such that join CD' where D'

is the point of intersection of side AD with the circle. Now, ABCD' is cyclic \Rightarrow $\angle3+$ $\angle4=180^\circ$

Now, it is given that the sum of pair opposite angles of a quadrilateral ABCD is 180° . Therefore, $\angle 2 + \angle 4 = 180^{\circ}$

From the above two equations, we get $\angle 3 + \angle 4 = \angle 2 + \angle 4 \Rightarrow \angle 3 = \angle 2$ Now, in triangle CDD', by external angle property

 $\angle 3 = \angle 1 + \angle 2$

 $\Rightarrow \angle 1 = 0$, hence the side CD' and CD coincides

Check Your Concept – 2

 \Rightarrow Point D lies on the circle



(ii) If O is the centre of the circle, find the value of x in each of the following figures:







Solved Examples

(1) Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 13 cm.

Solution: Let AB be a chord of a circle with centre 0 and radius 13 cm. Draw $OL \perp AB$. Join OA. Here, OL = 5 cm, OA = 13 cm In the right triangle OLA, $OA^2 = OL^2 + AL^2$ $13^2 = 5^2 + AL^2$ $AL^2 = 144$ AL = 12Since the perpendicular from the centre to a chord bisects the chord. Therefore,

 $AB = 2AL = 2 \times 12 = 24 \text{ cm}$

(2) AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm, If the chords are on the opposite sides of the centre and the distance between them is 17 cm, the radius of the circle is

Solution: $r^2 = x^2 + (5)^2$

Also, $r^2 = (17 - x)^2 + (12)^2$ So, $x^2 + (5)^2 = (17 - x)^2 + (12)^2$ $x^2 + 25 = (17 - x)^2 + 144$ $\Rightarrow x = 12$ $\therefore r^2 = 144 + 25 = 169$ $\Rightarrow r = 13 \text{ cm}$

(3) In the adjoining figure OD is perpendicular to the chord AB of a circle with centre O. If BC is a diameter, show that AC \parallel DO and AC = 2 \times OD.

Solution: Perpendicular from the centre of a circle to a chord bisects the chord

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We know that OB \perp AB

From the figure, we know that D is the midpoint of AB

We get, AD = BD

We also know that O is the midpoint of BC

We get, OC = OB

Consider \triangle ABC

Using the midpoint theorem

We get OB \parallel AC and

OD = 12 \times AC

By cross multiplication

AC = 2 \times OD

Therefore, it is proved that AC \parallel DO and AC = 2 \times OD
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(4) If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure). Solution: We know that, OA = OD and OB = OC.

They are the radius of respective circles. In \triangle OBC, we know that OB = OC, so \angle OBC = \angle OCB $\therefore \angle$ OCD = \angle OBA In \triangle OAD, we know that OA = OD, so \angle OAD = \angle ODA Since, \angle OCD = \angle OBA and \angle OAD = \angle ODA, we get \angle AOB in \triangle OAB is equal to \angle COD in \triangle OCD. \therefore From SAS congruency, we can say that \triangle OAB and \triangle OCD are congruent. So, **AB** = **CD**.

(5) Prove that the line joining the mid-points of two parallel chords of a circle passes through the centre.

Solution: Let AB and CD be two parallel chords having P and Q as their mid-points respectively. Let O be the centre of the circle. Join OP and OQ and though O, draw XY || AB or CD.

Now, P is the mid-point of AB OP \perp AB, \angle BPO = 90° But, XY || AB. Therefore, \angle XOQ= \angle BPO [Corresponding angles] \angle XOQ = 90° Similarly, Q is the mid-point of CD OQ \perp CD, \angle DQO = 90° But, XY || CD. Therefore, \angle POX = \angle DQO = 90° [Corresponding angles] \angle POX + \angle XOQ = 90° + 90° = 180° POQ is a straight line. Hence, PQ is a straight line passing through the centre of the circle.











