

INTRODUCTION TO COORDINATE GEOMETRY



Concepts Covered

- Introduction to Coordinate Geometry, Cartesian System
- Plotting of a Point on Coordinate Axes, Distance of Point from Axes
- Distance Between Two Points, Application of Distance Formula
- Mid-point formula

Introduction

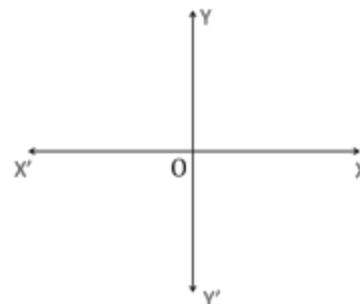
It is a branch of Mathematics in which geometric problems are solved through algebra by using the coordinate system. So, it is known as **coordinate geometry**.

Coordinate axes

Let $X'OX$ and YOY' be two mutually perpendicular lines intersecting at point O in a plane.

These two lines are called **reference lines** or **coordinate axes**.

The horizontal reference line $X'OX$ is called **X-axis** and the vertical reference line YOY' is called **Y-axis**.



Origin

The point of intersection of these two axes, i.e., O is called the **origin**. It has zero distance from both the axes so that its x-coordinate and y-coordinate are both zero. Therefore, the coordinates of origin are $(0,0)$.

The plane containing the coordinate axes is called **coordinate plane** or **XY-plane**.

Coordinates of a Point

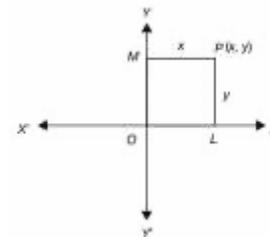
Let P be a point in the XY -plane. Draw perpendiculars PL and PM to X -axis and Y -axis, respectively.

Let $PL = y$ and $PM = x$.

Then point P is taken as (x, y) .

Here, x and y are called the **rectangular cartesian coordinates** or coordinates of the point P .

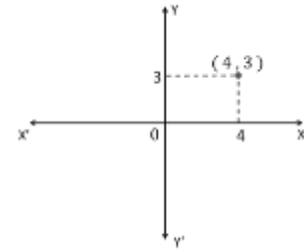
x is called x-coordinate or **abscissa** and y is called y-coordinate or **ordinate** of the point P .



Locating a Point on the Coordinate Plane

To plot a point (4,3) on the coordinate plane, follow the steps given below:

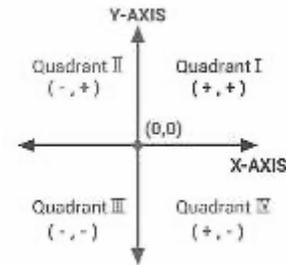
- Step 1:** Draw two perpendicular lines, name them as X-axis and Y-axis.
Step 2: Start from the origin. Move 4 units to the right, along the positive X-axis and draw a perpendicular dotted line to X-axis.
Step 3: Move 3 units up, along the positive Y-axis and draw perpendicular dotted line to Y-axis.
Step 4: Mark the point of intersection, this point of intersection will be your point (4, 3).



Quadrant

In the cartesian system, a plane is divided into four regions by a horizontal line called X-axis and a vertical line called Y-axis. These four regions are known as **quadrants**.

A quadrant can be defined as a region/part of a cartesian plane which is obtained when the two axes intersect each other. It is used to determine the position of a point in a plane. Observe the figure given adjacent which shows a cartesian plane which is divided into 4 quadrants by the two axes.



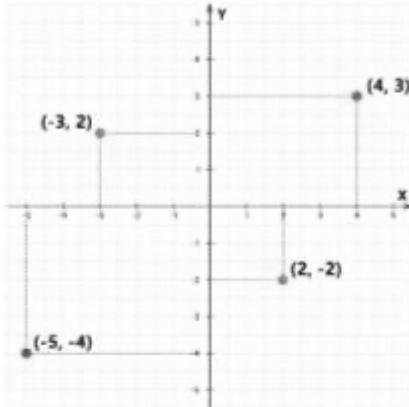
Convention of Signs

| Quadrant | x-coordinate | y-coordinate | Point |
|-----------------|--------------|--------------|--------|
| First quadrant | + | + | (+, +) |
| Second quadrant | - | + | (-, +) |
| Third quadrant | - | - | (-, -) |
| Fourth quadrant | + | - | (+, -) |

Example:

Plot the points A (4, 3), B (2, -2), C (-5, -4), and D (-3, 2) in the XY-plane.

