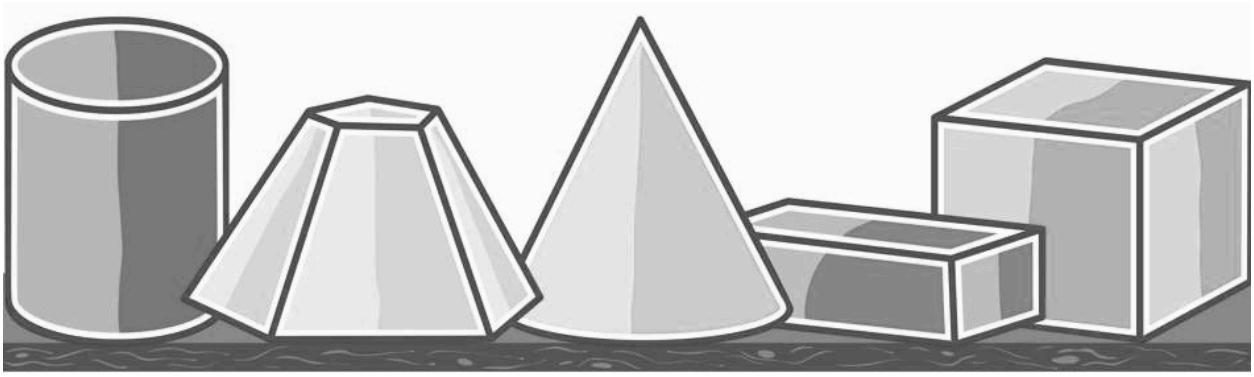


SURFACE AREA AND VOLUME

Concepts Covered

- Surface Area of Cube
- Volume of Cube
- Surface Area of Cuboid
- Volume of Cuboid

Introduction

You can draw some shapes on your notebook and blackboard completely. These are called plane figures or two-dimensional figures. We know what rectangles, squares, and circles are, what we mean by their perimeters and areas, and how we can find them. It would be interesting to see what happens if we cut out many of these plane figures of the same shape and size from cardboard sheets and stack them up in a vertical pile. Through this process, we shall obtain some solid figures such as a prism, a cuboid, a cylinder, etc.

Surface Area and Volume

Any object which is occupying space is called a solid. The total area enclosed by all the bounding surfaces of the solid is called the total surface area of the solid. It is measured in square units. Also, the amount of space enclosed by the bounding surface or surfaces of a solid is called the volume of the solid. It is measured in cubic units.

Lateral surface area

The **lateral surface** area of an object is the area of all the surfaces of the object, excluding its base and top (if there are any). The lateral surface area is the area of the lateral surface.

Curved surface area

The total area of all the curved surfaces in the object is the curved surface area.

Prism

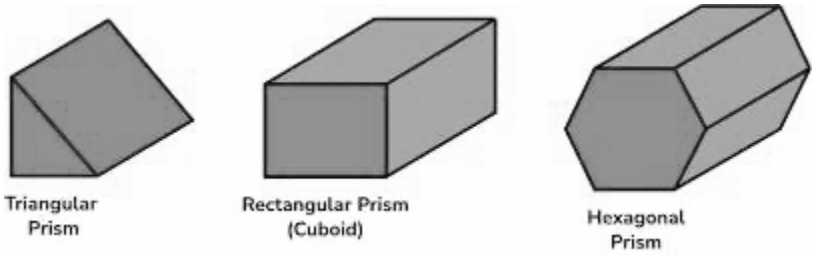
A prism is a solid with identical ends, flat surfaces, and the same cross-sectional area all along its length. The top and the bottom surfaces are two congruent and parallel polygons(base). The lateral faces are parallelograms.

The line joining the centres of the top and the bottom parallel polygons is called the height or axis of the prism.

If two parallel and congruent polygons are regular and, if the axis is perpendicular
to the base, then the prism is called a right prism. The lateral surfaces of a right prism are rectangles.

