LINEAR EQUATIONS IN TWO VARIABLES

Concepts Covered

- Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Linear Equation in One Variable:

An equation is an equality that involves one or more unknown quantities. These quantities are known as variables. A linear equation in one variable is that equation in which only single variable with degree one occurs. This can be written in the form ax + b = c, where a, b and c are numbers, a ≠ 0 and x is the variable.

Example:

$$\frac{17-3x}{5}-\frac{4x+2}{3}=5-6x+\frac{7x+14}{3}$$

Solution: Multiplying both sides by 15 i.e. the LCM of 5 and 3,

we get 3(17 – 3x) – 5(4x + 2) = 15 (5 – 6x) + 5 (7x + 14)

⇒ 51 – 9x – 20x – 10 = 75 – 90x + 35x + 70

⇒ 41 – 29x = 145 – 55x

⇒ –29x + 55x = 145 – 41

⇒ 26x = 104

$\Rightarrow \frac{26x}{26}=\frac{104}{26}$

⇒ x = 4

Thus, x = 4 is the solution of the given equation.

Example:

Solve: $(2x + 3)^2 + (2x – 3)^2 = (8x + 6) (x – 1) + 22$

Solution: We have,

$(2x + 3)^2 + (2x – 3)^2 = (8x + 6) (x – 1) + 22$ [Using: $(a + b)^2 + (a – b)^2 = 2(a^2 + b^2)$ on LHS]

$\Rightarrow 2\{(2x)^2 + 3^2\} = x (8x + 6) – (8x + 6) + 22$

$\Rightarrow 2(4x^2 + 9) = 8x^2 + 6x – 8x – 6 + 22$

$\Rightarrow 8x^2 + 18 = 8x^2 – 2x + 16$

⇒ 2x = –2

⇒ x = –1

Hence, x = –1 is the solution of the given equation.

Linear Equation in Two Variables:

An equation is a statement in which one expression equals to another expression. An equation of the form ax + by + c = 0, where a, b and c are real numbers such that a ≠ 0 and b ≠ 0, is called a linear equation in two variables. The process of finding solution(s) is called solving an equation.
The solution of a linear equation is not affected when
(i) The same number is added to (subtracted from) both sides of the equation,
(ii) Both sides of the equation are multiplied or divided by the same non-zero number.

Solution of a Linear Equation in Two Variables:

A pair of values, one for x and the other for y that satisfy the linear equation, is known as a solution of the equation.
For Example, x = 2 and y = 5 is the solution of the equation 3x + 5y = 31
For every value of x. there is a corresponding value of y.
Thus, a linear equation in two variables has infinitely many solutions.
The graph of every linear equation in two variables is a straight line and every point on the graph (straight line) represents a solution of the linear equation. Thus, every solution of the linear equation can be represented by a unique point on the graph of the equation.

Note:

- The graphs of x = a and y = a are lines parallel to the y-axis and x-axis, respectively.

- The graph of y = 0 is x-axis.
- The graph of the equation y = mx is a line passing through origin.
- Every point on the graph of a linear equation in two variables is a solution of the linear equation.

Example:

Find out which of the following equations have x = 2, y = 1 as a solution:
(i) 2x + 5y = 9
(ii) 5x + 3y = 14
(iii) 2x + 3y = 7
Solution:

(i) 2x + 5y = 9
Putting x = 2 and y = 1 on the LHS
2(2) + 5(1) = 4 + 5 = 9 = RHS
x = 2, y = 1 is a solution of the given equation.
(ii) 5x + 3y = 14
Putting x = 2 and y = 1 on the LHS
5(2) + 3(1) = 10 + 3 = 13, which is not equal to RHS
x = 2, y = 1 is not a solution of the given equation.
(iii) 2x + 3y = 7
Putting x = 2 and y = 1 on the LHS
2(2) + 3(1) = 4 + 3 = 7 = RHS
x = 2, y = 1 is a solution of the given equation

Example:

Find solutions of the form x = a, y = 0 and x = 0, y = b for each of the following pairs of equations. Do they have any common such solution?
(i) 3x + 2y = 6 and 5x – 2y = 10.
(ii) 5x + 3y = 15 and 5x + 2y = 10
(iii) 9x + 7y = 63 and x – y = 10.
Solution: (i) 3x + 2y = 6 and 5x – 2y = 10.