6





(6) In figure, BD = DC and \angle DBC = 25°, find the measurement of \angle BAC. Solution: It is given that,

BD = DC ⇒ ∠CBD = ∠BCD (angles opposite to equal side) Now, In∆BCD we have ⇒ ∠CBD + ∠BCD + ∠BDC = 180° (sum of angles of a triangle) ⇒ ∠BDC = 180° - 50° = 130° (as ∠CBD = ∠BCD = 25°) From the property of the circle, we know that the angle subtended by the same chord on the minor segment is twice the angle on the major segment. So,

 $\Rightarrow 2\angle BAC = \angle BDC \Rightarrow \angle BAC = \frac{130^{\circ}}{2} = 65^{\circ}$



(7) In fig., \triangle PQR is an isosceles triangle with PQ = PR and m \angle PQR = 35°. Find m \angle QSR and m \angle QTR.

Solution: Given: \triangle PQR is an isosceles triangle with PQ = PR and m \angle PQR = 35°

In \triangle PQR: \angle PQR = \angle PRQ = 35° (Angle opposite to equal sides) Again, by angle sum property \angle P + \angle Q + \angle R = 180° \angle P + 35° + 35° = 180° \angle P + 70° = 180° \angle P = 180° - 70° \angle P = 110° Now, in quadrilateral SQTR, \angle QSR + \angle QTR = 180° (Opposite angles of a quadrilateral) 110° + \angle QTR = 180°

(8) In the given figure, O is the centre of the circle. Find \angle CBD.

Solution: ABCP is a cyclic quadrilateral

 $\angle APC = \angle CBD$ $\angle APC = \frac{1}{2} \angle AOC$ $\Rightarrow \angle APC = 50^{\circ}$ $\angle APC = angle CBD$ $\Rightarrow angle CBD = 50^{\circ}$



(9) Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution: Also, AP = PB = 2.5 cm and CQ = QD = 5.5 cm (Perpendicular from the centre to a chord of the circle bisects the chord.) In the right triangles QAP and OCQ, we have $OA^2 = OP^2 + AP^2$ and $OC^2 = OO^2 + CO^2$ and $r^2 = (6 - x)^2 + (5.5)^2 \dots \dots \dots (2)$ $\Rightarrow x^{2} + (2.5)^{2} = (6 - x)^{2} + (5.5)^{2}$ $\Rightarrow x^{2} + 6.25 = 36 - 12x + x^{2} + 30.25$ 12x = 60 $\therefore x = 5$ Join OA and OC. Let the radius of the circle be r cm and 0 be the centre Draw OP \perp AB and OQ \perp CD. We know, $OQ \perp CD$, $OP \perp AB$ and $AB \parallel CD$. Therefore, points P, 0 and 0 are collinear. So, PQ = 6 cm. Let OP = x. Then, 00 = (6 - x)cmAnd 0A = 0C = rPutting x = 5 in (1), we get $r^2 = 52 + (2.5)^2 = 25 + 6.25 = 31.25 \Rightarrow r^2 = 31.25 \Rightarrow r = 5.6$ Hence, the radius of the circle is 5.6 cm Putting x = 5 in (1), we get $r^2 = 52 + (2.5)^2 = 25 + 6.25 = 31.25 \Rightarrow r^2 = 31.25 \Rightarrow r = 5.6$ Hence, the radius of the circle is 5.6 cm





(10) The length of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?

Solution: AB = 6 cm, CD = 8 cmLet $\text{OL} \perp \text{AB}$ and $\text{OM} \perp \text{CD}$ \therefore OL = 4 cm Let OM = x cmLet r be the radius of the circle Now in right \triangle OLA $OA^2 = OL^2 + AL^2 = 4^2 + \left(\frac{6}{2}\right)^2$ $r^2 = 16 + 9 = 25 \dots \dots \dots \dots (i)$ and in right $\triangle OMC$ $0C^2 = 0M^2 + CM^2$ $r^2 = x^2 + \left(\frac{8}{2}\right)^2$ $= x^{2} + (4)^{2} = x^{2} + 16 \dots \dots \dots \dots \dots (ii)$ From (i) and (ii), $x^2 + 16 = 25$ $\Rightarrow x^{2} = 25 - 16 = 9$ $\Rightarrow x^{2} = (3)^{2}$ $\therefore x = 3 \text{ cm}$ \therefore Distance = 3 cm



(11) Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita threw a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Nisha is 24 cm each, what is the difference between Ishita and Nisha.