Solution:



Example:

In which quadrant do the following points lie?

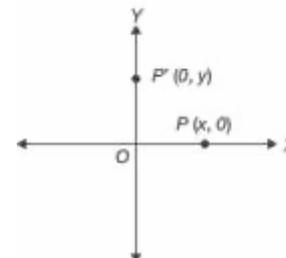
- (i) (3, 2) (ii) (-2, 1) (iii) (-1, -3) (iv) (5, -1)

- Solution:** (i) In the point (3, 2), abscissa and ordinate are both positive. So, it lies in the first quadrant.
(ii) In the point (-2, 1), abscissa is negative and ordinate is positive. So, it lies in the second quadrant.
(iii) In the point (-1, -3), abscissa and ordinate are both negative. So, it lies in the third quadrant.
(iv) In the point (5, -1), abscissa is positive, and ordinate is negative. So, it lies in the fourth quadrant.

Point on the Axes

Let P be a point on X-axis, so that its distance from X-axis is zero. Hence, point P can be taken as (x, 0).

Let P' be a point on Y-axis, so that its distance from Y-axis is zero. Hence, point P' can be taken as (0, y).



Example:

Which of the following points lie on the x-axis?

(i) (1, 1) (ii) (1, 0) (iii) (0, 1) (iv) (0, 0)

Solution: Points of the form $(a, 0)$, i.e., the points in which ordinate is 0, those points lie on the x -axis and the points in which abscissa is 0 lie on the y -axis.

(ii) (1, 0), (iv) (0, 0); these points have their ordinate 0, so they lie on x -axis.

Example:

Write the axis on which the given point lies.

(i) (2, 0) (ii) (0, -5) (iii) (-4, 0) (iv) (0, -1)

Solution: (i) (2,0)

The ordinate of the point (2,0) is zero.

Hence, the (2,0) lies on the x -axis.

(ii) (0, -5)

The abscissa of the point (0, -5) is zero.

Hence, the (0, -5) lies on the y -axis.

(iii) (-4,0)

The ordinate of the point (-4,0) is zero.

Hence, the (-4,0) lies on the x -axis.

(iv) (0, -1)

The abscissa of the point (0, -1) is zero.

Hence, the (0, -1) lies on the y -axis.

Example:

For each of the following points, write the quadrant in which it lies

(i) (-6,3) (ii) (-5,-3) (iii) (11,6) (iv) (1,-4)

Solution: (i) (-6,3)

Points of the type $(-, +)$ lie in the II quadrant.

Hence, the point lies (-6,3) in the II quadrant.

(ii) (-5,-3)

Points of the type $(-, -)$ lie in the III quadrant.

Hence, the point lies (-5,-3) in the III quadrant.

(iii) (11,6)

Points of the type $(+, +)$ lie in the I quadrant.

Hence, the point lies (11,6) in the I quadrant.

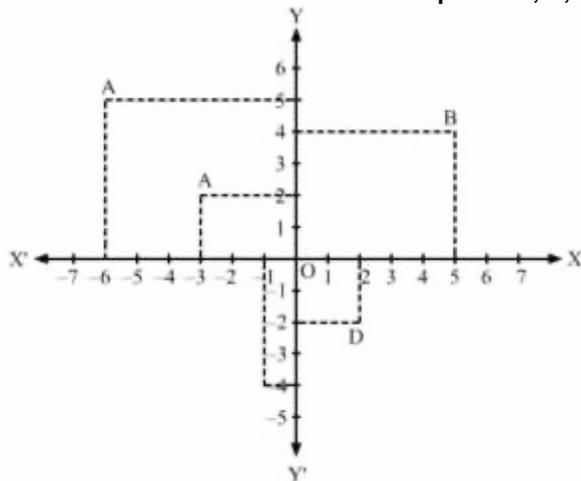
(iv) (1,-4)

Points of the type $(+, -)$ lie in the IV quadrant.