Notes: The following points hold good for all prisms.
1. The number of lateral faces = The number of sides of the base.
2. The number of edges of a prism = The number of sides of the base × 3.
3. The sum of the lengths of the edges = 2(The perimeter of the base) + The number of sides× Height.

4. For a prism, whose base is a polygon, the Number of vertices + the Number of faces = Number of edges + 2.
This is known as Euler's formula, i.e., V + F = E + 2.



Prism

Surface Area and Volume of a Cube and a Cuboid

Cuboid

In a right prism if the base is a rectangle, then the right prism is called a cuboid.
There are 3 dimensions in all solids and so does the cuboid,
the three dimensions of the cuboid are its length (l), breadth (b) and height(h).
1. The lateral surface area of a cuboid = 2(l + b)h sq. units,
2. The total surface area of a cuboid = LSA + 2(Base area) = 2(l + b)h + 2lb = 2(lb + bh + lh) sq. units.
3. The volume of a cuboid = (lb)h = lbh cubic units,
4. Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.

Cuboid

Cube

In a cuboid, if its length, breadth and height are equal, then it is called a cube. All the edges of a cube are equal. Thus, the edge in itself is completely sufficient to determine the size of the cube.
If the edge of a cube is 'a' units, then
1. The lateral surface area of the cube = $4a^2$.
2. The total surface area of the cube = LSA + 2(Area of base) = $4a^2 + 2a^2 = 6a^2$ units.
3. The volume of the cube = a^3.
4. The diagonal of the cube = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2}$ units.

Cube

Example:

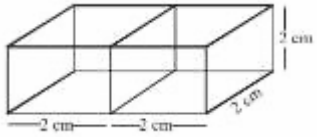
The dimensions l × b × h of a room are 12 m × 7 m × 5 m. Find:
(a) The diagonal of the room.
(b) The cost of flooring at the rate of Rs 2 per m^2.
(c) The cost of white washing the inside of the room excluding the floor at the rate of Rs 3 per m^2.
Solution: (a)The diagonal of the room = $\sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 7^2 + 5^2} = \sqrt{144 + 49 + 25} = \sqrt{218}$ m.
(b)To find the cost of flooring, we should know the area of the base.
Base area = l x b = 12 × 7 = 84 m^2. ∴ Cost of flooring = 84 × 2 = Rs 168.
(c)Total area to be whitewashed
= LSA + area of roof = 2(l + b) x h + l x b
= 2(12 + 7) x 5 + 12 × 7 = 2(19)(5) + 84 = 190 + 84 = 274 m^2.
∴ Cost of white washing = 274 × 3 = Rs 822.

Example:

A box is in the form of a cube. Its outer edge is 5 m long. Find:
(a) The total length of the edges.
(b) Cost of painting the outside of the box, on all surfaces, at the rate of Rs 5 per m^2.
Solution: (a) Length of edges = Number of edges × 3 × Length of each edge = 4 × 3 × 5 = 60 m.
(b) To find the cost of painting the box, we need to find the total surface area.
TSA = $6a^2 = 6 \times 5^2 = 6 \times 25 = 150$ m^2.
∴ Cost of painting = 150 × 5 = Rs 750.

Example:

Two cubes of side 2 cm each are joined end to end, find the volume of the cuboid so formed.
Solution: When two cubes of side 2 cm each are joined end to end then,
l = (2 + 2) = 4 cm, b = 2 cm & h = 2 cm
∴ volume = l × b × h = 4 × 2 × 2 = 16 cm^3



Check Your Concept - 1

(i) Three cubes of sides 3 cm, 4 cm and 5 cm, respectively, are melted and formed into a larger cube. What is the side of the cube formed?

(ii) Find the number of soaps of size 2.1 cm × 3.7 cm× 2.5 cm that can be put in a cuboidal box of size 6.3 cm × 7.4 cm × 5 cm.

Solved Examples

(1) The length, breadth and height of a cuboid are in the ratio 6 : 4 : 5. If the total surface area of the cuboid is 2368 cm^2 ; find its dimension.