Solution: Let Ishita at A, Isha at b and Nisha at C.

 $\therefore AB = BC = 24 \text{ cm.....(given)}$ and ROA = OC = 20 m (radii) Let OD = x m $\therefore BD = OB - OD = (20 - x)m$ Let AC = (2y)mHence, AD = (y)mIn △ODA, $OD^2 + DA^2 = OA^2$ $\mathbf{x}^2 + \mathbf{y}^2 = (20)^2 \dots \dots$ (Pythagoras Theorem) $\Rightarrow x^2 + y^2 = 400$ In \triangle ADB, AD² + BD² = AB² \Rightarrow y² + (20 - x)² = (24)² $\therefore y^2 + 400 - 40x + x^2 = 576$ \therefore (x² + y²) + 400 - 576 = 40x $\therefore 400 + 400 - 576 = 40x$ $\Rightarrow x = \frac{224}{40} = 5.6 \text{ m}$ Substituting this value in $x^2 + y^2 = 400$ we get $(5.6)^2 + y^2 = 400$ $\therefore 31.36 + y^2 = 400$ $\Rightarrow y^2 = 400 - 31.36$ \Rightarrow y² = 368.64 \Rightarrow y = 19.2 m Now, $AC = 2y = 2 \times 19.2 \text{ m} = 38.4 \text{ m}$ Hence, the distance between Ishita and Nisha = 38.4 m





(12) In the given figure, $\angle ABC = 69^{\circ}$ and $\angle ACB=31^{\circ}$. Find $\angle BDC$. Solution: The correct option is D 80°

Given that $\angle ABC = 69^{\circ}$ and $\angle ACB = 31^{\circ}$. Using the angle sum property in $\triangle ABC$, we have $\angle BAC + \angle ABC + \angle ACB = 180^{\circ}$ i.e., $\angle BAC + 69^{\circ} + 31^{\circ} = 180^{\circ}$ $\Rightarrow \angle BAC = 180^{\circ} - 100^{\circ} = 80^{\circ}$ Now, since the angles in the same segment are equal, $\angle BAC = \angle BDC$. $\therefore \angle BDC = \mathbf{80}^{\circ}$

(13) In the figure, $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.

Solution: Given: $\angle PQR = 100^{\circ}$ $\angle POR = 2 \times \angle PQR = 2 \times 100^{\circ} = 200^{\circ}$ $\therefore \angle POR = 360^{\circ} - 200^{\circ} = 160^{\circ}$ $\ln \triangle OPR$ $\Rightarrow OP = OR$... Radii of the circle $\Rightarrow \angle OPR = \angle ORP$ Now, $\angle OPR + \angle ORP + \angle POR = 180^{\circ}$...Sum of the angles in a triangle $\Rightarrow \angle OPR + \angle OPR + 160^{\circ} = 180^{\circ}$ $\Rightarrow 2\angle OPR = 180^{\circ} - 160^{\circ}$ $\Rightarrow \angle OPR = 10^{\circ}$

(14) In fig., O is the centre of the circle. If $\angle CEA = 30^\circ$, find the values of x, y and z.

P O O R



(15) In fig., O is the centre of the circle and $\angle DAB = 50^{\circ}$. Calculate the values of x and y.

Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part of the

Solution: 0 is the centre of the circle and $\angle DAB = 50^{\circ}$.

Hence, $x = 150^\circ$, $y = 60^\circ$ and $z = 30^\circ$

 \Rightarrow x + 30° = 180° \Rightarrow x = 180° - 30° = 150°

Solution: $\angle AEC$ and $\angle ADC$ are in the same segment.

ABCD is a cyclic quadrilateral.

 $\therefore \angle AOC = 2 \angle D = 2 \times 30^{\circ} = 60^{\circ}$

 $\therefore \angle AEC = \angle ADC = 30^{\circ}$

 $\therefore \angle B + \angle D = 180^{\circ}$ $\Rightarrow x + z = 180^{\circ}$

 $\therefore z = 30^{\circ}$

circle.