Hence, the point lies (1,-4) in the IV quadrant.

Example:

Write down the coordinates of each of the points A, B, C, D and E shown below:



Solution: (i) Distance of A from the Y -axis = $OL = -6$ units Distance of A from the X -axis = $AL = 5$ units. Hence, the coordinates of A are $(-6,5)$.

(ii) Distance of B from the Y -axis = $OM = 5$ units Distance of B from the X -axis = $BM = 4$ units. Hence, the coordinates of B are $(5,4)$.

(iii) Distance of C from the Y -axis = $ON = -3$ units Distance of C from the X -axis = $CN = 2$ units. Hence, the coordinates of C are $(-3,2)$.

(iv) Distance of D from the Y -axis = $OP = 2$ units Distance of D from the X -axis = $DP = -2$ units. Hence, the coordinates of D are $(2, -2)$.

(v) Distance of E from the Y -axis = $OL = -1$ units Distance of E from the X -axis = $AL = -4$ units. Hence, the coordinates of E are $(-1, -4)$.

Solved Examples

(1) Without plotting the point on a graph, state in which quadrant or on which axis do the following point lies $(-23, 4)$.

Solution: X coordinate – negative
Y coordinate – positive
 \therefore 2nd quadrant.

(2) Name the quadrant in which the point lies:

(a) $A(1, 1)$ (b) $B(2, 4)$ (c) $C(-3, -10)$ (d) $D(-1, 2)$
(e) $E(1, -1)$ (f) $F(-2, -4)$ (g) $G(-3, 10)$ (h) $H(1, -2)$

Solution: (a) $A(1, 1)$ lies in the 1st quadrant,
(b) $B(2, 4)$ lies in the 1st quadrant.
(c) $C(-3, -10)$ lies in the 3rd quadrant.
(d) $D(-1, 2)$ lies in the 2nd quadrant.
(e) $E(1, -1)$ lies in the 4th quadrant.
(f) $F(-2, -4)$ lies in the 3rd quadrant.
(g) $G(-3, 10)$ lies in the 2nd quadrant.
(h) $H(1, -2)$ lies in the 4th quadrant.

(3) Plot the following points and check whether they are collinear or not:

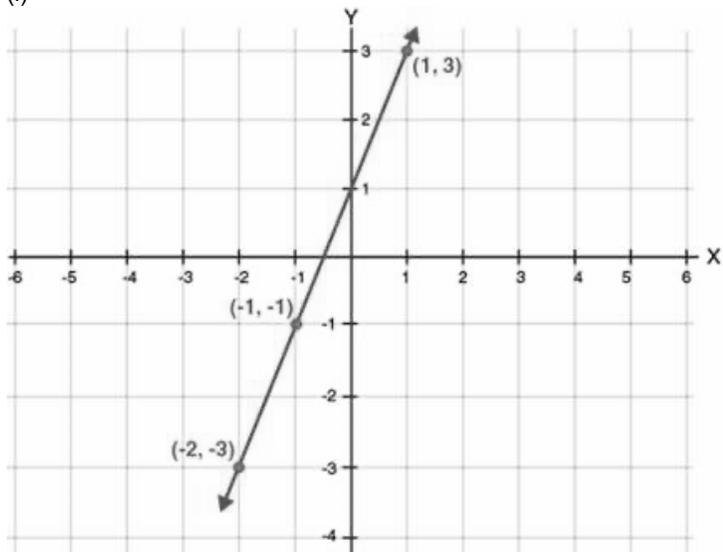
(i) $(1, 3)$, $(-1, -1)$, $(-2, -3)$

(ii) $(1, 1)$, $(2, -3)$, $(-1, -2)$

(iii) $(0, 0)$, $(2, 2)$, $(5, 5)$

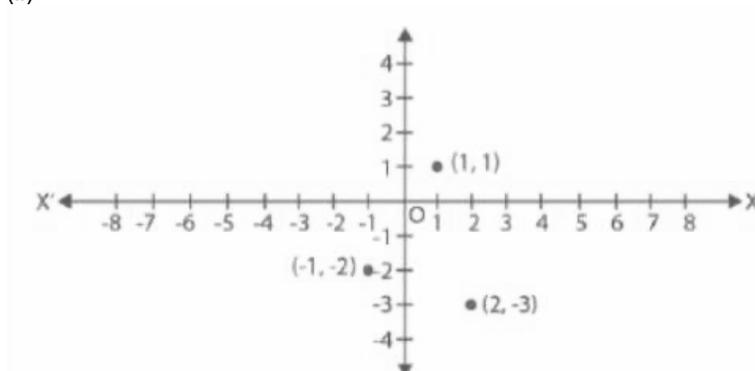
Solution:

(i)



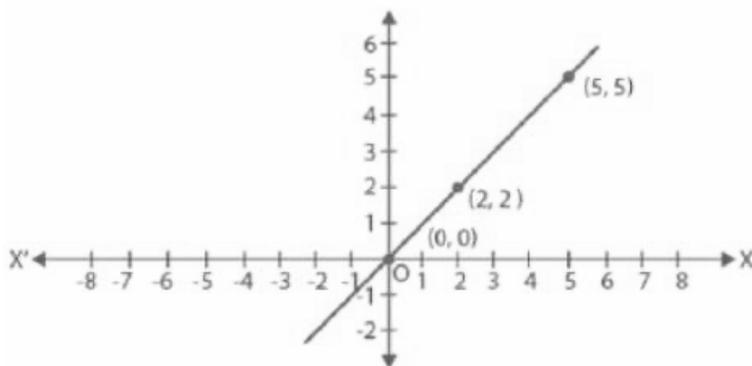
The points $(1, 3)$, $(-1, -1)$, $(-2, -3)$ lie in a straight line,
Hence, the points are collinear.

(ii)



The points $(1, 1)$, $(2, -3)$, $(-1, -2)$ do not lie in a straight line,
Hence, the points are not collinear.

(iii)

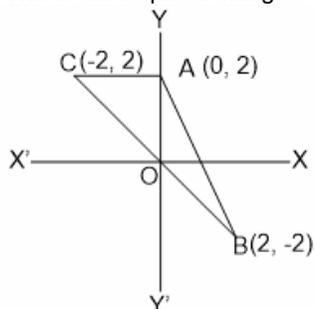


The points $(0, 0)$, $(2, 2)$, $(5, 5)$ lie in a straight line,
Hence, the points are collinear.

(4) Draw a triangle ABC where vertices A, B and C are $(0, 2)$, $(2, -2)$ and $(-2, 2)$ respectively.

Solution: Plot the point A by taking its abscissa = 0 and ordinate = 2.

Similarly, plot points B and C taking abscissa 2 and -2 and ordinates -2 and 2 respectively. Join A, B and C. This is the required triangle.

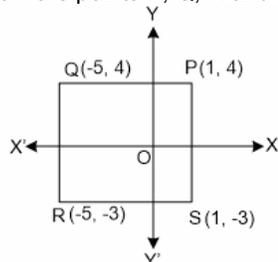


(5) Draw a rectangle PQRS in which vertices P, Q, R and S are $(1, 4)$, $(-5, 4)$, $(-5, -3)$ and $(1, -3)$ respectively.

Solution: Plot the point P by taking its abscissa 1 and ordinate 4.

Similarly, plot the points Q, R and S taking abscissa as -5 , -5 and 1 and ordinates as 4, -3 and -3 respectively.

Join the points P, Q, R and S. PQRS is the required rectangle.

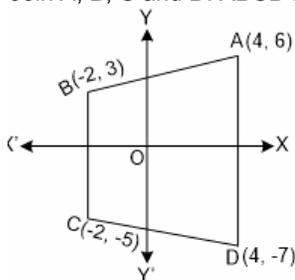


(6) Draw a trapezium ABCD in which vertices A, B, C and D are $(4, 6)$, $(-2, 3)$, $(-2, -5)$ and $(4, -7)$ respectively.

Solution: Plot the point A taking its abscissa as 4 and ordinate as 6.