Solution: Let the length $(\ell) = 6x$ cm, breadth $(b) = 4x$ cm and height $(h) = 5x$ cm,

$\therefore$ Total surface area $= 2(\ell \times b + b \times h \times h \times \ell)$

$= 2(6x \times 4x + 4x \times 5x + 5x \times 6x)cm^2$

$= 2(24x^2 + 20x^2 + 30x^2)cm^2$

$= 148x^2\ cm^2$

Total surface area $= 2368\ cm^2$

$\Rightarrow x^2 = \frac{2368}{148} = 16$ and $x = \sqrt{16} = 4$

$\therefore$ length $= 6x\ cm = 6 \times 4\ cm = 24\ cm$, breadth $= 4x\ cm = 4 \times 4\ cm = 16\ cm$ and height $= 5x\ cm = 5 \times 4\ cm =$ **20 cm**.

(2) A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

(i) The area of the sheet required for making the box.

(ii) The cost of the sheet for it if a sheet measuring 1 m^2 cost Rs. 20.

Solution: Given: length $(\ell) = 1.5$ m, breadth $(b) = 1.25$ m and depth i.e., height $(h) = 65\ cm = 0.65$ m.

(i) Since, the box is open at the top it has five faces of which four faces are the walls forming lateral surface area and one face is the base.

$\therefore$ The area of the sheet required for making the box.

= Lateral surface area of the box + area of its base

$= 2(\ell + b) \times h + \ell \times b$

$= 2(1.5\ m + 1.25\ m) \times 0.65\ m + 1.5\ m \times 1.25\ m$

$= 2 \times 2.75\ m \times 0.65\ m + 1.875\ m^2$

$= 3.575\ m^2 + 1.875\ m^2 = 5.45\ m^2$

(ii) Since, a sheet measuring $1\ m^2$ costs Rs. 20

$\therefore$ The cost of the sheet for the box $= 5.45 \times$ Rs $20 =$ **Rs. 109**

(3) The length, breadth and height of a room are 5m, 4m and 3m respectively. Find the cost of whitewashing the walls of the room and the ceiling at the rate of Rs. 7.50 per m^2.

Solution: Given. $\ell = 5$ m, $b = 4$ m and $h = 3$ m

Since the area of the walls of the room

$=$ its lateral surface area $= 2(\ell + b) \times h$

And, the area of the ceiling of the room $= \ell \times b$

$\therefore$ Total area to be whitewashed = Area of the walls + area of the ceiling

$= 2(\ell + b) \times h + \ell \times b.$

$= 2(5\ m + 4\ m) \times 3\ m + 5\ m \times 4\ m$

$= 2 \times 9\ m \times 3\ m + 20\ m^2$

$= 54\ m^2 + 20\ m^2 = 74\ m^2$

Since the rate of whitewashing = Rs. 7.50 per m^2

$\therefore$ Cost of whitewashing $= 74 \times$ Rs. $7.50 =$ **Rs. 555**

(4) The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs.15,000, find the height of the hall.

Solution: We know, the perimeter of a rectangle $= 2($ length + breadth $) = 2(\ell + b)$

And, given the floor of a rectangular hall has perimeter 250 m

$\Rightarrow 2(\ell + b) = 250$ m

Area of the four walls of the hall = Lateral surface area of the hall

$= 2(\ell + b) \times h = 250\ m \times h\ m = 250h\ m^2$

Since the rate of painting four walls is Rs. 10 per m^2.
$\therefore$ Cost of painting $250\mathrm{h}\ \mathrm{m}^2 = 250\mathrm{h} \times$ Rs. 10
According to the given statements :
$250\mathrm{h} \times$ Rs. $10 =$ Rs. 15,000
$\Rightarrow \mathrm{h} = \dfrac{15{,}000}{250 \times 10}\ \mathrm{m} = 6\ \mathrm{m}$
$\therefore$ The height of the hall $= \mathbf{6\ m}$

(5) A matchbox measures $6\ \mathrm{cm} \times 4\ \mathrm{cm} \times 2.5\ \mathrm{cm}$. What will be the volume of a packet containing 24 such boxes?