 $\therefore y = 60^{\circ}$

OA = OB (Radii of a circle) $\Rightarrow \angle OBA = \angle OAB = 50^{\circ}$ In \triangle OAB, we have: $\angle OAB + \angle OBA + \angle AOB = 180^{\circ}$ $\Rightarrow 50^{\circ} + 50^{\circ} + \angle AOB = 180^{\circ}$ $\Rightarrow \angle AOB = (180^\circ - 100^\circ) = 80^\circ$ Since AOD is a straight line, we have: $\therefore x = 180^{\circ} - \angle AOB$ $=(180^{\circ}-80^{\circ})=100^{\circ}$ i.e., x = 100° The opposite angles of a cyclic quadrilateral are supplementary. ABCD is a cyclic quadrilateral. Thus, $\angle DAB + \angle BCD = 180^{\circ}$ $\angle BCD = (180^{\circ} - 50^{\circ}) = 130^{\circ}$ $\therefore v = 130^{\circ}$ Hence, $x = 100^{\circ}$ and $y = 130^{\circ}$



(16) In fig., OD and OE are respectively perpendiculars to chords AB and AC of a circle whose centre is O. If OD = OE, prove that ADE is an isosceles triangle.

Solution: Given: In the figure, AB and AC are two equal chords of a circle whose centre is

0. 0D \perp AB and OE 1AC. To Prove: ADE is an isosceles triangle. Proof: \because AB = AC \therefore 0D = 0E \because Equal chords are equidistant from the centre \therefore In \triangle 0DE \angle 0DE = \angle 0ED Angle opposite to equal sides \Rightarrow 90° - \angle 0DE = 90° - \angle 0ED $\Rightarrow \angle$ 0DA - \angle 0DE = \angle 0EA - \angle 0ED $\Rightarrow \angle$ ADE = \angle AED \therefore AD = AE Sides opposite to equal angles $\therefore \triangle$ ADE is an isosceles triangle.





Exercise

FILL IN THE BLANKS

(1) All points lying inside/outside a circle are called_ points/ points.

(2) Circles having the same centre and different radii are called circles.

- (3) A point whose distance from the centre of a circle is greater than its radius lies in of the circle.
- (4) A continuous piece of a circle is of the circle.
- (5) The longest chord of a circle is ______ of the circle.
 (6) An arc is a ______ when its ends are the ends of a diameter.
- _of the circle. (7) Segment of a circle is the region between an arc and
- (8) A circle divides the plane, on which it lies, in_ parts.

TRUE OR FALSE

(1) A circle is a plane figure.

(2) Line segment joining the centre to any point on the circle is a radius of the circle.

(3) If a circle is divided into three equal arcs each is a major arc.

(4) A circle has only a finite number of equal chords.

(5) A chord of a circle, which is twice as long as its radius is the diameter of the circle.

(6) Sector is the region between the chord and its corresponding arc.

(7) The degree measure of an arc is the complement of the central angle containing the arc.

(8) The degree measure of a semi-circle is 180°.

OBJECTIVE TYPE QUESTIONS

 (1) The region between an arc and the two ratio (A) Segment (C) Minor arc 	adii joining the centre of the end points of the arc is cal (B) Semi circle (D) Sector	lled a
 (2) In a circle with centre O and a chord BC, (A) 25° (C) 75° 	the point D lies on the same side BC as O. If \angle BOC = (B) 100° (D) 150°	50°, then ∠BDC =
(3) The region between the chord and either(A) A Sector(C) A Segment	of the arc is called (B) A Semicircle (D) A Quarter circle	
 (4) AB is a chord of a circle with radius 'r'. If angle, then AB is equal to (A) 3r (C) 2r 	P is any point on the circle such that ∠APB is a right (B) r (D) r2	
 (5) In the figure, triangle ABC is an isosceles 50°. Then the measure of angle BDC and an (A) 60°,100° (C) 50°,100° 	s triangle with AB = AC and measure of angle ABC = ngle BEC will be (B) 80°,100° (D) 40°,120°	в 50° С
 (6) In the given figure if OA = 5 cm, AB = 8 c (A) 4 cm (C) 5 cm 	cm and OD is perpendicular to AB, then CD is equal to (B) 3 cm (D) 2 cm	
(7) Segment of a circle is the region betweer(A) Perpendicular(C) Chord	n an arc andof the circle. (B) Radius (D) Secant	СВ
 (8) The degree measure of a semicircle is (A) 0° (C) 360° 	(B) 90° (D) 180°	Ď
 (9) If chords AB and CD of congruent circles (A) AB = CD (C) AB < AD 	subtend equal angles at their centres, then (B) AB > CD (D) None of the above	
 (10) The angle subtended by the diameter of (A) 90 (C) 180 	f a semi-circle is (B) 45 (D) 60	