For 3x + 2y = 6
x = 2, y = 0 and x = 0, y = 3 is a solution.
For 5x – 2y = 10
x = 2, y = 0 and x = 0, y = –5 is a solution.
 For this pair (2, 0) is a common solution.
(ii) 5x + 3y = 15 and 5x + 2y = 10
For 5x + 3y = 15
x = 3, y = 0 and x = 0, y = 5 is a solution
x = 3, y = 0 and x = 0, y = 5 is a solution
For 5x + 2y = 15
x = 2, y = 0 and x = 0 y = 5 is a solution
For this pair (0,5) is a common solution.
(iii) 9x + 7y = 63 and x – y = 10
For 9x + 7y = 63

Example:

Find a value for a so that each of the following equations may have x = 1, y = 1 as a solution:
(i) 3x + ay = 6
(ii) ax – 2y = 10
(iii) 5x + 3y = a
Solution: (i) 3x + ay = 6.

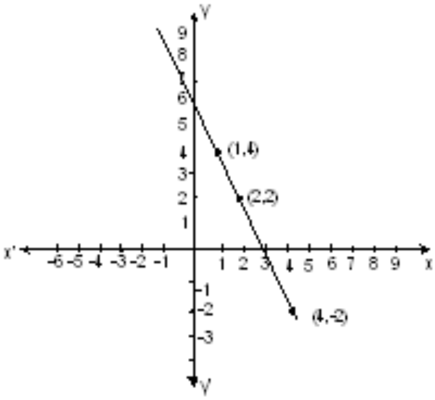
If x = 1, y = 1 is a solution, then it must satisfy the equation.
$\therefore$ 3(1) + a(1) = 6 $\Rightarrow$ a = 6 – 3 = 3.
(ii) ax – 2y = 10
$\therefore$ a(1) – 2(1) = 10 $\Rightarrow$ a = 10 + 2 = 12
(iii) 5x + 3y = a
$\therefore$ 5(1) + 3(1) = a $\Rightarrow$ a = 8

Example:

Draw the graph of each of the following equations. Read a few solutions from the graph and verify the same by actual substitution. In each case, find the points where the line meets the two axes.
(i) 2x + y = 6
(ii) x – 2y = 4
Solution: (i) 2x + y = 6

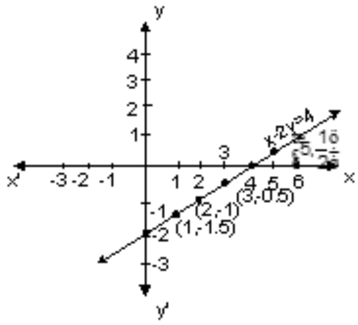
Check that x = 1, y = 4 and x = 2, y = 2 are solutions of the given equation.
So, we use the following table to draw the graph.

X	1	2
Y	4	2



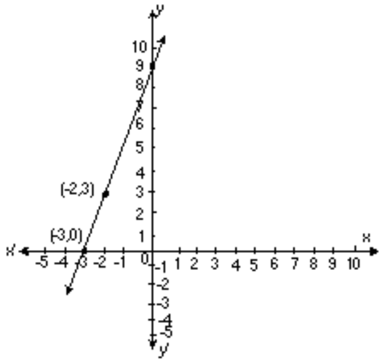
We draw the graph by plotting the two points then by joining the same. We find from the graph that x = –1, y = 8 and x = 4, y = –2 are also the solutions.
Verification: Putting x = –1, and y = 8 on the LHS
2(–1) + 8 = 6 = RHS
Putting x = 4 and y = –2 on LHS
2 (4) + (–2) = 8 – 2 = 6 = RHS
The line meets the x–axis at (3, 0) and y–axis at (0, 6).
(ii) x – 2y = 4