Solution: The shape of a matchbox is a cuboid
$\therefore$ Volume of 1 matchbox
= its length $\times$ breadth $\times$ height
$= 6 \times 4 \times 2.5 = 60\ \mathrm{cm}^3$
$\Rightarrow$ Volume of a packet containing 24 such boxes = Volume of 24 matchboxes
$= 24 \times 60 = \mathbf{1440\ cm^3}$

(6) A cuboidal water tank is $6\ \mathrm{m}$ long, $5\ \mathrm{m}$ wide and $4.5\ \mathrm{m}$ deep. How many litres of water can it hold? $(\mathbf{1\ m^3 = 1000\ell})$

Solution: Volume of water which tank can hold = Volume of the tank
= Its length $\times$ width $\times$ height
$= 6\ \mathrm{m} \times 5\ \mathrm{m} \times 4.5\ \mathrm{m} = 135\ \mathrm{m}^3$
$= 135 \times 1000\ell = \mathbf{135000\ell}$

(7) Find the volume of a solid cube of side $12\ \mathrm{cm}$. If this cube is cut into 8 identical cubes, find :
(i) Volume of each small cube.
(ii) Side of each small cube.
(iii) Surface area of each small cube.

Solution: Since, the side (edge) of the given solid cube $= 12\ \mathrm{cm}.3$
$\therefore$ The volume of a given solid cube
$= (\text{ edge})^3 = (12\ \mathrm{cm})^3 = 1728\ \mathrm{cm}^3$ Ans.
(i) As the given cube is cut into 8 identical cubes.
$\Rightarrow$ Vol. of 8 small cubes obtained = Vol. of given cube $= 1728\ \mathrm{cm}^3$
$\Rightarrow$ Volume of each small cube $= \dfrac{1728}{8} = 216\ \mathrm{cm}^3$
(ii) If edge (side) of each small cube $= \mathrm{x}\ \mathrm{cm}$
$\therefore$ Volume $= (\text{edge})^3$
$\Rightarrow \mathrm{x}^3 = 216 = 6 \times 6 \times 6 = 6^3 \Rightarrow \mathrm{x} = 6\ \mathrm{cm}$
$\therefore$ Side of each small cube $= 6\ \mathrm{cm}$
(iii) Surface area of each small cube $= 6 \times (\text{edge})^2$
$= 6 \times (6\ \mathrm{cm})^2 = \mathbf{216\ cm^2}$

(9) A river $3\ \mathrm{m}$ deep and $40\ \mathrm{m}$ wide is flowing at the rate of $2\ \mathrm{km}$ per hour. How much water will fall into the sea in a minute?

Solution: Volume of water that flows through a river, canal or pipe, etc., in unit time
= Area of cross-section $\times$ Speed of water through it.
Since the area of cross-section of the river = depth $\times$ width
$= 3\ \mathrm{m} \times 40\ \mathrm{m} = 120\ \mathrm{m}^2$
And the speed of flow of water through the river
$= 2\ \mathrm{km/hr} = 2 \times \dfrac{5}{18}\ \mathrm{m/s} = \dfrac{5}{9}\ \mathrm{m/s}$
$\therefore$ Vol. of water that flows through it in $1\mathrm{sec}.$
= Area of cross-section $\times$ speed of water through it $= 120 \times \dfrac{5}{9} = \dfrac{200}{3}\ \mathrm{m^3/sec}$
$\Rightarrow$ Volume of water that flows through it in $1\ \mathrm{min.}(60\mathrm{sec.}) = \dfrac{200}{3} \times 60 = 4000\ \mathrm{m}^3$
$\therefore$ The volume of water that will fall into the sea in a minute. $= \mathbf{4000\ m^3}$

(10) The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively $2.5\ \mathrm{m}$ and $10\ \mathrm{m}$.