(11) The center of the circle lies in (A) Interior (C) Circumference	(B) Exterior (D) None of the above	
(12) Equal of the congruent circl (A) Segments (C) Arcs	es subtend equal angles at the centers. (B) Radii (D) Chords	
(13) In a circle with centre 0 and a chord BC $\angle BEC =$ (A) 80° (C) 160°	, points D and E lie on the same side of BC. (B) 20° (D) 40°	Then, if $\angle BDC = 80^{\circ}$, then
 (14) A regular octagon is inscribed in a circle (A) 45° (C) 90° 	e. The angle that each side of the octagon s (B) 75° (D) 60°	ubtends at the centre is
 (15) AD is the diameter of a circle and AB is centre of the circle is (A) 6 cm (C) 5 cm 	a chord. If AD = 50 cm, AB = 48 cm, then th (B) 8 cm (D) 7 cm	e distance of AB from the
(16) A chord of a circle which is twice as long(A) Diameter(C) Arc	g as its radius is a of the circle (B) Perpendicular (D) Secant	
 (17) If there are two separate circles drawn a have (A) 0 (C) 2 	apart from each other, then the maximum n (B) 1 (D) 3	umber of common points they
(18) If a line intersects two concentric circles (A $AB = CD$ (C) $AB < CD$	with centre O at A, B, C and D, then (B) AB > CD (D) None of the above	
(19) The length of the tangent drawn from a(A) 7 cm(C) 10 cm	point 8 cm away from the centre of a circle (B) 72 cm (D) 5 cm	of radius 6 cm is
 (20) A tangent PQ at a point P of a circle of that OQ = 12 cm. Length of PQ is (A) 12 cm (C) 8.5 cm 	radius 5 cm meets a line through the centre (B) 13 cm (D) 119 cm	O at a point Q, so
 (21) If tangents PA and PB from a point P then ∠POA is equal to (A) 50° (C) 70° 	to a circle with centre O are inclined to e (B) 60° (D) 80°	ach other at an angle of 800
 (22) Two circles touch each other externally (A) 60° (C) 30° 	at C and AB is a common tangent to the cir (B) 45° (D) 90°	rcle. Then ∠ACB=
 (23) ABC is a right-angled triangle, right-an inscribed in △ ABC. The radius of the circle is (A) 1 cm (C) 3 cm 	gled at B such that BC = 6 am and AB = 8 s (B) 2 cm (D) 4 cm	3 cm. A circle with centre O is
(24) In the given figure, $\angle BAD = 78^\circ$, $\angle DCF =$	x and ∠DEF = y. The value of $\frac{2x-y}{9}$ is	(IMO – 2021-22)
(A) 7° (C) 6°	(B) 8° (D) 4°	$ \begin{array}{c} A \\ \hline 78^{\circ} \\ B \\ \hline C \\ \hline F \end{array} $



(25) Read the following statements carefully and choose the correct option.

(IMO – 2021-22)

Statement-I: In a cyclic quadrilateral ABCD, if $\angle A - \angle C = 60^\circ$, then the smaller of two is 60° . Statement-II: The angle subtended by an arc of a circle at the centre is half the angle subtended by it at any point on the remaining part of the circle.

- (A) Both Statement-I and Statement-II are true.
- (B) Both Statement-I and Statement-II are false.
- (C) Statement-I is true but Statement-II is false.
- (D) Statement-I is false but Statement-II is true.