x	2	3
y	–1	–0.5



Check that x = 2, y = –1 and x = 3, y = –0.5 are solutions of the given equations.
So, we draw the graph using the table.
Verification: Putting x = 1, y = –1.5 on LHS
1 – 2 (–1.5) = 1 + 3 = 4 = RHS
Putting x = 5, y = $\frac{1}{2}$ on LHS
5 – 2 $\left(\frac{1}{2}\right)$ = 5 – 1 = 4 = RHS
x = 1, y = –1.5 and x = 5,
y = $\frac{1}{2}$ are other solutions from the graph.
The line meets x-axis at (4, 0) and y-axis at (0, -2)

(1) Draw the graph of each of the following equations. Read a few solutions from the graph and verify the same by actual substitution. In each case, find the points where the line meets the two axes.
(i) y – 3x = 9
(ii) 2(x + 3) – 3(y + 1) = 0
(iii) (x – 4) – y + 4 = 0
Solution: (i) y – 3x = 9

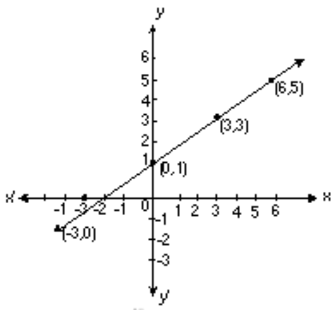


x	–2	– 3
y	3	0

As x = –2, y = 3 and x = –3, y = 0 are the solutions.
So, we use this table to draw the graph.
The line meets x-axis at (–3, 0) and y-axis at (0, 9).
Also from the graph x = –1, y = 6
Verification LHS y – 3x = 6 – 3 (–1)
= 6 + 3 = 9
= RHS
(ii) 2(x + 3) – 3(y + 1) = 0
or 2x + 6 –3y – 3 = 0
or 2x – 3y + 3 = 0

x	3	–3
y	3	–1

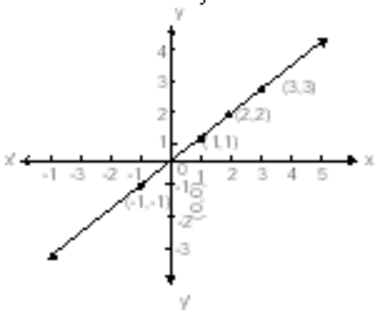
The other solutions on the graph are (6,5) and (0, 1)
(i) 2 (6) – 3 (5) + 3 = 12 – 15 + 3 = 0 = RHS
So, (6, 5) is a solution.
(ii) 2(0) – 3 (1) + 3 = –3 + 3 = 0 = R.H.S.
So (0, 1) is also a solution.
The line meets the x-axis at (–1.5, 0) and y-axis at (0, 1)



(iii) (x – 4) – y + 4 = 0
 x – 4 – y + 4 = 0
 x – y = 0

x	1	2
y	1	2

The other points lying on the line are (–1, –1), (3, 3)
Now, we verify that these are solutions of equation x – y = 0
Verification:
(i) (–1, –1)
(–1) – (–1) = –1 + 1 = 0 = RHS
So, (–1, –1) is a solution.
(ii) (3, 3)
3 – 3 = 0 = RHS
So, (3, 3) is also a solution.
Now, this line passes through the origin and as we can see from the graph, it does not meet the two axes.