Similarly, plot the points B, C and D taking abscissa as -2 , -2 and 4 and ordinates as 3, -5 , and -7 respectively.

Join A, B, C and D. ABCD is the required trapezium.



(7) Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

Solution: Let the coordinates of point A be (x, y). As AB is diameter and O is centre of circle. Then O will be mid-point of AB.

∴ Coordinates of O = Coordinates of mid-point of AB

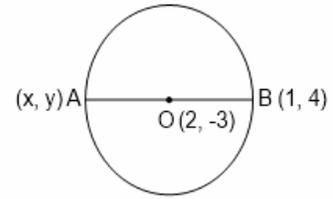
$$\Rightarrow (2, -3) = \frac{x+1}{2}, \frac{y+4}{2}$$

$$\Rightarrow 2 = \frac{x+1}{2}$$

$$\Rightarrow x + 1 = 4 \Rightarrow x = 3$$

$$\text{and } -3 = \frac{y+4}{2} \Rightarrow y + 4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A are (3, -10).



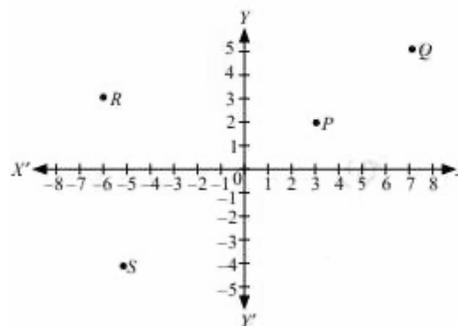
Exercise

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- (1) If $x > 0$ and $y < 0$, then the point $(x, -y)$ lies in _____ quadrant.
- (2) Which point among $(2, 3)$, $(-3, -4)$ and $(1, -7)$ is nearest to the origin? _____.
- (3) The points $(0, 0)$, $(0, 4)$ and $(4, 0)$ form a/an _____ triangle.
- (4) If (x, y) represents a point and $x > 0$ and $y < 0$, in which quadrant(s) can the point lie? _____.
- (5) The points (p, p) , $(-p, p)$ and $(-\sqrt{3}p, \sqrt{3}p)$ form the vertices of a/an _____ triangle.
- (6) If coordinates of point A are (x, y) such that $x < 0$, $y = 0$, A lies on _____.

OBJECTIVE TYPE QUESTION

- (1) The endpoints of the longest chord of a circle are $(-4, 2)$ and $(-6, -8)$. Find its centre.
 (A) $(-\frac{10}{3}, -2)$ (B) $(-5, -2)$
 (C) $(-5, -4)$ (D) $(-5, -3)$
- (2) In a parallelogram PQRS, $P(15, 9)$, $Q(7, 10)$, $R(-5, -4)$, then the fourth vertex S is
 (A) $(3, -2)$ (B) $(3, -4)$
 (C) $(9, -5)$ (D) $(3, -5)$
- (3) If the points A $(5, 3)$, B $(-2, 3)$ and D $(5, -4)$ are the three vertices of a square ABCD, then find the coordinates of the vertex C. (IMO 2021-22 Set A)
 (A) $(4, 2)$ (B) $(-2, -4)$
 (C) $(-2, 4)$ (D) $(4, -2)$
- (4) Study the given graph carefully. Sum of ordinate of point P and abscissa of point R is _____. (IMO 2021-22 Set B)
 (A) 4 (B) 6
 (C) 8 (D) -4



- (5) Select the appropriate option for P, Q, R, S and T. (IMO 2021-22 Set A)
 The point of intersection of the axes is known as P.
 The signs of abscissa and ordinate will be different in Q and R quadrants.
 The graph of line $x = 4$ is parallel to S at a distance of T units from the origin.