Solution: The capacity of the cuboidal tank is 50000 litres of water.
$\Rightarrow$ Volume of the tank $= 50000$ litres
$= \dfrac{50000}{1000}\ \mathrm{m}^3\ [\because 1\ \mathrm{m}^3 = 1000$ litres $]$
$= 50\ \mathrm{m}^3$
$\Rightarrow$ Length of the tank $\times$ its breadth $\times$ its height $= 50\ \mathrm{m}^3$
$\Rightarrow 2.5\ \mathrm{m} \times$ breadth $\times 10\ \mathrm{m} = 50\ \mathrm{m}^3$
$\Rightarrow$ Breadth $= \dfrac{50}{2.5 \times 10}\ \mathrm{m} = \mathbf{2\ m}$

Exercise

FILL IN THE BLANKS

(5) If eight cubes are stacked to form a big cube, then the percentage decrease in the total surface area is__.
(13) Three cubes, each of side 3.2 cm, are joined end-to-end. The total surface area of the resulting cuboid _____
(15) The ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube_____.
(17) The areas of three adjacent faces of a cuboid are a, b and c. Then the volume is_____.
(18) Two cubes each of volume 64 cm^3 are joined end to end. The volume of the resulting cuboid is_____.

OBJECTIVE TYPE QUESTIONS

(1) The dimensions of a cuboidal container are 12 cm × 10 cm × 8 cm. How many bottles of syrup can be poured into the container if each bottle contains 20 cm^3 of syrup?
(A) 46 (B) 54
(C) 48 (D) 58

(2) A classroom is 5 m long, 2.5 m broad and 3.6 m high. If each student is given 0.5 m2 of the floor area, then how many cubic metres of air would each student get?
(A) 1.4 (B) 1.8
(C) 1.2 (D) 1.6

(3) How many cubes, each of total surface area 54 sq.dm, can be made from a cube of edge 1.2 metres?
(A) 64 (B) 81
(C) 125 (D) 25

(4) A swimming pool, $30\ m$ long has a depth of water of $80\ cm$ at one end and $2.4\ m$ at the other end. Find the area of each vertical cross-section of the pool along the length. **[IMO 2021-22]**
(A) $54\ m^2$ (B) $48\ m^2$
(C) $36\ m^2$ (D) $42\ m^2$

(5) A metallic sheet of rectangular shape having dimensions $48\ cm \times 36\ cm$. From each one of its corners, a square of $8\ cm$ is cut off. An open box is made up of the remaining sheet. Find the volume of the box. **[IMO 2020-21]**
(A) $4280\ cm^3$ (B) 2050 cm3
(C) $5120\ cm^3$ (D) $4690\ cm^3$

(6) The length of a hall is $20\ m$ and the width $16\ m$. The sum of the areas of the floor and the roof is equal to the sum of the areas of the four walls. Find the height of the hall. **[IMO 2020-21]**
(A) $6.45\ m$ (B) $7.18\ m$
(C) $8.89\ m$ (D) $9.20\ m$

(7) A cupboard made up of wood has inner measures of $120\ cm, 70\ cm$ and $45\ cm$. If the thickness of the wood is $2.5\ cm$, then find the volume of the wood. **[IMO 2019-20]**
(A) $90750\ cm^3$ (B) $82125\ cm^3$
(C) $47000\ cm^3$ (D) $85000\ cm^3$

(8) The paint in a certain container is sufficient to paint an area equal to $10.865\ m^2$. How many bricks of dimensions $31.5\ cm \times 10\ cm \times 5.5\ cm$ can be painted from that paint? **[IMO 2019-20]**
(A) 150 (B) 200
(C) 100 (D) 250

Answer Key

CHECK YOUR CONCEPT

(1) **(i)** 6cm **(ii)** 12

FILL IN THE BLANKS

(1) 50% **(2)** 143.36 cm^2 **(3)** 6 : π **(4)** $\sqrt{abc}$ **(5)** 128 cm^3

OBJECTIVE TYPE QUESTIONS

(1)	(C)	**(3)**	(A)	**(5)**	(C)	**(7)**	(A)
(2)	(B)	**(4)**	(B)	**(6)**	(C)	**(8)**	(C)