(2) Find a value for a so that each of the following equations may have x = 1, y = 1 as a solution :
(i) 5x + 2ay = 3a
(ii) 9ax + 12ay = 63
(iii) x – y = a
Solution: (i) 5x + 2ay = 3a
 5(1) + 2a(1) = 3a
 5 = 3a – 2a
 a = 5
(ii) 9ax + 12ay = 63
 9a(1) + 12a (1) = 63
 21a = 63

$$a = \frac{63}{21} = 3$$

(iii) x – y = a
 1 – 1 = a
a = 0

(3) Solve graphically the system of equations:

2x + y − 5 = 0
5x − 2y + 1 = 0
Solution: Given equations:

2x + y − 5 = 0 ...(1)

5x − 2y + 1 = 0 ...(2)

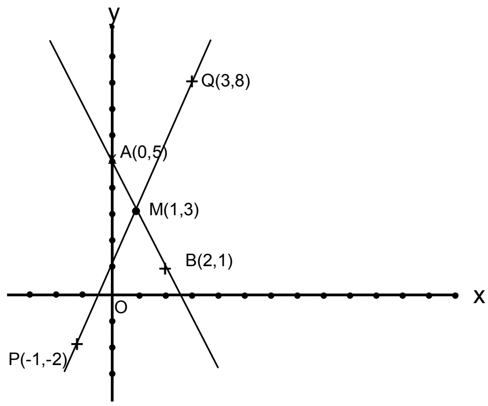
To draw the graph of equation (1)

x	0	2
y	5	1

To draw the graph of equation (2)

x	−	3
y	−	8

Plotting the points A (0, 5) and B (2,1) and drawing a line joining them, we get line AB representing equation
(1). Similarly by plotting the points P (-1, -2) and Q (3, 8), we get line PQ representing equation (2)



We observe that there is a point M (1,3) common to the lines AB and PQ. This point gives a solution to both of the equations (1) and (2). Hence, the solution of the given equations is x = 1 and y = 3.

(4) Solve the system of equations graphically.
x + 2y − 2 = 0
3x + 6y + 6 = 0
Solution: Given system of equations are

x + 2y − 2 = 0 …(1)

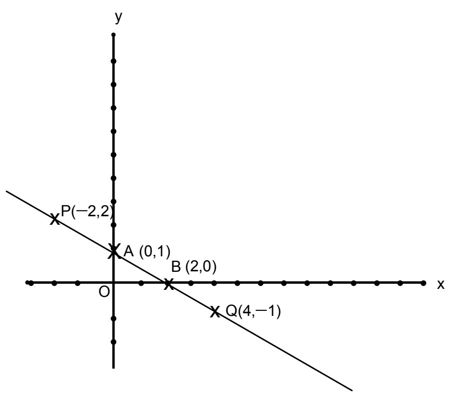
3x + 6y + 6 = 0 …(2)

To draw the graph of equation (1)

x	0	2
y	1	0

To draw the graph of equation (2)

x	−	4
y	2	−



Plotting the points A (0, 1) and B (2,0) and joining them with a line, we get line AB representing equation (1) and line PQ by joining the points P (−2,2) and Q (4, −1) which represents equation (2).
The two lines AB and PQ are coincident. Hence every point on the line determines a common solution. Thus the system of equations has infinitely many common solutions.

(5) Find common solution of the pair of equations graphically
3x − y − 2 = 0
6x − 2y + 6 = 0
Solution: The given system is:

$3x - y - 2 = 0$ ....(1)

$6x - 2y + 6 = 0$ ....(2)

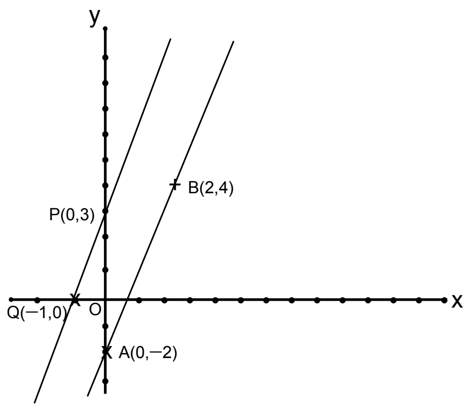
For line (1):

x	0	2
y	−	4

For line (2):

x	0	−
y	3	0

Plotting the points A (0, −2) and B (2, 4) and joining the points with a straight line, we get line AB representing equation (1) and similarly line PQ passing through the points P (0,3) and Q (−1, 0) representing equation (2)



The lines AB and PQ are parallel and do not intersect at any point. Therefore the given system of equations do not have a solution. Hence the given system is inconsistent.