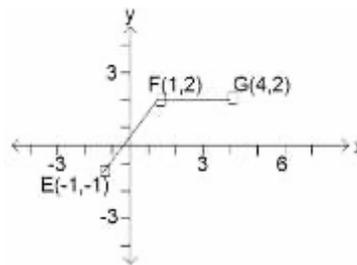
| | P | Q | R | S | T |
|-----|--------|----|-----|--------|---|
| (A) | Origin | I | III | x-axis | 2 |
| (B) | Origin | II | III | y-axis | 4 |
| (C) | Origin | II | IV | x-axis | 3 |
| (D) | Origin | II | IV | y-axis | 4 |
- (6) Read the following statements carefully and state 'T' for True and 'F' for False. (IMO 2019-20 Set A)
 P. Two points having same abscissae but different ordinates, lies on a line parallel to y-axis.
 Q. The abscissa of a point on y-axis is always equal to its ordinate.
 R. The point at which the two coordinate axes meet is called the origin.
 S. The signs of abscissae and ordinates are same in the quadrant I and IV.
 T. The perpendicular distance of the point $(2, 8)$ from y-axis is 8 units.

| | P | Q | R | S | T |
|-----|---|---|---|---|---|
| (A) | T | T | T | F | T |
| (B) | F | T | F | T | F |
| (C) | T | F | T | F | F |
| (D) | T | F | T | F | T |

- (7) Consider the given statements carefully. (IMO 2018-19 Set A)
 I. Any point on the x-axis is of the form $(0, a)$.
 II. The point $(0, 0)$ lies on both the axes.
 III. The point $(3, -2)$ lies in the III quadrant.
 Which of the above statements is/are true?
 (A) Both I and II (B) Only I
 (C) Only II (D) Both II and III

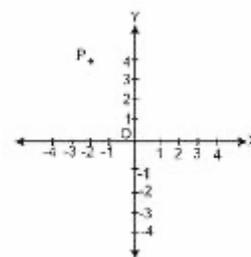
- (8) The three vertices of a square ABCD are A $(4, 3)$, B $(-3, 3)$ and C $(-3, -4)$. Find: (IMO 2018-19 Set B)
 I. The coordinates of D.
 II. The area of square ABCD.
 I II
 (A) $(-4, -4)$ 49 sq. units
 (B) $(3, -4)$ 25 sq. units
 (C) $(2, -4)$ 36 sq. units
 (D) $(4, -4)$ 49 sq. units

- (9) What would be the coordinates of point H for points E, F, G and H to form a parallelogram?
 (A) $(2, -1)$ (B) $(3, -1)$
 (C) $(6, -1)$ (D) $(-1, 2)$



- (10) On plotting the points $O(0, 0)$, $A(3, 0)$, $B(3, 4)$, $C(0, 4)$ and joining OA, AB, BC and CO which of the following figure is obtained?
 (A) Square (B) Rectangle
 (C) Trapezium (D) Rhombus
- (11) If $P(-1, 1)$, $Q(3, -4)$, $R(1, -1)$, $S(-2, -3)$ and $T(-4, 4)$ are plotted on the graph paper, then the point(s) in the fourth quadrant are
 (A) P and T (B) Q and R
 (C) Only S (D) P and R
- (12) If the coordinates of the two points are $P(-2, 3)$ and $Q(-3, 5)$, then (abscissa of P) - (abscissa of Q) is
 (A) -5 (B) 1
 (C) -1 (D) -2

- (13) In Fig. coordinates of P are
 (A) $(-4, 2)$ (B) $(-2, 4)$
 (C) $(4, -2)$ (D) $(2, -4)$



- (14) If $(3, -4)$ and $(-6, 5)$ are the extremities of the diagonal of a parallelogram and $(-2, 1)$ is third vertex, then its fourth vertex is
 (A) $(-1, 0)$ (B) $(0, -1)$
 (C) $(-1, 1)$ (D) None of these

Answer Key

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- | | |
|-------------------------------------|--------------------------|
| (1) first | (4) IV quadrant |
| (2) $(2, 3)$ | (5) equilateral triangle |
| (3) right-angled isosceles triangle | (6) negative x-axis |

OBJECTIVE TYPE QUESTIONS

- | | | | | | | |
|---------|---------|----------|----------|----------|----------|----------|
| (1) (D) | (2) (D) | (3) (B) | (4) (D) | (5) (D) | (6) (C) | (7) (C) |
| (8) (D) | (9) (A) | (10) (B) | (11) (B) | (12) (B) | (13) (B) | (14) (A) |