(26) The figure given below is made up of one big circle, two identical medium circles and two identical small circles. The ratio of the radius of the small circle to the radius of the medium circle is 2 : 3. (IMO – 2021-22)



(a) What is the total area of the shaded part in the figure?

(b) What fraction of the big circle is unshaded?



- (A) $44\pi \text{cm}^2$ 5/18
- (B) $40\pi \text{cm}^2$ 5/18
- (C) $40\pi \text{cm}^2$ 13/18
- (D) $44\pi \text{cm}^2$ 13/18

(27) In the given figure, if 0 is the centre of the circle, then x = ?



(B) 40° (D) 38°

(28) In the given figure, 0 is the centre of the circle and $\angle BAC = 56^\circ$. The measure of $\angle BDC$ is (IMO – 2020-21)





(29) Find the value of x in the given figures.

	(i)	(ii)
(A)	180°	40°

- (B) 210° 35°
- (C) 230° 22.5°
- (D) 200° 30



(IMO - 2020-21)

(IMO - 2020-21)



(30) ABCD is a cyclic quadrilateral. If $\angle BCX = 70^{\circ}$ and $\angle ADX' = 80^{\circ}$, then find the values of x and y respectively. (IMO – 2019-20)



(B) 70°, 70°(D) None of these

(31) Read the statements carefully and select the correct option.

(IMO - 2018-19)

(IMO - 2017-18)

Statement-I: If two circles with centres A and B intersect each other at points M and N, then the line joining the centres AB bisects the common chord MN at right angle.

Statement-II: Two circles of radii 10 cm and 8 cm intersect each other and the length of the common chord is 12 cm. Then the distance between their centres is 8 cm.

- (A) Both Statement-I and Statement-II are true.
- (B) Both Statement-I and Statement-II are false.
- (C) Statement-I is false but Statement-II is true.
- (D) Statement-I is true but Statement-II is false.

(32) In the given figure, $\angle POT = 150^{\circ}$ and 0 is the centre of circle. Find the measure of

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(C) 105°, 105°

(B) 105°,75° (D) 75° ,105°

(33) Read the given statements carefully. State 'T' for 'True' and 'F' for 'False'. (i) In a rhombus, the diagonals bisect each other at right angle. (IMO - 2017-18)

(ii) A line drawn through the midpoint of a side of a triangle, parallel to another side bisects the third side.

(iii) Two chords are equal if the angle subtended by two chords at the centre of a circle are equal.

(iv) The angle in a semi-circle is an acute angle.

	(i)	(ii)	(iii)	(iv)
(A)	Т	F	F	Т
(B)	F	Т	Т	F
(C)	Т	F	Т	F
(D)	Т	Т	Т	F



Answer Key

CHECK YOUR CONCEPT

(1) (i) 9 cm (ii) 8 cm

(iii) 12 cm

(i) ∠ADC=50° and ∠ABC=130° (2)

(ii) (a) $22 \frac{1^0}{2}$,(b) 60° , (c) 60°

(iii) 135°, 60°

FILL IN THE BLANKS

- Interior/Exterior (5) Diameter (1)
- (2) Semi-Circle (6) Concentric
- (3) (4) (7) Chord The Exterior
- (8) Arc Three

TRUE OR FALSE

- True True (5) (1)
- (2) True (6) False
- (3) False (7) False
- (4) (8) True False

OBJECTIVE TYPE QUESTIONS

(1)	(D)	(6)	(D)	(11)	(A)	(16)	(A)	(21)	(A)	(26)	(C)	(31)	(D)
(2)	(A)	(7)	(Ċ)	(12)	(D)	(17)	(A)	(22)	(D)	(27)	(Ċ)	(32)	(Β)
(3)	(C)	(8)	(D)	(13)	(A)	(18)	(A)	(23)	(B)	(28)	(C)	(33)	(D)
(4)	(C)	(9)	(A)	(14)	(A)	(19)	(B)	(24)	(C)	(29)	(D)		
(5)	(B)	(10)	(C)	(15)	(D)	(20)	(D)	(25)	(C)	(30)	(A)		