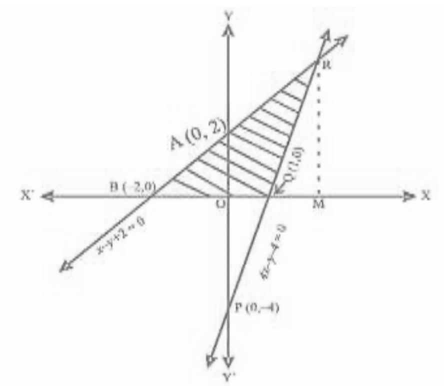
(6) Draw the graphs of the pair of linear equations x – y + 2 = 0 and 4x – y – 4 = 0. Calculate the area of the triangle formed by the lines so drawn and the x-axis.
Solution: For drawing the graphs of the given equations, we find two solutions of each of the equations, as given in the Table.

Table

x	0	–2
y = x + 2	2	0

x	0	1
y = 4x – 4	– 4	0

Plot the points A (0, 2), B (–2, 0), P (0, –4) and Q (1, 0) on the graph paper, and join the points to form the lines AB and PQ as shown in Fig.



We observe that there is a point R (2, 4) common to both the lines AB and PQ.

The triangle formed by these lines and the x- axis is BQR.
The vertices of this triangle are B (–2, 0), Q (1, 0) and R (2, 4).
We know that

Area of triangle $= \frac{1}{2}$ Base × Altitude

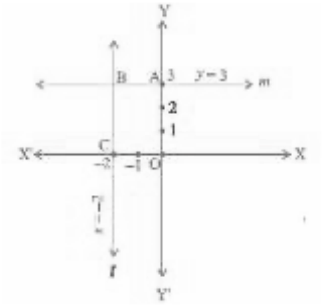
Here, Base = BQ = BO + OQ = 2 + 1 = 3 units.
Altitude = RM = Ordinate of R = 4 units.

So, area of Δ BQR $= \frac{1}{2}$ □ 3 □ 4 = 6 sq. units.

(7) Draw the graphs of the lines x = –2 and y = 3. Write the vertices of the figure formed by these lines, the x-axis and the y-axis. Also, find the area of the figure.

Solution:

We know that the graph of x = –2 is a line parallel to y-axis at a distance of 2 units to the left of it.
So, the line l is the graph of x = –2 [see Fig.]
The graph of y = 3 is a line parallel to the x-axis at a distance of 3 units above it. So, the line m is the graph of y = 3.
The figure enclosed by the lines x = –2, y = 3, the x-axis and the y-axis is OABC, which is a rectangle. (Why?)
A is a point on the y-axis at a distance of 3 units above the x-axis. So, the coordinates of A are (0, 3).
C is a point on the x-axis at a distance of 2 units to the left of y-axis. So, the coordinates of C are (–2, 0)



B is the solution of the pair of equations x = –2 and y = 3. So, the coordinates of B are (–2, 3)
So, the vertices of the rectangle OABC are O (0, 0), A (0, 3), B (–2, 3), C (–2, 0)
The length and breadth of this rectangle are 2 units and 3 units, respectively.
As the area of a rectangle = length × breadth, the area of rectangle OABC = 2 × 3 = 6 sq. units.

OBJECTIVE TYPE QUESTION

(1) The linear equation 2x – 5y = 7 has
(A) A unique solution
(B) Two solutions
(C) Infinitely many solutions
(D) No solution

(2) The equation 2x + 5y = 7 has a unique solution, if x, y are:
(A) Natural numbers
(B) Positive real numbers
(C) Real numbers
(D) Rational numbers

(3) If (2, 0) is a solution of the linear equation 2x + 3y = k, then the value of k is
(A) 4
(B) 6
(C) 5
(D) 2

(4) Any solution of the linear equation 2x + 0y + 9 = 0 in two variables is of the form
(A) $(-\frac{9}{2}, m)$
(B) $(n, -\frac{9}{2})$
(C) $(0, -\frac{9}{2})$
(D) (– 9, 0)

(5) The graph of the linear equation 2x + 3y = 6 cuts the y-axis at the point
(A) (2, 0)
(B) (0, 3)
(C) (3, 0)
(D) (0, 2)

(6) The equation x = 7, in two variables, can be written as
(A) 1.x + 1.y = 7
(B) 1.x + 0.y = 7
(C) 0.x + 1.y = 7
(D) 0.x + 0.y = 7

(7) Any point on the x-axis is of the form
(A) (x, y)
(B) (0, y)
(C)(x, 0)
(D) (x, x)

(8) Any point on the line y = x is of the form
(A) (a, a)
(B) (0, a)
(C) (a, 0)
(D) (a, – a)

(9) The equation of x-axis is of the form
(A) x = 0
(B) y = 0
(C) x + y = 0
(D) x = y

(10) The graph of y = 6 is a line
(A) Parallel to x-axis at a distance 6 units from the origin
(B) Parallel to y-axis at a distance 6 units from the origin
(C) Making an intercept 6 on the x-axis.
(D) Making an intercept 6 on both the axes.

(11) x = 5, y = 2 is a solution of the linear equation
(A) x + 2 y = 7
(B) 5x + 2y = 7
(C) x + y = 7
(D) 5x + y = 7

(12) If a linear equation has solutions (–2, 2), (0, 0) and (2, – 2), then it is of the form
(A) y – x = 0
(B) x + y = 0
(C) –2x + y = 0
(D) –x + 2y = 0

(13) The positive solutions of the equation ax + by + c = 0 always lie in the
(A) 1st quadrant
(B) 2nd quadrant
(C) 3rd quadrant
(D) 4th quadrant

(14) The graph of the linear equation 2x + 3y = 6 is a line that meets the x-axis at the point
(A) (0, 2)
(B) (2, 0)
(C) (3, 0)
(D) (0, 3)

(15) The graph of the linear equation y = x passes through the point
(A) $\frac{3}{2}, \frac{-3}{2}$
(B) $0, \frac{3}{2}$
(C) (1, 1)
(D) $\frac{-1}{2}, \frac{1}{2}$

(16) If we multiply or divide both sides of a linear equation with a non-zero number, then the solution of the linear equation:

(A) Changes
(B) Remains the same
(C) Changes in case of multiplication only
(D) Changes in case of division only

(17) How many linear equations in x and y can be satisfied by x = 1 and y = 2?

(A) Only one
(B) Two
(C) Infinitely many
(D) Three

(18) The point of the form (a, a) always lies on:

(A) x-axis
(B) y-axis
(C) On the line y = x
(D) On the line x + y = 0

(19) The point of the form (a, – a) always lies on the line

(A) x = a
(B) y = – a
(C) y = x
(D) x + y = 0

(20) The linear equation y = 2x + 3 cuts the y-axis at

(A) (0, 3)
(B) (0, 2)
(C) $\left(\frac{3}{2}, 0\right)$
(D) $\left(\frac{2}{3}, 0\right)$

Answer Key

OBJECTIVE TYPE QUESTIONS

(1)	(C)	**(6)**	(B)	**(11)**	(C)	**(16)**	(B)
(2)	(A)	**(7)**	(C)	**(12)**	(B)	**(17)**	(C)
(3)	(A)	**(8)**	(A)	**(13)**	(A)	**(18)**	(C)
(4)	(A)	**(9)**	(B)	**(14)**	(C)	**(19)**	(D)
(5)	(D)	**(10)**	(A)	**(15)**	(C)	**(20)**	(A)